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Fully kinetic theory of drift-mirror modes in space plasmas. Electron drift-mirror instability

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The main goal of the present paper is to develop a fully kinetic theory of drift-mirror instability accounting for the plasma inhomogeneity and finite electron temperature effects. We consider an inhomogeneous high-β plasma immersed in nonuniform external magnetic field $\mathbf{B}$. We introduce a local Cartesian system of coordinates with the z axis along the external magnetic field, the x axis in the direction opposite to the perpendicular gradient of this field, $-\nabla_\perp B$, and y axis to completing the triad. In a high-β plasma the magnetic field gradient $\kappa_B = d_x \ln B$ is directly connected to the plasma density gradient $\kappa_n = d_x \ln n$ via equilibrium condition $\kappa_B = -(\beta_\perp/2)\kappa_n$, where $\beta_\perp$ is the perpendicular plasma beta and n is the plasma density. Then, we assume that the wavelength in the $\hat{x}$-direction is smaller than the characteristic length of the plasma and magnetic field inhomogeneity. This allows the use of the WKB approximation. In the linear approximation, all perturbed quantities are assumed to vary as $\exp(-i\omega t + i\int k_x(x')dx' + ik_y y + ik_z z)$, where ω is the wave frequency and $\mathbf{k} = (k_x, k_y, k_z)$ is the wave vector. For simplicity, we neglect the curvature of the magnetic field lines choosing a straight magnetic field line geometry. The general kinetic dispersion relation $\Lambda_{\alpha\beta}\Psi_\beta = 0$, for low frequency, $\omega \ll \omega_{ci}$, oscillations (ω_{ci} is the ion cyclotron frequency) in nonuniform high-β plasma has been given by *Pokhotelov et al.* [1985]. Here $\Lambda_{\alpha\beta}$ is the dispersion tensor of the magnetoactive plasma with α, $\beta = 1, 2, 3$ and the vector Ψ_β is a column vector with components $\Psi_\beta = \left(\Psi_{\rm A}, \Psi_{\rm M}, \Psi_\parallel\right)$. The potentials $\Psi_{\rm A}$, $\Psi_{\rm M}$ and $\Psi_\parallel$ are defined by

$$\Psi_{\rm A} = -\frac{k_\perp}{\omega}\phi, \qquad \Psi_{\rm M} = \frac{(\mathbf{k}\times\mathbf{A})_z}{k_\perp} - \frac{ik_y\kappa_B}{k_\perp\omega}\phi, \qquad \Psi_\parallel = A_z - \frac{k_\parallel}{\omega}\phi, \tag{1}$$

where the indices A and M refer to the shear Alfvén and magnetosonic waves which are obtained in the low-β approximation. In high-β plasmas, the potentials $\Psi_{\rm A}$ and $\Psi_{\rm M}$ both comprise transverse and compressional oscillations of the magnetic field The vector and scalar potentials $\mathbf{A}$ and ϕ are connected with the wave electric and magnetic fields through the ordinary relations $\mathbf{E} = -i\mathbf{k}\phi + i\omega\mathbf{A}$, $\delta\mathbf{B} = i\mathbf{k} \times \mathbf{A}$ and the vector potential $\mathbf{A}$ satisfies the Coulomb gauge condition $\mathbf{k} \cdot \mathbf{A} = 0$.

By setting the determinant of the system to zero, det $\Lambda_{\alpha\beta} = 0$, we obtain the dispersion equation that describes the eigenmode spectrum of inhomogeneous plasma in a magnetic field. For mirror type perturbations with $k_\parallel \ll k_\perp$, and in the limit of small ion Larmor radius, $k_\perp \rho_i \ll 1$, the off-diagonal components of the dielectric tensor ε_{12} and ε_{21} are much smaller than ε_{22}. Accordingly, in the mirror approximation ($\omega \ll k_\parallel (n^{-1} < v_\parallel^2 F_i >)^{1/2}$) and after some algebraic calculations the following dispersion is obtained,

$$\mathcal{D}(\omega, \mathbf{k}) \equiv \Delta + \frac{\left[\sum_j q_j \left\langle \mu B \frac{\partial F_j}{\partial W} (\omega - \hat{\omega}_{j*}) \right\rangle\right]^2}{2p_{i\perp} \sum_j q_j^2 \left\langle \frac{\partial F_j}{\partial W} (\omega - \hat{\omega}_{j*})(\omega - \omega_{Dj}) \right\rangle}$$

