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# TWO DIMENSION BOUNCE- AVERAGED FOKKER-PLANCK SOLVER WITH NONLINEAR COLLISION OPERATOR FOR THE SIMULATION OF THE ION HEATING PROBLEMS

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**1-Introduction.** Numerical simulations of heating and current drive experiments in the Ion Cyclotron Resonance Frequency range (ICRH) usually rely upon different numerical codes, one group for the description of the launching, propagation and absorption of the waves and another one for the time evolution of the distribution function. Interfacing these different codes would give a unified numerical package for solving self-consistently the various aspects of the heating and current drive problems but would also lead to additional demands to the numerical solvers. For instance, self-consistently requires an appropriate treatment of the non-linear contribution of the collision operators in the Fokker-Planck solvers. A two-dimensional bounce-averaged ion Fokker-Planck solver for the ion distribution QLIFOPS is presented. It is based on a fast electron solver QLEFOPS [1] which has recently been coupled to the full-wave code TORIC to provide a tool for current drive simulations in the ion cyclotron frequency range [2]. Preliminary tests have been done on various JET heating scenarios using data provided by TORIC and quasilinear coefficients as derived in Ref. [3].

**2- The approach of the kinetic equation.** The basic equation describing the ion distribution function can be written as:

$$\frac{\partial f_\alpha}{\partial t} = \sum_\beta C^{\alpha/\beta}[f_\beta, f_\alpha] + Q^{RF}[f_\alpha] - \frac{Z_\alpha e}{m_\alpha} E_{||} \frac{\partial f_\alpha}{\partial v_{||}}, \tag{1}$$

where $\sum_\beta C^{\alpha/\beta}[f_\beta, f_\alpha]$ and $Q^{RF}[f_\alpha]$ are the collision and the quasilinear operators, respectively whereas the last term is the ohmic contribution. As for the quasilinear diffusion operator we use the simplified expression provided by Stix [4]:

$$Q^{RF}[f_\alpha] = \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\Big(v_\perp C_n |J_{n-1}(\frac{\kappa_\perp v_\perp}{\omega_c})|^2 \frac{\partial f_\alpha}{\partial v_\perp}\Big), \tag{2}$$

where $J_{n-1}$ is the Bessel function of the order $n-1$, $\kappa_\perp$ is the perpendicular wave number of the RF wave and $C_n$ is some normalization constant. This expression of the heating operator is sufficient for our purpose to demonstrate the important role of the nonlinear contribution of the collision operator.

The collision term $C^{\alpha/\beta}$, which describes the collisions of the test particles $\alpha$ with the background particles $\beta$, can be written using the Rosenbluth potentials $\phi_\beta$ and $\psi_\beta$:

$$\begin{aligned} C^{\alpha/\beta} &= \frac{1}{v^2}\frac{\partial}{\partial v}\Big[v^2\Big(A_1^\beta f_\alpha + A_2^\beta \frac{\partial f_\alpha}{\partial v}\Big)\Big] + \frac{1}{v^2}\frac{\partial}{\partial v}\Big(A_3^\beta \frac{\partial f_\alpha}{\partial \theta_0}\Big) \\ &+ \frac{1}{v^2 sin\theta_0}\frac{\partial}{\partial \theta_0}\Big(sin\theta_0 B_2^\beta \frac{\partial f_\alpha}{\partial v}\Big) + \frac{1}{v^2 sin\theta_0}\frac{\partial}{\partial \theta_0}\Big[sin\theta_0\Big(B_1^\beta f_\alpha + B_3^\beta \frac{\partial f_\alpha}{\partial \theta_0}\Big)\Big], \end{aligned} \tag{3}$$

where

$$A_1^\beta(v,\theta_0) = 2\kappa_\beta \frac{m_\alpha}{m_\beta}\Big\langle \frac{\partial \phi_\beta}{\partial v}\Big\rangle_b, \tag{4}$$

