29th EPS Conference on Plasma Phys. and Contr. Fusion Montreux, 17-21 June 2002 ECA Vol. **26B**, P-4.107 (2002)

Forced Magnetic Reconnection Characterized by Resistive-Kink Time Scale in Cylindrical Tokamak

A. Ishizawa, S. Tokuda[1)] and M. Wakatani

Graduate School of Energy Science Kyoto University, Uji, Kyoto, 611-0011, Japan
[1)]*Naka Fusion Research Establishment, Japan Atomic Energy Research Institute, Naka, Ibaraki, 311-0193, Japan*

Abstract

The linear initial-value problem, newly formulated by using the asymptotic matching method, is successfully solved on forced magnetic reconnection. Numerical solutions reveal that plasma inertia in a singular-current layer plays the dominant role in the reconnection process even for a weak boundary perturbation, resulting in the resistive-kink time scale. The inertia also constructs a non-constant-psi structure in the current layer. This non-constant-ψ structure forces the reconnection by inducing a current density just on a resonant surface in the layer, even if a total current in the layer indicates stable tearing modes.

1 Introduction

In plasma confinement, forced reconnection is caused by an externally applied perturbation such as error fields, even if an equilibrium is stable against tearing modes [1, 2, 3, 4]. This reconnection can cause mode-locking in a rotating plasma. It also models seed islands formation for neo-classical tearing modes.

In this paper, we solve directly a temporal evolution equation derived from the non-constant-ψ matching, and show the reconnection occurs on the resistive-kink time scale [5]. This time scale originates from a similarity in the structure between the forced reconnection and resistive-kink modes. This non-constant-ψ structure forces the time evolution of current density just on a resonant surface in the layer to deviate from the time evolution of total current density of the layer. Therefore, a current density is induced on the resonant surface, and even if the total current indicates the equilibrium is stable against tearing modes, the reconnection is forced.

2 Boundary layer analysis of forced reconnection

The forced reconnection occurs as a response to an applied external magnetic perturbation. We calculate the response of the cylindrical plasma to the external perturbation as an initial-value problem by use of the boundary layer analysis. We assume that the time scale of the external perturbation τ_e is much slower than the Alfvén time scale so that the response of the plasma to the external magnetic perturbation is governed by the static ideal MHD equations except the resonant surface. This quasi-static region is referred as outer region. In the vicinity of the resonant surface the inertial and resistivity of plasma is important. This narrow current layer is referred as inner layer.

Here, we describe the response to the external magnetic perturbation in the outer region. The external magnetic perturbation is modeled as the boundary perturbation with poloidal and toroidal numbers m and n, $\psi_1(r=a) = \psi_e(t/\tau_e)\exp(im\theta - in\zeta)$). The magnetic perturbation is written as $\boldsymbol{b} = \nabla\zeta \times \nabla\psi_1$, where ζ is the toroidal angle and ψ_1 denotes the perturbed magnetic flux. The boundary perturbation produces the magnetic perturbation $\psi_1 = \psi_1(r,t)\exp(im\theta - in\zeta)$, which has a rational surface at $r = r_s$ where $q(r_s) = rB_\zeta/R_0B_\theta = m/n$. In the outer region, this response to a given external perturbation is governed by the marginal ideal MHD equations,

$$\frac{d}{dr}\left(r\frac{d\psi_1}{dr}\right) - \frac{m^2}{r}\psi_1 - \frac{mq\psi_1}{(m-nq)}\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}\frac{r^2}{q}\right) = 0. \tag{1}$$

The solution to Eq. (1) can be written as $\psi_1(r,t) = \psi_1(r_s,t)f(r) + \psi_e(t)g(r)$, where $f(r)$ and $g(r)$ satisfy Eq. (1). These functions are subject to the boundary conditions $f(0) = 0$, $f(r_s) = 1$, $f(a) = 0$ and $g(0) = 0$, $g(r_s) = 0$, $g(a) = 1$[2]. The time evolution of the outer solution which describes the quasi-static equilibrium, is determined only by the outer-reconnected flux, $\psi_1(r_s,t)$, because the time dependence of the external magnetic perturbation, $\psi_e(t/\tau_e)$, is a given function.

In order to obtain the time dynamics of the reconnection we should investigate the dynamics in the inner layer where the resistivity and the inertia of the plasma are important. Since the amplitude of the external magnetic perturbation is assumed to be small, the perturbed quantities obey the linearized reduced MHD equations. We apply the Laplace transform $\tilde{F}(r,s) = \int_0^\infty F(r,t)e^{-st}dt$ to the linearized reduced MHD equations. The initial conditions for the perturbed part of the magnetic potential and the static potential $\varphi = \varphi_1(r,t)\exp(im\theta - in\zeta)$ are $\psi_1(r,0) = \varphi_1(r,0) = 0$. By stretching the variables in the vicinity of the rational surface as $\hat{x} = (r - r_s)/(\varepsilon r_s)$, $\hat{s} = \tau_c s$, we have the equation in the inner layer