$$-\frac{i\pi^2}{k_\parallel m_i p_{i\perp}} \int_0^\infty dW (W - W_0)^2 \frac{\partial F_{res}}{\partial W} (\omega - \hat{\omega}_{i*}) = 0, \tag{2}$$

which describes the drift mirror modes and may be applied to any particle distributions. The angular braces $\langle ... \rangle$ denote the averaging over the full velocity space, while we have passed from the variables $v_\parallel$ and $v_\perp$ to particle energy $W = m_j(v_\perp^2 + v_\parallel^2)/2$ and magnetic moment $\mu = m_j v_\perp^2 / 2B$. The term describing the resonant interaction of ions with the low frequency wave perturbations contain the derivative $\partial F_{res}/\partial W \equiv (\partial F_i/\partial W)_{\mu = W/B}$. The resonant particles do not move along the magnetic field line, while $v_\parallel = 0$, and the condition $\mu = W/B$ holds for them. The reference energy W_0 in Eq.(2) is given by

$$W_0 = \frac{(\omega - \omega_{Di}^{res}) e \sum_j q_j \left\langle \mu B \frac{\partial F_j}{\partial W} (\omega - \hat{\omega}_{j*}) \right\rangle}{\sum_j q_j^2 \left\langle \frac{\partial F_j}{\partial W} (\omega - \hat{\omega}_{j*})(\omega - \omega_{Dj}) \right\rangle}, \tag{3}$$

while $\hat{\omega}_{j*}$ stands for the diamagnetic drift frequency,

$$\hat{\omega}_{j*} = \frac{(\mathbf{k}_\perp \times \nabla F_j) \cdot \hat{\mathbf{z}}}{m_j \omega_{cj} \partial F_j / \partial W}. \tag{4}$$

and $\omega_{Di}^{res} \equiv \omega_{Di} \mid_{\mu = W/B}$ is the magnetic drift frequency of the resonant ions. The parameter Δ is given by $\Delta = A - {\beta_{i\perp}}^{-1} - \alpha$, where $\alpha = (k_\parallel^2/k_\perp^2)\beta_{i\perp}^{-1}[1 + (\beta_\perp - \beta_\parallel)/2]$ and A denotes the generalized anisotropy factor defined as

$$A = -\frac{1}{2p_{i\perp}} \sum_j \left\langle \mu^2 B^2 \frac{\partial F_j}{\partial W} \right\rangle - \frac{p_\perp}{p_{i\perp}}. \tag{5}$$

In what follows we analyze the general drift mirror (DM) dispersion relation (2) for the case of Maxwellian electron distribution F_e with an isotropic temperature T_e, and bi-Maxwellian ion distribution F_i with perpendicular and parallel temperatures $T_\perp$ and $T_\parallel$.

Hydrodynamic stability. First, we analyze the hydrodynamic (HD) stability of the plasma, as described by the first two terms on the left in Eq. (2). In this limit, the kinetic term being small as $\omega_n/k_\| v_{T_\|}$ is neglected and the dispersion relation receives the following solution

$$\omega_\pm = \omega_e \frac{\alpha \pm [\alpha K \Delta / P]^{1/2}}{K - \alpha} \quad \text{where} \quad \omega_e = \frac{\beta_\perp A T_e}{2(T_\| + T_e)}\omega_n \tag{6}$$

Here the quantity K is defined as $K = A - \beta_{i\perp}{}^{-1} - P$, the anisotropy factor A, identically equals to the ion temperature anisotropy factor A_i, i.e. $A \equiv A_i = T_\perp/T_\| - 1$ and P is given by $P = A^2 T_e T_\| / \left[2T_\perp(T_e + T_\|)\right]$. From Eq. (6a) it follows that in the range of ion temperature anisotropy bounded by $\beta_{i\perp}^{-1} + \alpha < A < A_{cr}$ plasma is hydrodynamically unstable. The value A_{cr} corresponds to the condition $K = 0$ and is given by

$$A_{cr} = \frac{\beta_{i\perp}^{-1} + \left[1 + \beta_{i\perp}^{-2} + 2\beta_{i\perp}^{-1} / \left(1 + T_e/T_\|\right)\right]^{1/2} - 1}{1 + \left(1 + T_e/T_\|\right)^{-1}}. \tag{7}$$

The unstable mode corresponds to the lower sign in Eq.(6) for negative values of K, in contrast to the ion DM instability which appears for $K \geq 0$. The maximum instability growth rate is attained at $\alpha = \alpha_{\max} = K(K+P)/(K-P)$ and is given by $\gamma_{\max} = \omega_e(K+P)/2P$. In order to outline the close association of this HD instability with the finite electron temperature and the plasma inhomogeneity we term it as hydrodynamic electron drift mirror instability (HEDM).