$$A_2^\beta(v,\theta_0) = -2\kappa_\beta \Big\langle \frac{\partial^2 \psi_\beta}{\partial v^2}\Big\rangle_b, \tag{5}$$

$$A_3^\beta(v,\theta_0) = B_2^\beta(v,\theta_0) = 2\kappa_\beta \Big\langle \frac{1}{\sqrt{\eta(\vartheta)}}\frac{cos\theta}{cos\theta_0}\Big(\frac{1}{v}\frac{\partial\psi_\beta}{\partial\theta} - \frac{\partial^2\psi_\beta}{\partial v\partial\theta}\Big)\Big\rangle_b, \tag{6}$$

$$B_1^\beta(v,\theta_0) = 2\kappa_\beta \frac{m_\alpha}{m_\beta}\Big\langle \frac{1}{\sqrt{\eta(\vartheta)}}\frac{cos\theta}{cos\theta_0}\frac{\partial \phi_\beta}{\partial \theta}\Big\rangle_b, \tag{7}$$

$$B_3^\beta(v,\theta_0) = -2\kappa_\beta \frac{1}{v}\Big\langle \frac{1}{\eta(\vartheta)}\frac{cos^2\theta}{cos^2\theta_0}\Big(\frac{1}{v}\frac{\partial^2\psi_\beta}{\partial\theta^2} + \frac{\partial\psi_\beta}{\partial v}\Big)\Big\rangle_b \tag{8}$$

and

$$\kappa_\beta = \frac{3}{2}\sqrt{\frac{\pi}{2}}\frac{n_\beta}{n_e}\sqrt{\frac{m_e}{m_\alpha}}\frac{ln\Lambda^{\alpha/\beta}}{ln\Lambda^{e/e}}Z_\alpha^2 Z_\beta^2, \tag{9}$$

where $\eta(\vartheta)$ is the dimensionless magnetic field normalized by its value at the mid-plane. The subscript 0 indicates an evaluation of the pitch-angle at the mid-plane and the brackets $< *** >_b$ represents a bounce averaging.

The nonlinear part of the collision operator is obtained by a truncated expansion of the distribution function and of the Rosenbluth potentials in Legendre polynomials [5]:

$$\{f_\alpha;\phi_\alpha;\psi_\alpha\} \approx \{f_\alpha^0;\phi_\alpha^0;\psi_\alpha^0\} + cos\theta\,\{f_\alpha^1;\phi_\alpha^1;\psi_\alpha^1\}. \tag{10}$$

So far, the coefficients $f_\alpha^m(v)$ of the expansion of the distribution function are related to the coefficients of the Rosenbluth potential expansion $\phi_\alpha^m(v)$ and $\psi_\alpha^m(v)$ in the following way [6]:

$$\phi_\alpha^m(v) = -\frac{1}{2m+1}\Big[\int_0^v \frac{w^{m+2}}{v^{m+1}} f_\alpha^m(w)dw + \int_v^\infty \frac{v^m}{w^{m-1}} f_\alpha^m(w)dw\Big], \tag{11}$$

$$\psi_\alpha^m(v) = \frac{1}{2(4m^2-1)}\Big[\int_0^v \frac{w^{m+2}}{v^{m-1}}\Big(1 - \frac{(m-1/2)w^2}{(m+3/2)v^2}\Big) f_\alpha^m(w)dw + \int_v^\infty \frac{v^m}{w^{m-3}}\Big(1 - \frac{(m-1/2)v^2}{(m+3/2)w^2}\Big) f_\alpha^m(w)dw\Big] \tag{12}$$

thus the ion-ion collision contribution in (3) becomes:

$$A_1^\alpha(v,\theta_0) = 2\kappa_\alpha \Big\langle \frac{\partial\phi_\alpha^0}{\partial v} + cos\theta\frac{\partial\phi_\alpha^1}{\partial v}\Big\rangle_b, \tag{13}$$

