

## Interactions of Relativistically Intense Electromagnetic Waves With Cold Strongly Underdense Plasmas

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**Abstract.** A fully nonlinear formalism is presented that describes the interactions between relativistically intense laser pulses and cold strongly underdense plasmas in 1D geometry. Asymptotic solutions to the Maxwell and cold electron fluid dynamics equations are developed in the Compton limit, where the ratio of the plasma and the optical field frequencies is a small parameter. Lowest order expressions for the optical field frequency shift and self-modulated envelope, perturbed plasma electron concentration, and the energy of the plasmons, generated by the propagating laser pulses, are derived.

### 1. Introduction and Basic Equations

The interactions of relativistically intense ( $I > I_r \cong 10^{18}$  W/cm<sup>2</sup>) laser radiation with plasmas represent one of the most interesting and rapidly evolving areas of contemporary theoretical and experimental physics (for example, see Ref. 1 and the references therein). At such intensities, a rapid nonlinear ionization at the laser pulse front creates a plasma, into which the optical field propagates. The dynamics of this field is determined by the interplay of the following nonlinearities: the relativistic nonlinearity arising from the plasma dielectric response variation due to the increase in the masses of free electrons, driven by the superstrong electromagnetic field at velocities comparable to the speed of light; the plasma charge-displacement nonlinearity; and the nonlinearity, originating from the excitation of plasma waves by the propagating laser radiation. An important issue is the development of a formal, fully nonlinear theory of the propagation of relativistically intense optical fields in cold underdense plasmas, and, in particular, of their interactions with low frequency electron

fluid oscillations. Below, such a theory is presented for the slab geometry case. An expansion in the small parameter, equal to the ratio of the unperturbed plasma and laser pulse frequencies, is used to develop asymptotic solutions to the Maxwell and cold plasma relativistic electron fluid dynamics equations, which describe the propagation of high intensity laser pulses into plasmas. As a development of a previous study<sup>2</sup>, the proposed formalism makes it possible to establish solutions, corresponding to laser pulses of finite duration and longitudinal size. These new solutions describe the propagation of waveforms comprising a relativistically intense nonmonochromatic optical field and a plasmon coupled to it.

The interactions of superintense laser radiation with cold underdense plasmas are described by the Maxwell and relativistic electron fluid dynamics equations (for ultrashort laser pulses, it can be assumed that the ions are an immobile background due to their high inertia)<sup>1</sup>

$$\mathbf{A}_{xx} - \mathbf{A}_{tt} = \frac{n}{\gamma} \mathbf{A}, \quad \varphi_{xx} = n - 1, \quad p_t = (\varphi - \gamma)_x, \quad n_t + \left( \frac{n}{\gamma} p \right)_x = 0.$$

Here  $\gamma = \sqrt{1 + |\mathbf{A}|^2 + p^2}$  is the relativistic mass factor;  $\mathbf{A}$  and  $\varphi$  are the electromagnetic field vector and scalar potentials normalized by  $mc^2/e$  ( $m$  is the electron rest mass);  $p$  is the plasma electron longitudinal momentum normalized by  $mc$ ; and  $n$  is the plasma electron concentration normalized by its unperturbed value. The longitudinal coordinate  $x$  and time  $t$  are normalized by  $c/\omega_p$  and  $\omega_p^{-1}$ ;  $\omega_p$  is the unperturbed plasma electron frequency.

## 2. Asymptotic Technique

In what follows, linearly polarized laser radiation is considered, so that  $A_2 = 0$ . Let us introduce new variables defined by the following relations:  $\xi = x - qt$ ,  $\eta = \varepsilon(x - q^{-1}t)$ ,  $\tau = (\varepsilon/2q)t$ ,  $\Theta = f(\xi, \eta)/\varepsilon$ , where  $q = \sqrt{1 + \varepsilon^2}$  is the electromagnetic radiation “linear” phase velocity. Then the “linear” group velocity makes  $q^{-1}$ , and the parameter  $\varepsilon$  is small in case the plasma is substantially underdense. In the linear theory  $\varepsilon$  is the ratio of the plasma frequency to that of the optical field. Since the group velocity defined above is close to the speed of light, a Lorentz transform is associated with the use of a co-moving reference frame, which explains the scaling factor in the second and third of the above relations.

