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Saturation of beam-plasma instability is sensitive to discrete particle effects

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A single wave model describes the self-consistent interaction of an electrostatic potential perturbation Φ and N électrons, in a 1-D plasma with length L and periodic boundary conditions. For the potential, one harmonic spatial mode is dominant,

$$\Phi(z,\tau) = \phi_k(\tau) \exp i(kz - \omega_k \tau) + \text{c.c.} \quad (1)$$

(c.c. = complex conjugate) with (k, ω_k) related by $\epsilon(k, \omega_k) = 0$. In normalized variables $t = \alpha\tau$, $x_l = kz_l - \omega_k \tau$, with $\alpha^3 = ne^2/[m\epsilon_0(\partial\epsilon/\partial\omega_k)]$, $V = (ek^2\phi_k)/(\alpha^2 m)$, this system is described by the self-consistent hamiltonian

$$H(x_l, p_l, \zeta, \zeta^*) = \sum_{l=1}^{N} \Big(\frac{p_l^2}{2} - N^{-1/2}\zeta \mathrm{e}^{ix_l} - N^{-1/2}\zeta^* \mathrm{e}^{-ix_l}\Big), \quad (2)$$

where $\zeta = N^{1/2}V$. This hamiltonian conserves momentum $\mathsf{P} = \sum_l p_l + |\zeta|^2$. For $N \to \infty$, the particle distribution is represented by a density $f(x,p,t)$, and the canonical equations of (2) give the Vlasov-wave system

$$\frac{dV}{dt} = i \int \exp(-ix) f(x,p,t) dx dp \quad (3)$$

$$\frac{\partial f}{\partial t} + p\frac{\partial f}{\partial x} + \left(iV\mathrm{e}^{ix} - iV^*\mathrm{e}^{-ix}\right)\frac{\partial f}{\partial p} = 0. \quad (4)$$

Theorem : For any $T > 0$ et for any initial condition at finite N going to a density f for $N \to \infty$, solutions of (2) go to solution of the Vlasov-wave system (3)-(4) on the whole interval $0 \le t \le T$.

A symplectic code (resp. semi-lagrangian code) is used to numerically solve the N particles model (resp. kinetic model) equations.

As the cold beam case is globally well understood, we will focus on the warm beam case. We consider the three initial distribution functions shown on Fig. 1.

For the linearly damped case : $f'(0) < 0$ the thermodynamic equilibrium between the wave and the particles predicts a finite intensity for the wave, with $\omega_b \sim N^{-1/2}$. On the contrary, kinetic theory implies that ω_b decreases to 0 or a value smaller than the initial value. We

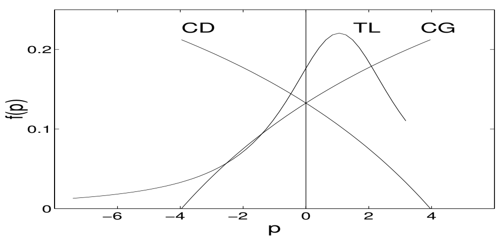

Figure 1: Warm beam initial distributions.

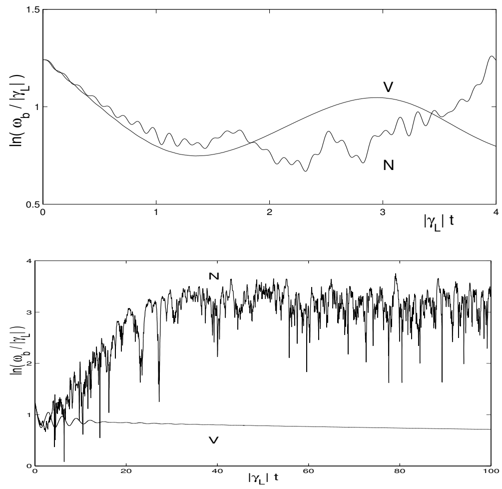

Figure 2: Time evolution of $\ln(\omega_{\mathrm{b}}(t)/|\gamma_{\mathrm{L}}|)$ for the CD distribution and an initial wave intensity lower than the thermal equilibrium value : (N) $N = 32000$ particules, (V) kinetic scheme on a 32×512 grid in (x, p)

check (Fig. 2) that, for $N < \infty$, the N particles system starts to evolve like the kinetic system. However when trapping effects set in :

- the N particles system evolves chaotically toward a thermal equilibrium, due to spontaneous wave emission by particles,
- the kinetic system gives wave oscillations (non linear Landau damping decribed by Mazitov and O'Neil), but these oscillations are damped.

For a warm beam with positive slope in $p_0 = 0$, we compare (Fig. 3) the time evolution for two N values and two semi-lagrangian grids with an initial CG distribution and an instability rate $\gamma_{\mathrm{r}} = 0.08$. The agreement is excellent in the linear regime and when saturation by trapping oscillations starts (reached for $\omega_{\mathrm{b}}/\gamma_{\mathrm{r}} \approx 3.1$). The thermal equilibrium level for the wave intensity is well above this saturation level. Beyond the first trapping oscillation, the simulations depart from each other.

- kinetic simulation : secondary instability for a coarse grid (V1), trapping oscillations relaxation for the finer grid (V2).
- N particles : secondary instability, weaker for N larger ; we have also observed that the particles distribution function gets uniform on a larger and larger domain as the wave grows.

The strength of the secondary instability also depends on the shape of the initial velocity distribution. It is weaker for the TL than for the CG distribution : the number of fast particles decreases more rapidly for TL than for CG.