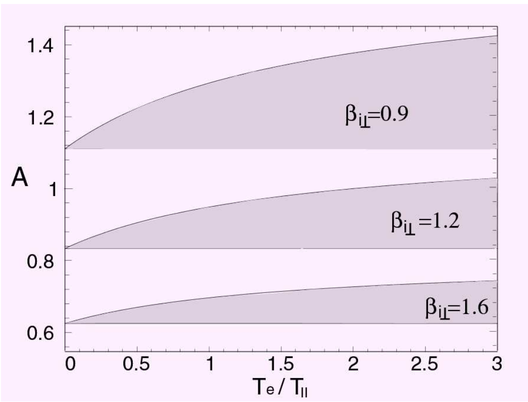

Figure 1: Regions of the HEDM instability. The instability region becomes wider with electron temperature increase and $\beta_{i\perp}$ decrease.

Kinetic stability. When $K > 0$, the plasma is hydrodynamically stable and the term in the dispersion relation, which describes the kinetic effects, should be retained. For the marginal conditions, $K \ll 1$, the dispersion relation obtains the form of a cubic equation for ω, describing three wave modes. In the limit of zero $k_{\|}$, it decouples into the zero frequency mode $\omega = 0$ and the ion DM instability. However, incorporation of finite $k_{\|}$ results in the appearance of an additional kinetic instability. The obtained equation, can be solved analytically in two different limits. For frequencies close to the kinetic ion DM frequency $\omega \simeq \omega_{DM}$, where

$$\omega_{DM} = \omega_n \left\{ 1 - \beta_\perp A \left[\frac{3}{2} + \frac{A(T_e/T_\perp)(1+T_e/T_{perp})}{(1+T_e/T_\|)^2 + (1+T_e/T_\perp)^2} \right] \right\}, \tag{8}$$

we obtain the solution

$$\omega = \omega_{DM} + \frac{i}{b} \left[K - \alpha \left(1 + \frac{\omega_e}{\omega_{DM}} \right)^2 \right], \tag{9}$$

describing the known kinetic ion DM instability which develops for $K > 0$, including the small correction term ω_e/ω_{DM}. The parameter b,

$$b = \frac{\sqrt{\pi}}{k_\| v_{T_\|}} \frac{T_\perp}{T_\|} \frac{(1+T_e/T_\|)^2 + (1+T_e/T_\perp)^2}{2(1+T_e/T_\|)^2}. \tag{10}$$

describes the leading order kinetic effects. In the low frequency limit, $\omega \ll (|\omega_{DM}|, |\omega_e|)$, we obtain the following solutions

$$\omega_\pm = \pm \frac{(\alpha/2)^{1/2}\omega_e}{(K^2 + b^2\omega_{DM}^2)^{1/2}} \left\{ \left[(K^2 + b^2\omega_{DM}^2)^{1/2} + K \right]^{1/2} + \right.$$
$$\left. i\mathrm{sign}(\omega_{DM}) \left[(K^2 + b^2\omega_{DM}^2)^{1/2} - K \right]^{1/2} \right\}. \tag{11}$$

Since $\omega_{DM} < 0$, it follows that the lower sign corresponds to an instability. This instability appears only in the presence of kinetic effects for inhomogeneous plasma with finite electron temperature. In order to distinguish it from the hydrodynamic instability, we term it *kinetic EDM instability*. For smaller values of $A - A_{cr} (> 0)$, the kinetic EDM is the dominant kinetic instability whereas, for higher values of $A - A_{cr}$, the ion kinetic DM instability dominates.

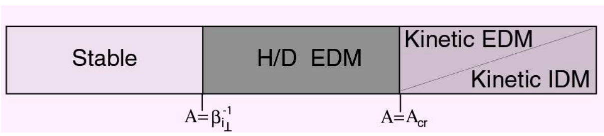

Figure 2: The drift mirror instabilities

References

Pokhotelov, O. A., V. A. Pilipenko, and E. Amata, Drift anisotropy instability of a finite β magnetospheric plasma, *Planet. Space. Sci.*, *33*, 1229, 1985.