$$A_2^\alpha(v,\theta_0) = -2\kappa_\alpha \Big\langle \frac{\partial^2\psi_\alpha^0}{\partial v^2} + cos\theta\frac{\partial^2\psi_\alpha^1}{\partial v^2}\Big\rangle_b, \tag{14}$$

$$A_3^\alpha(v,\theta_0) = B_2^\alpha(v,\theta_0) = 2\kappa_\alpha \Big\langle \frac{1}{\sqrt{\eta(\vartheta)}}\frac{cos\theta}{cos\theta_0} sin\theta\Big(\frac{\partial\psi_\alpha^1}{\partial v} - \frac{\psi_\alpha^1}{v}\Big)\Big\rangle_b, \tag{15}$$

$$B_1^\alpha(v,\theta_0) = -2\kappa_\alpha \Big\langle \frac{1}{\sqrt{\eta(\vartheta)}}\frac{cos\theta}{cos\theta_0} sin\theta\phi_\alpha^1\Big\rangle_b, \tag{16}$$

$$B_3^\alpha(v,\theta_0) = -2\kappa_\alpha \frac{1}{v}\Big\langle \frac{1}{\eta(\vartheta)} \frac{cos^2\theta}{cos^2\theta_0} \frac{d\psi_\alpha^0}{dv} \Big\rangle_b$$
$$-2\kappa_\alpha \frac{1}{v}\Big\langle \frac{1}{\eta(\vartheta)} \frac{cos^2\theta}{cos^2\theta_0} cos\theta \Big( \frac{d\psi_\alpha^1}{dv} - \frac{\psi_\alpha^1}{v} \Big) \Big\rangle_b. \qquad (17)$$

The coefficients (13-17) still have a complicated structure since we retain the possibility of strong departure from Maxwellian distribution functions this is necessary, as will be demonstrated below, in cases of very strong heating.

The second order contributions in (13-17), proportional to the functions $\phi_\alpha^1$ and $\psi_\alpha^1$, can be derived semi-analytically. It is not so for the contributions proportional to $\phi_\alpha^0$ and $\psi_\alpha^0$, which need extensive numerical efforts. The problem is that the Fokker-Planck solver calculates the distribution function using the variables $v$ and $\theta_0$ evaluated at the midplane. But the expansion of the distribution function in Legendre polynomials (10) is carried out in local variables $v$ and $\theta$. Details of the numerical constructions of the coefficients (13-17) will be given elsewhere to save space for the numerical results.

**3- Nonlinear versus linear collision operator in numerical simulations.** Simulation results of plasmas heating during ICRH using the nonlinear and the linear description of the collision operator will be compared below. The coupling of the Fokker-Planck code with the wave code will not be used here. But we may already indicate that in contrast with the usual procedure, where the amplitudes of the diffusion coefficients as obtained from the wave code are normalized to the absorbed power deduced from experimental heating rates, we intend to use the numerical link between the wave code and the Fokker-Planck solver to avoid any renormalization of the amplitude of the electromagnetic field.

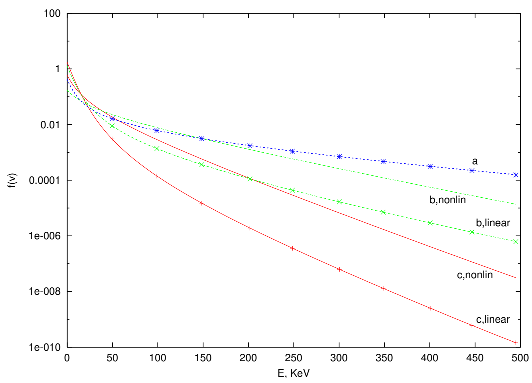


Figure 1: *The dependence of the isotropic distribution f(v) on the energy* $E = mv^2/2$. *The calculations have been carried out for the plasma parameter of the section 3a and three values of the deuterium concentration: a)* 2%*; b)* 5%*; c)* 10%. *The curves with the symbols correspond to the linear description of the collision operator.*