The mechanism controlling the long time system evolution is the trapping-detrapping of particles by the pulsating separatrix. The particle motion equation

$$\ddot{x} = -\omega_{\mathrm{b}}^2(t) \sin(x - \theta(t)) \tag{5}$$

(a)

(b)

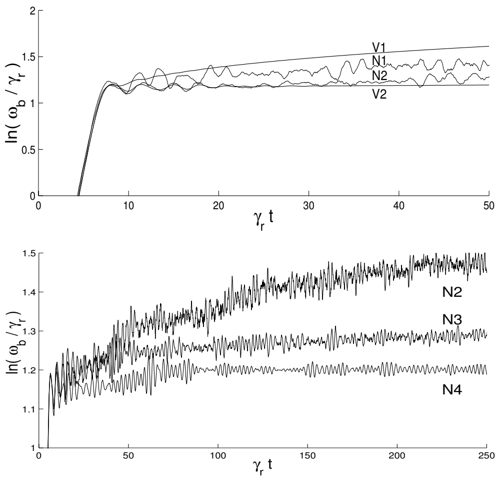

Figure 3: Time evolution of $\ln(\omega_{\mathrm{b}}(t)/\gamma_{\mathrm{r}})$. (a) CG initial distribution : kinetic scheme with a (V1) 32×128, (V2) 256×1024 grid in (x, p) ; N particles system for (N1) $N = 128000$, (N2) $N = 512000$. (b) Comparison of CG (N2) with TL initial distribution for (N3) $N = 64000$, (N4) $N = 2048000$.

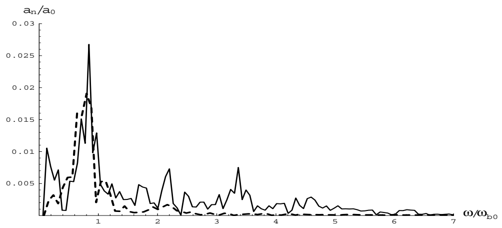

Figure 4: Fourier coefficients a_n/a_0 ($n \geq 1$) of $\omega_{\mathrm{b}}^2(t)$ when $38 \leq \gamma_{\mathrm{L}} t \leq 65$ for $N = 48000$ (continuous line) and $N = 768000$ (traits) particles, versus frequency.

is the same as for a pendulum, with variable gravity intensity and orientation. If V oscillates with a pulsation very different from ω_{b}, one can define an adiabatic invariant for the particle motion. The oscillations of V mainly affect the particles motion close to the instantaneous separatrix in (x, p).

We study the wave intensity for the TL case on two time intervals :

- T_1 : just after non linear saturation , $38 \leq \gamma_{\mathrm{L}} t \leq 65$,
- T_2 : much later, $250 \leq \gamma_{\mathrm{L}} t \leq 302$,

for $N = 48000$ and $N = 768000$ particles. The low frequency components associated to the slow growth are filtered. The Fourier decomposition (with $2\pi/T \ll \omega_{\mathrm{b}0}$ and $T = T_1$ or T_2) can be written

$$\omega_{\mathrm{b}}^2(t) = a_0 \left(1 + \sum_{n=1}^{\infty} \frac{a_n}{a_0} \cos(\omega_n t + \varphi_n) \right) \quad (6)$$

where

$$a_0 = \omega_{\mathrm{b}0}^2 = \frac{1}{T} \int_{t_0}^{t_0+T} \omega_{\mathrm{b}}^2(t) dt \quad (7)$$

and $\gamma_{\mathrm{L}} t_0 = 38$ or 250. Figs 4 and 5 show the coefficients a_n/a_0 versus ω normalized by $\omega_{\mathrm{b}0}$ (to eliminate N et T dependent scales).

In the T_1 window (Fig. 4), the time evolution appears to be dominated by a sharp frequency spectrum around the trapping frequency). This suggests that chaos for this system is mainly associated to the separatrix pulsation. There is no "second frequency", to which one could associate a second resonance inducing chaos for the particles by resonance overlap. Trapping-detrapping belongs to **slow chaos**. In the T_2 window (Fig. 5), both N values give more similar spectra, with a larger spread of frequencies and a weaker bump. Chaos is less powerful to mix particles. In the trapping oscillations regime, the

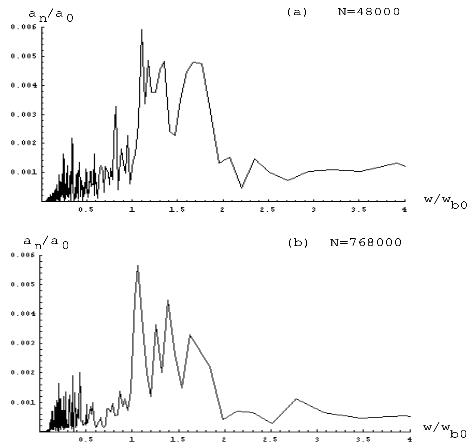

Figure 5: Fourier coefficients a_n/a_0 ($n \geq 1$) of $\omega_\mathrm{b}^2(t)$ when $250 \leq \gamma_\mathrm{L} t \leq 302$ for (a) $N = 48000$ et (b) $N = 768000$ particules, versus frequency.

anisochronicity of the trapped particles motion induces a filamentation of f. When filaments have a size lower than the grid, the numerical entropy grows, and the simulation looses the reversibility of the ideal vlasovian model: $|V|$ or ω_b saturate. But trapping-detrapping of particles is a chaotic process which is not described by the continuous ideal Vlasovian model and the secondary growth is missed.

To conclude, we have exhibited, for an elementary model of a basic wave-particle interaction phenomenon, finite N effects which do not result from numerical errors and are not present in numerical simulations of the kinetic approach. Their understanding implies the analysis of phase space geometry and the description of chaos in the particles dynamics. As a corollary, we cannot consider that the limits $t \to \infty$ and $N \to \infty$ generally commute.

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