

Stabilization of Farley-Buneman Modes in the Presence of Radar Beams

P. K. Shukla, L. Stenflo, M. Rosenberg, and D. P. Resendes

Department of Plasma Physics, Umeå University, SE-90187 Umeå, Sweden

Abstract

The nonlinear interaction between an intense radar beam and the Farley-Buneman (FB) modes in the Earth's atmosphere is considered. It is found that the FB instability can be stabilized by the radar beam. The implications of our investigation to the suppression of the plasma turbulence in dusty meteor trails and polar mesospheric clouds are discussed.

Coherent radar beams probing the equatorial E-region ionosphere and the mesosphere between 80 and 130 km frequently receive echoes from plasma trails that are left behind ablating meteors or some other sources. The behavior of collision-dominated ionospheric plasmas and meteor trails has been discussed in many books and review articles [1,2]. It has been recognized that the plasma structuring [3] in the Earth's upper atmosphere could be due to FB and gradient drift instabilities. However, the presence of charged dust [4] in the meteor trails and in the mesosphere can affect [5,6] the threshold conditions for the FB and ion wave two-stream instabilities which are triggered by the electron $\mathbf{E}_0 \times \mathbf{B}_0$ drift, where \mathbf{E}_0 and \mathbf{B}_0 are the dc electric and magnetic fields, respectively. The European Incoherent Scatter (EISCAT) radar is also being used [7] for determining the properties of the turbulence at the mesospheric altitudes. In the present paper, we show that the FB instability can be stabilized by intense radar beams due to parametric interactions [8].

Let us consider the nonlinear propagation of intense radar beams in the Earth's mesospheric/ionospheric plasma in the geomagnetic field $B_0 \hat{\mathbf{z}}$, where B_0 is the magnetic field strength and $\hat{\mathbf{z}}$ is the unit vector along the z axis. The plasma also has an equilibrium

electric field $\mathbf{E}_0 = E_0 \hat{\mathbf{x}}$ so that the electrons have a drift velocity $u_{e0} \hat{\mathbf{y}} = -\hat{\mathbf{y}}(cE_0/B_0)$, where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are the unit vectors along the x and y axes, respectively. The constituents of our plasma are electrons, ions, micron-sized massive charged dust particles, and neutrals. The wave frequency ω_0 and the wavevector \mathbf{k}_0 of the radar beam satisfy $\omega_0^2 = k_0^2 c^2 + \omega_{pe}^2 \omega_0 / (\omega_0 + i\nu_e)$, where ω_0 is supposed to be much larger than the electron gyrofrequency $\omega_{ce} = eB_0/m_e c$ and the electron-neutral collision frequency ν_e . Here c is the speed of light in vacuum, $\omega_{pe} = (4\pi n_e e^2/m_e)^{1/2}$ is the electron plasma frequency, n_e is the electron number density, e is the magnitude of the electron charge, and m_e is the electron mass.

The wave equation for a radar beam in the presence of an electron density perturbation $n_{e1} (= n_e - n_{e0})$ of the FB mode is

$$(\partial_t + \nu_e) (\partial_t^2 - c^2 \nabla^2) \mathbf{A} + \omega_p^2 \left(1 + \frac{n_{e1}}{n_{e0}}\right) \partial_t \mathbf{A} \approx 0, \quad (1)$$

where $\omega_p = (4\pi n_{e0} e^2/m_e)^{1/2}$ is the unperturbed plasma frequency.

We now present the relevant equations for the FB mode in the presence of the ponderomotive force of the radar beams. The dynamics of the FB mode is controlled by the magnetized electrons and unmagnetized ions, since the mode frequency is much smaller (larger) than the electron (ion) gyrofrequency. Furthermore, for the FB mode $|(\partial_t + \nu_e + u_{e0} \partial_y) n_{e1}| \ll \omega_{ce} n_{e1}$. Hence, for our purposes, we have [8]

$$\left\{ \partial_t + \frac{u_{e0} \delta}{(\delta + \Psi)} \partial_y + \frac{\Psi}{\nu_i (\delta + \Psi)} \left[\partial_t^2 - V_{Ti}^2 \nabla_{\perp}^2 \left(3 + \frac{T_e \delta}{T_i} \right) \right] \right\} \frac{n_{e1}}{n_{e0}} = \frac{\nu_e \delta}{2B_0^2 (\delta + \Psi)} \nabla_{\perp}^2 |\mathbf{A}|^2, \quad (2)$$

where $\Psi = \nu_e \nu_i / \omega_{ce} \omega_{ci}$, $\omega_{ci} = eB_0/m_i c$, $\delta = n_{i0}/n_{e0}$, T_e and T_i are the electron and ion temperatures, ν_i is the ion-neutral collision frequency, and $V_{Ti} = (T_i/m_i)^{1/2}$.