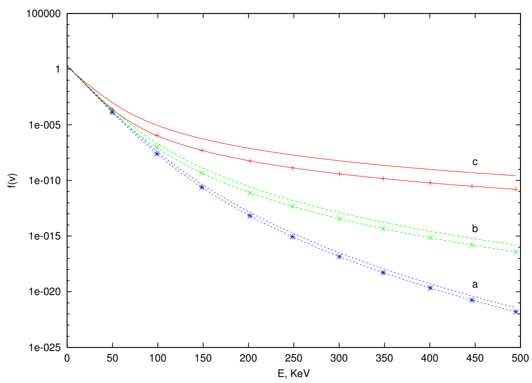


Figure 2: *The dependence of the isotropic distribution f(v) on the energy* $E = mv^2/2$. *The calculations have been carried out for the plasma parameter of the section 3b and three values of the constant* $C_2$ *in the quasi-linear operator: a)* $2.4 \cdot 10^{16} W/cm^3$*; b)* $4.8 \cdot 10^{16} W/cm^3$*; c)* $1.2 \cdot 10^{17} W/cm^3$. *The curves with the symbols correspond to the linear description of the collision operator.*

**3a- Minority ion heating at the fundamental harmonic of the cyclotron resonance.** The code has been tested on the heating at the fundamental harmonic of the cyclotron resonance (Stix's problem) and has demonstrated good correspondence to the analytical results given by formulas (34) and (38) in [4]. It is clear that the nonlinear description of the collision operator is not essential for Stix's example of minority deuterium heating mainly due to the low concentration (5%). The plasma parameters considered here are: D-T plasma, $n_e = 5 \cdot 10^{13} cm^{-3}$, $T_e = T_T = 5KeV$. For the fundamental harmonic heating $J_0 = 1$ and the constant $C_1$ has been chosen to be equal to $3 \cdot 10^{16} cm^2/s^3$ in order to provide the an absorbed power level of the order of $0.1W/cm^3$. The results of the test are shown in figure 1 both for the linear (curves with symbols) and for the nonlinear collision operator. The deuterium densities are $n_D/n_e$: a) 2%; b) 5%; c) 10%. The linear collision operator underestimates the effective temperature of the ion tail by 9% in case b) and by 12% in case c).

**3b- Heating at second harmonic of the cyclotron resonance.** The hydrogen heating at second cyclotron harmonic requires a nonlinear description of the collision operator when the absorbed power is large enough to change the averaged temperature of the distribution. The results of the simulations are presented in figure 2. Here $J_1 \approx (\kappa_\perp v_\perp / 2\omega_c)^2$ for second harmonic heating. The following parameters have been chosen for the simulation: $n_e = n_H = 5 \cdot 10^{13} cm^{-3}$, $T_e = 5KeV$, $\kappa_\perp = 0.5cm^{-1}$ and $\omega_c = 1.5 \cdot 10^8 rad/s$. The curves have been built for three values of the constant $C_2$: a) $2.4 \cdot 10^{16} cm^2/s^3$; b) $4.8 \cdot 10^{16} cm^2/s^3$; c) $1.2 \cdot 10^{17} cm^2/s^3$. The three cases provided the following values of the absorbed power: a) $0.02W/cm^3$; b) $0.04W/cm^3$; c) $0.1W/cm^3$. The linear collision operator underestimate the effective temperature of the ion tail only by 1% in case b) and by 3% in case c) but the averaged temperature of the distribution is underestimated by 8% in case b) and by 21% in case c).

**Conclusions** The importance of the nonlinear description of the collision operator in the self-consistent simulations of realistic heating scenarios has been demonstrated using a newly developed bounce-averaged Fokker-Planck solver. Heating scenarios with the absorbed power $0.05W/cm^3$ in the case of hydrogen heating at the fundamental harmonic on the one hand and 5% deuterium concentration in the case of the minority heating at the second harmonic on the other hand already show a important deviation due to the nonlinear description of the collision operator as compared to the linear one. This conclusion is thus of some relevance in the future exploitation of the self-consistent wave-particle numerical codes.

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