Solutions to the basic equations presented in the previous Section can be developed as asymptotic series in the small parameter  $\varepsilon$ . To the lowest order, the result is:

$$A_{1,0} = a(\varphi_0, \eta) \sin(\Theta + \lambda(\eta)\tau), \quad a(\varphi_0, \eta) = g_0(\eta) \left[ \frac{\lambda^2(\eta)}{4} + \varphi_0^{-1} \right]^{-1/4},$$

$$f_\xi = \frac{\lambda(\eta)}{2} + \sqrt{\frac{\lambda^2(\eta)}{4} + \varphi_0^{-1}}, \quad n_0 = \frac{1}{2} \left( \frac{1 + a^2/2}{\varphi_0^2} + 1 \right) - \frac{a^2}{4\varphi_0^2} \cos 2(\Theta + \lambda(\eta)\tau), \quad p_0 = \varphi_0(n_0 - 1),$$

and the scalar potential  $\varphi_0$  is found from the following ordinary differential equation:

$$\varphi_{0\xi\xi} = \frac{1}{2} \left( \frac{1 + a^2/2}{\varphi_0^2} - 1 \right).$$

Functions  $\lambda(\eta)$  and  $g_0(\eta)$  are determined by the boundary and initial conditions. The normalized averaged energy of a plasmon coupled to a relativistically intense optical field is

$$K_0 = \frac{\varphi_0}{4} \left[ \left( 1 + \frac{1 + a^2/2}{\varphi_0^2} \right)^2 + \frac{a^4}{8\varphi_0^4} \right] - 1.$$

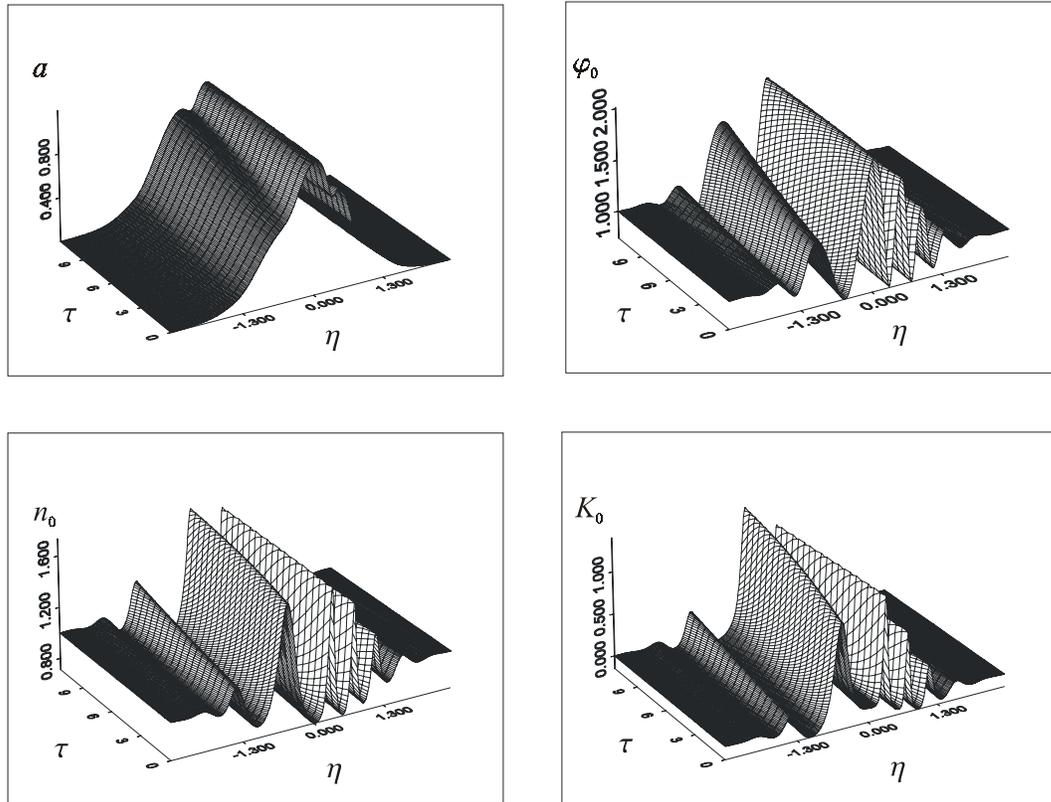
### 3. Nonlinear Self-Modulation of a Relativistically Intense Laser Pulse in a Plasma and Plasmon Generation

An example of the nonlinear self-modulation of a relativistically intense ultrashort laser pulse in a cold underdense plasma and of the corresponding plasmon generation is considered below. Here  $g_0(\eta) = \exp(-\eta^2)$  and  $\lambda(\eta) \equiv 0$ . The calculated distributions of the spatially and temporally localized self-modulated optical field vector potential amplitude and the scalar potential, as well as the plasma electron concentration and plasmon energy, are shown in Fig. 1. The plot of the scalar potential demonstrates the excitation of plasma oscillations by the propagating laser pulse. Besides, the self-modulation of the vector potential distribution due to the interaction of the optical field with the plasma wake it generates is also illustrated by Fig. 1.

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2. A.V. Borovsky, A.L. Galkin, V.V. Korobkin, and O.B. Shiryaev, *Phys. Rev. E* **59**, 2253 (1999).



**Fig. 1.** The self-modulation of a relativistically intense laser pulse in a cold underdense plasma. Spatial distributions of the normalized vector potential amplitude, scalar potential, electron concentration, and plasmon energy.