The parametric stabilization of the FB mode by a constant amplitude radar pump (ω_0, \mathbf{k}_0) can be demonstrated from eqs. (1) and (2). The pump generates radar sidebands $(\omega_{\pm}, \mathbf{k}_{\pm})$ by interacting with the FB mode (ω, \mathbf{k}) . Hence, we decompose the vector potential as $\mathbf{A} = \mathbf{A}_{0+} \exp(-i\omega_0 t + i\mathbf{k}_0 \cdot \mathbf{r}) + \mathbf{A}_{0-} \exp(i\omega_0 t - i\mathbf{k}_0 \cdot \mathbf{r}) + \sum_{+,-} \mathbf{A}_{\pm} \exp(-i\omega_{\pm} t + i\mathbf{k}_{\pm} \cdot \mathbf{r})$, where \mathbf{A}_{\pm} is the vector potential of the sidebands, $\omega_0 \approx (k_0^2 c^2 + \omega_p^2)^{1/2}$, $\omega_{\pm} = \omega \pm \omega_0$ and $\mathbf{k}_{\pm} = \mathbf{k} \pm \mathbf{k}_0$. Supposing that the electron density perturbation varies as $n_{e1} = N_e \exp(-i\omega t + i\mathbf{k} \cdot \mathbf{r})$, we Fourier analyze (1) and (2) and obtain

$$\left[(k_{\pm}^2 c^2 - \omega_{\pm}^2) (\omega_{\pm} + i\nu_e) + \omega_{\pm} \omega_p^2 \right] \mathbf{A}_{\pm} = \mp \omega_0 \omega_p^2 \frac{N_e}{n_{e0}} \mathbf{A}_{0\pm}, \quad (3)$$

and

$$\begin{aligned} & \left\{ \omega - k_y U_0 - i \frac{\Psi}{\nu_i (\delta + \Psi)} \left[\omega^2 - k^2 V_{Ti}^2 \left(3 + \frac{T_e \delta}{T_i} \right) \right] \right\} \frac{N_e}{n_{e0}} \\ & = -i \frac{\nu_e \delta k_y^2}{B_0^2 (\delta + \Psi)} (\mathbf{A}_{0-} \cdot \mathbf{A}_+ + \mathbf{A}_{0+} \cdot \mathbf{A}_-). \end{aligned} \quad (4)$$

where $U_0 = u_{e0} \delta / (\delta + \Psi)$.

By using eq. (3) we can eliminate \mathbf{A}_{\pm} from eq. (4). The resultant nonlinear dispersion relation is [8]

$$\omega - k_y U_0 - i \frac{\Psi}{\nu_i (\delta + \Psi)} \left[\omega^2 - k^2 V_{Ti}^2 \left(3 + \frac{T_e \delta}{T_i} \right) \right] = \frac{i \nu_e \delta \omega_p^2 k_y^2 |\mathbf{A}_0|^2}{B_0^2 (\delta + \Psi)} \sum_{\pm} D_{\pm}^{-1}, \quad (5)$$

where $|\mathbf{A}_0|^2 = \mathbf{A}_{0-} \cdot \mathbf{A}_{0+}$, $D_{\pm} \approx \mp 2\omega_0 (\omega - \mathbf{k} \cdot \mathbf{v}_g + i\Gamma \mp \Delta)$, $\mathbf{v}_g = \mathbf{k}_0 c^2 / \omega_0$, $\Gamma = \nu_e \omega_p^2 / 2\omega_0^2$, and $\Delta = k^2 c^2 / 2\omega_0$. Assuming that both sidebands are resonant, viz. $D_{\pm} \approx 0$, we can rewrite eq. (5) as

$$\begin{aligned} & \omega - k_y U_0 - i \frac{\Psi}{\nu_i (\delta + \Psi)} \left[\omega^2 - k^2 V_{Ti}^2 \left(3 + \frac{T_e \delta}{T_i} \right) \right] \\ & = i \frac{\nu_e \delta \Delta}{\omega_0} \frac{k_y^2 |\mathbf{A}_0|^2}{B_0^2 (\delta + \Psi)} \frac{\omega_p^2}{[(\omega - \mathbf{k} \cdot \mathbf{v}_g + i\Gamma)^2 - \Delta^2]}. \end{aligned} \quad (6)$$