$$\left(1 + \frac{\hat{x}^2}{\hat{s}^2}\right)E - \frac{\hat{x}^2}{\hat{s}}\frac{d}{d\hat{x}}\left(\frac{1}{\hat{x}^2}\frac{dE}{d\hat{x}}\right) = \Psi_\infty, \tag{2}$$

where $E \equiv -\hat{s}d\tilde{\varphi}_{in}/d\hat{x} = \hat{x}^2 d/d\hat{x}(\tilde{\psi}_{in}/\hat{x}) + \Psi_\infty$, $\tilde{\psi}_{\rm in}(\hat{x},\hat{s}) = \tilde{\psi}_1(r,s)/B_\theta(r_s)r_s$, $\tilde{\varphi}_{\rm in}(\hat{x},\hat{s}) = \tilde{\varphi}_1(r,s)/v_A r_s$, $\varepsilon = (\tau_A/\tau_R k_m r_s)^{1/3}$, $\tau_c = \tau_A/\varepsilon k_m r_s$, $k_m \equiv m s_r/r_s$, $s_r = r_s q'(r_s)/q(r_s)$, $\tau_A = r_s/v_A$, $\tau_R = 4\pi a^2/\eta$, $v_A = B_\theta(r_s)/\sqrt{4\pi\rho}$ and $S = \tau_R/\tau_A$. The constants η and ρ denote the magnetic diffusivity and the density of the plasma. Following Ref.[5], we can obtain the solution to the inner equation (2). The singular nature of the outer solution is described by the discontinuity in the radial derivatives of g and f as $\Delta_s' = [dg/dr]_{r=r_s+0}^{r=r_s-0}$, $\Delta_0' = [df/dr]_{r=r_s+0}^{r=r_s-0}$. Since the equilibrium is supposed to be stable against the tearing mode, the tearing-mode stability parameter Δ_0' is negative. These two parameters Δ_s' and Δ_0' summarize the information required from the marginal ideal MHD equations to determine the time dynamics of the reconnection, which is represented by the inner-reconnected flux.

3 Evolution equation for inner-reconnected flux

We propose an integral equation which enables us to investigate the temporal evolution of inner-reconnected flux. Since the inner-reconnected flux is the same as true amount of reconnected flux, it gives reconnection rate and magnetic island width.

The non-constant-ψ asymptotic matching of the inner solution to the outer solution yields the matching conditions[4]. The inversion of the Laplace transform of these matching conditions gives the integral equation to the inner-reconnected flux

$$\psi_{in}(0,\hat{t}) = \int_0^{\hat{t}} \psi_1(r_s,\hat{\tau})H(\hat{t}-\hat{\tau})d\hat{\tau}, \tag{3}$$

$$\psi_1(r_s,\hat{t}) + \frac{1}{\hat{\tau}_\alpha}\int_0^{\hat{t}} \psi_1(r_s,\hat{\tau})G(\hat{t}-\hat{\tau})d\hat{\tau} = \frac{-\Delta'_s}{\Delta'_0}\psi_e(\hat{t}/\hat{\tau}_e), \tag{4}$$

where the kernel $H(\hat{t})$ is

$$\begin{aligned} H(\hat{t}) \;=\; & \frac{2\sqrt{2}}{3}\sum_{n=1}^{N}(-1)^n(4n-1)^{2/3}\frac{\Gamma(n-1/2)}{\Gamma(-1/2)n!}\exp(\frac{-\hat{t}}{2\hat{\tau}_n})\cos(\frac{\sqrt{3}}{2}\frac{\hat{t}}{\hat{\tau}_n}-\frac{\pi}{3}) \\ & -\frac{1}{\sqrt{2}\pi}\sum_{n=0}^{\infty}(-1)^n(4n-1)\frac{\Gamma(n-1/2)}{\Gamma(-1/2)n!}\int_0^\infty \frac{x^{3/2}e^{-x\hat{t}}}{(4n-1)^2+x^3}dx. \end{aligned} \tag{5}$$

where t and the ideal time scale $\tau_\alpha = -\Delta'_0\tau_A/(\pi k_m)$ are normalized by the resistive-kink time scale $\tau_c = \tau_A S^{1/3}/(k_m r_s)^{2/3}$ as, $\hat{t} = t/\tau_c$ and $\hat{\tau}_\alpha = \tau_\alpha/\tau_c$. Note that the amplitude of the tearing mode stability parameter Δ'_0 affects to the ideal time scale τ_α. The kernel $G(\hat{t})$ is the inverse Laplace transform of $\Delta'_{in}(s)$. It is written as

$$\begin{aligned} G(\hat{t}) \;=\; & -\frac{4}{3\pi}\sum_{n=1}^{N}\frac{\sqrt{n-1/4}}{n!}\Gamma(n-1/2)\exp(\frac{-\hat{t}}{2\hat{\tau}_n})\sin(\frac{\sqrt{3}}{2}\frac{\hat{t}}{\hat{\tau}_n}) \\ & +\frac{1}{3\pi^2}\int_0^\infty \sqrt{x}|\Gamma(ix-1/4)|^2\exp(-(4x)^{2/3}\hat{t}-\pi x)dx, \end{aligned} \tag{6}$$

where $\hat{\tau}_n = 1/(4n-1)^{2/3}$. The kernels G and H include only the resistive-kink time scale $\tau_c \propto \tau_A S^{1/3}$ and exclude the Sweet-Parker time scale and the tearing mode time scale, and thus we adopt normalized time $\hat{t} = t/\tau_c$ in these kernels. This resistive-kink time scale in the kernels leads to a typical time scale of the evolution of inner-reconnected flux, reconnection rate and magnetic islands width.

