Numerical Investigation of Anomalous Diffusion
in Nonuniform Plasma with Drift Waves

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A study of possible mechanism of an anomalous transport is carried out using the analysis of the dynamics of particles interacting with drift wave fluctuations. Estimates of the diffusion coefficient are based on the calculations of particle orbits.

In present work, estimations of anomalous diffusion/transport are based on detailed calculations of a particle motion a plasma. To use a such approach it is necessary to formulate special model of a non-stationary fields of an electrostatic/electromagnetic waves propagating in the plasma. The like approaches were discussed in Refs. [1, 2] for tokamak magnetic field. We consider a simple axisymmetric magnetic configuration (with magnetic field lines parallel to *z*-axis). The model under consideration takes into account only drift modes because drift waves are responsible for the anomalous transport [3].

In our model dispersion relations for the low-frequency drift (LFD) and lower hybrid drift (LHD) waves proceed from Refs. [4–6]. As parameters in dispersion relations depend on gradients of plasma density, plasma temperature, and magnetic field, we assume that plasma volume is divided on several layers transversally to gradients of the plasma parameters. In every layer gradients of corresponding parameters are supposed to be constants. Many modes of an electric wave potential in each layer propagate agreeing quasi-linear theory [7, 8]. As different modes have different phase velocities, different fluctuations of the electric field successive arise and decay in every layer. An interaction of plasma particles with strong fluctuations is the main cause of the anomalous transport in the framework of the proposed model.

In the presence of multimode perturbations, the motion of particles can become stochastic due to their interactions with fluctuations, in which case the particle confinement in a magnetic field is defined by collisionless diffusion [9–12].

Two dimensional (r, θ) configuration with magnetic field $B(r)$ depending on radius is considered. Wave vectors are directed along the azimuth θ. To obtain particle orbits the following dynamics equations are used:

$$m \, dv_r/dt = q \, [\tilde{E_r} + \mathrm{E_r}(\mathrm{r}) + v_\theta \, B], \tag{1}$$

$$m \, dv_\theta/dt = q \, (\tilde{E_\theta} - v_r \, B), \tag{2}$$

where m, q, v_r, v_θ are the mass, charge, radial and azimuthal velocities, respectively; $E_r(r)$ is the radial electrostatic field; $\tilde{E_\theta} = - (1/r) \, \partial\tilde{\varphi}/\partial\theta$, $\tilde{E_r} = - \partial\tilde{\varphi}/\partial r$ are the wave electric field components. Corresponding wave potential consists of many wave packets:

$$\tilde{\varphi} = \Sigma \varphi_{0s,n} \, g_s(r) \, \cos[n(\omega_s \, t - \theta) + \psi_{s,n}], \tag{3}$$

where s is the number of the packet, n is the azimuthal wave number, ω_s is the angular phase velocity, $g_s(r)$ is the radial profile of the packet, $\varphi_{0s,n}$ is the amplitude, and $\psi_{s,n}$ is the initial phase. The gradient of the magnetic field dB/dr is assumed to be a constant in a given plasma region. For these conditions the field reversal takes place if $dB/dr > B_0/a$, where $B_0 = B(a)$, a is the radius of a plasma.

In Figs. 1–3, examples of the results of the calculations of the ion dynamics under the action of LFD and LHD waves are presented. According the calculations for LFD and LHD waves the dependence of the coefficient of the anomalous diffusion is appears to be

$$D \sim \varepsilon^2 \, B^{-1}, \tag{4}$$

where $\varepsilon = |e\varphi_0|/(kT_e)$ is the relative level of the electric potential fluctuations, φ_0 is the order of maximum amplitude of the electric potential of the wave, e is the charge of the electron, k is the Boltzmann constant, T_e is the electron temperature.

From the consideration of single interaction of the particle with an electric field pulse the radial displacements of the particle can be calculated. Using these displacements and averaged time interval between two interactions one can estimate the diffusion coefficient in the stochastic regime [12], that gives for LFD and LHD the Bohm-like relation:

$$D \sim \varepsilon^2 \, k \, T_e \, (e \, B)^{-1}. \tag{5}$$

Note that the confinement time estimated from diffusion coefficient (5) agrees well with that previously suggested in Refs. [9, 11] for the field reversed magnetic configurations.