Letting $\omega = k_y U_0 + i\gamma$, where $\gamma, \sqrt{3}kV_{Ti}(1 + T_e \delta / 3T_i)^{1/2} < k_y U_0 \sim \mathbf{k} \cdot \mathbf{v}_g$, we obtain from eq. (6) the growth rate

$$\gamma \approx \frac{k_y^2 U_0^2 \Psi}{\nu_i (\delta + \Psi)} - \frac{\nu_e \delta \Delta}{\omega_0 (\delta + \Psi)} \frac{k_y^2 |\mathbf{A}_0|^2}{B_0^2} \frac{\omega_p^2}{\Gamma^2 + \Delta^2}, \quad (7)$$

which reveals suppression of the FB mode if

$$\frac{E_p^2}{B_0^2} > \frac{U_0^2 (\Gamma^2 + \Delta^2) \omega_0^3}{c^2 \omega_p^2 \omega_g^2 \Delta \delta}. \quad (8)$$

where $E_p = \omega_0 |\mathbf{A}_0| / c$ corresponds to the pump wave electric field and $\omega_g = (\omega_{ce} \omega_{ci})^{1/2}$ is the lower-hybrid resonance frequency. It turns out that the inequality (8) is satisfied for parameters ($E_p \sim 0.01$ V/cm, $B_0 \sim 0.5$ gauss, $\omega_0 / 2\pi = 4$ MHz, $u_{e0} \sim 10^5$ cm/s, $n_{i0} \sim 10^4$ cm $^{-3}$, $T_i \sim T_e = 0.02$ eV, $\delta = 4$, $\Psi \sim 4$, $\Delta \sim 10^5$ s $^{-1}$, $\omega_g \sim 3.5 \times 10^4$ s $^{-1}$, $\nu_e \sim 3 \times 10^5$ s $^{-1}$, and $\omega_{pi} \sim 2.5 \times 10^4$ s $^{-1}$), which are typical for the Earth's mesosphere at

an altitude of 90 km. Numerical estimates thus suggest that a sufficiently large amplitude radar beam can stabilize the FB mode. The stability of the latter may provide a clue to the reduction of the PMSE power when the EISCAT radar beams are used for probing the highly collisional magnetoplasma at the Earth's mesopause [7].

Acknowledgments

This work was partially supported by the The Royal Swedish Academy of Sciences as well as by the the European Commission (Brussels) through the contract No. HPRN-CT2000-00140. M. Rosenberg was supported in part by US DOE Grant No. DE-FG03-97ER5444.

References

- [1] M. C. Kelley, *The Earth's Ionosphere: Plasma Physics and Electrodynamics*, Academic Press, San Diego, 1989.
- [2] G. E. Thomas, *Rev. Geophys* **29**, 553 (1991); Cho, J. Y. N. and M. C. Kelley, *ibid.* **31**, 243 (1993).
- [3] J. Y. N. Cho, T. M. Hall, and M. C. Kelley, *J. Geophys. Res.* **97**, 875 (1992); J. D. Mitchell, J. D., C. L. Croskey, and R. A. Goldberg, *Geophys. Res. Lett.* **28**, 1423 (2001).
- [4] O. Havnes, J. Trøim , T. Blix, W. Mortensen, L. I. Næsheim, E. Thrane, and T. Tønnesen, *J. Geophys. Res.* **101**, 10839 (1996); O. Havnes, A. Brattli, T. Aslaksen, W. Singer, E. Latteck, T. Blix, E. Thrane , and J. Trøim, *Geophys. Res. Lett* **28**, 1419 (2001).
- [5] M. Rosenberg and V. W. Chow, *Planet. Space Sci.* **46**, 103 (1998).
- [6] M. Rosenberg and P. K. Shukla, *J. Geophys. Res.* **105**, 23135 (2000); M. Rosenberg, *IEEE Trans. Plasma Sci.* **29**, 261 (2001).
- [7] P. B. Chilson, E. Belova, M. T. Rietveld, S. Kirkwood, and U. Hoppe, *Geophys. Res. Lett.* **27**, 3801 (2000).
- [8] P. K. Shukla, L. Stenflo, M. Rosenberg, and D. P. Resendes, *J. Geophys. Res.* **107**, in press, (2002).