Here, we discuss an intrinsic unstable solution to the inner-layer equation (2) for a negative Δ'. The resistive-boundary layer theory is not reliable when $\hat{t} \ll 1$ i.e. $\hat{s} \gg 1$. The unstable solution is an eigen-function of a pole $\hat{s} = 1$ of the Gamma function $\Gamma(\hat{s}^{3/2}/4 - 1/4)$ and yields $|\Delta'_{in}| \to \infty$. We excluded the eigen function from the inner solution, and consequently, we excluded this pole from the kernels G and H. The eigen function of this pole corresponds to the unstable kink mode [5]. In addition, the theory yields false response that inner-reconnected flux exponentially grows on the ideal time scale. In order to exclude this false response we exclude the contributions of poles at $|\hat{s}| \gg 1$ to the kernels G and H by taking $N = 1$.

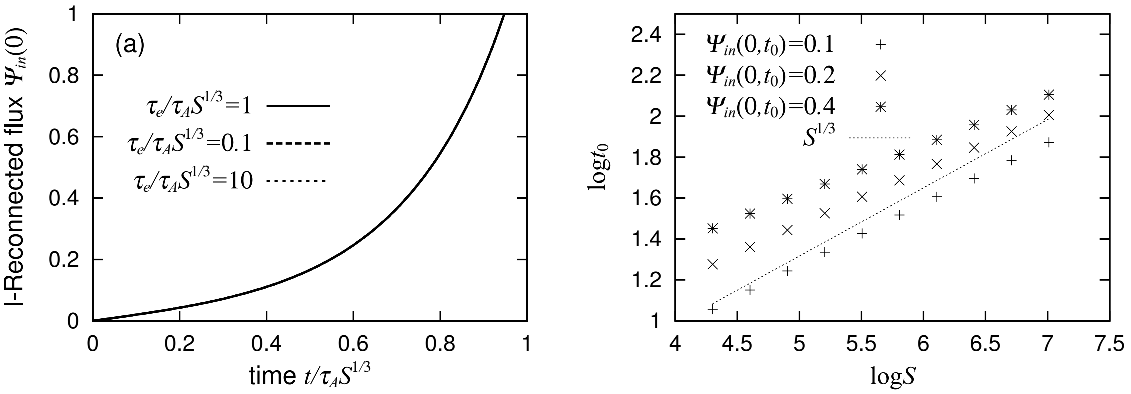

Figure 1: (a) Time evolution of the normalized inner-reconnected flux $\Psi_{in}(0) = \psi_{in}(0,t)/(-\Delta'_s\psi_e(t)/\Delta'_0)$ for $\psi_e(t) = \Psi_e t^2/\tau_e^2$, $N = 1$, $r_s\Delta'_0 = -5$. The inner-reconnected flux increases on the resistive-kink time scale $\tau_A S^{1/3}$. (b) The S dependence of the time t_0 when the inner-reconnected flux reaches on values $\Psi_{in}(0,t_0) = 0.1$, 0.2 and 0.4 for $N = 1$, $r_s\Delta'_0 = -5$.

The integral equation is valid when an amplitude of solution is less than $O(1)$. If the amplitude of the inner-reconnected flux exceeds unity, then it grows exponentially. This exponential growth is spurious, because we consider a response of stable plasma. Hence we show numerical solutions which are less than $O(1)$.

4 Summary

We have successfully solved the time evolution equation for the reconnection forced by boundary perturbations. The equation is derived from the asymptotic matching method taking into account the non-constant-ψ structure of a singular-current layer. It governs the time evolution of the inner-reconnected flux, which represents the amount of reconnected flux in the singular layer. Numerical solutions of the equation have revealed the features of reconnection; it is forced by the non-constant-ψ structure and evolves with the resistive-kink time scale. Such features are caused by the plasma inertia in the layer.

References

[1] T. S. Hahm and R. M. Kulsrud; Phys. Fluids **28**, 2412 (1985)

[2] R. Fitzpatrick and T. C. Hender; Phys. Fluids B **3**, 644 (1991)

[3] X. Wang and A. Bhattacharjee; Phys. Plasmas **4**, 748 (1997)

[4] A. Ishizawa, and S. Tokuda, Phys. Plasmas **7**, 875 (2000)

[5] G. Ara, B. Basu, B. Coppi, G. Laval, M. N. Rosenbluth and B. V. Waddell; Ann. Phys. **112**, 443 (1978)