In conclusion, we have analysed the ion dynamics under the conditions of a stochastic regime of anomalous diffusion. Both qualitative estimates and computational results demonstrate the anomalous diffusion regime.
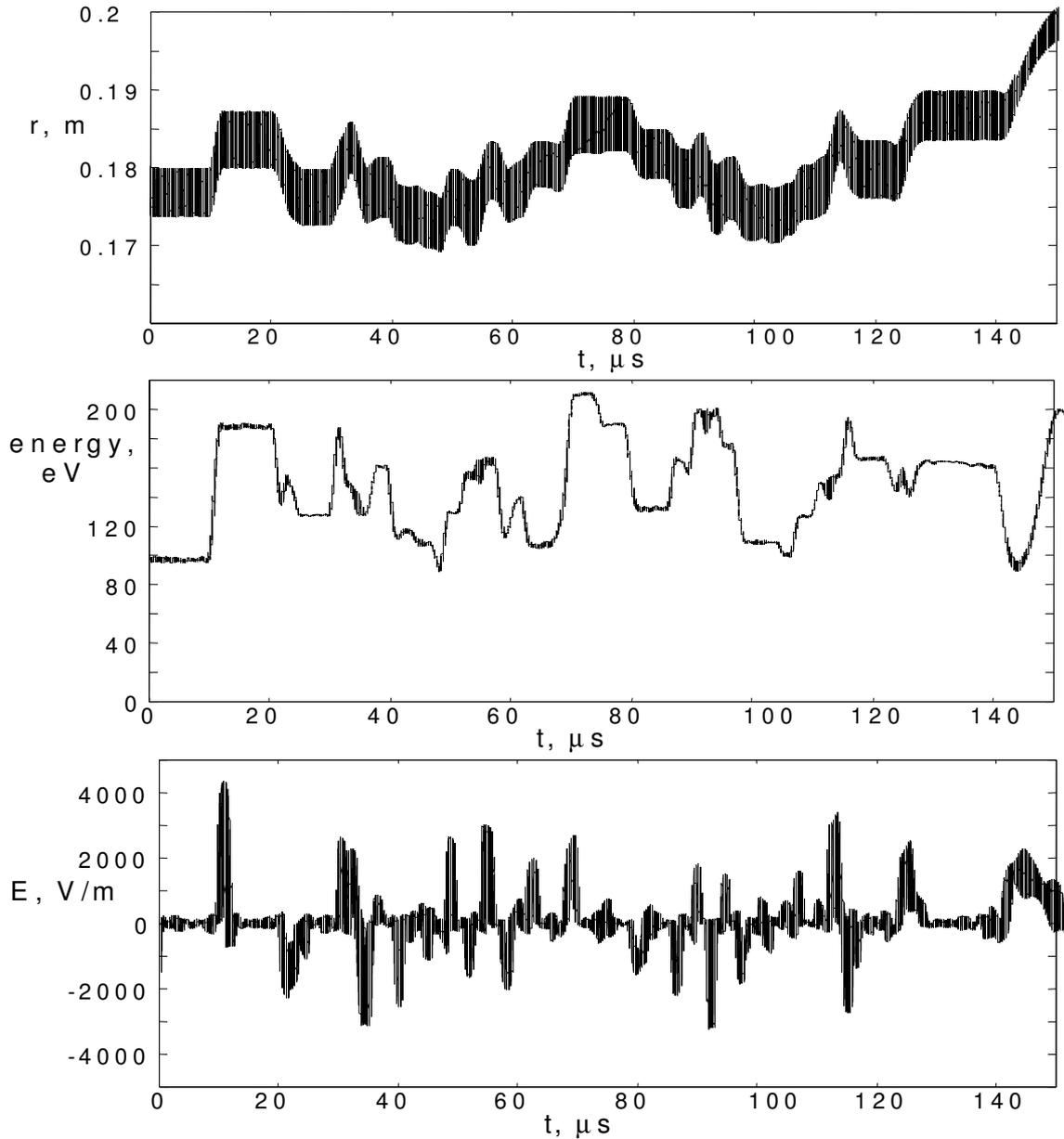
Fig. 1. Radial coordinate of the particle (proton), its energy, and the azimuthal component of the electric field acting on the particle (LFD). B_0=1 T, a=0.2 m, dB/dr=12 T/m, ε=0.05
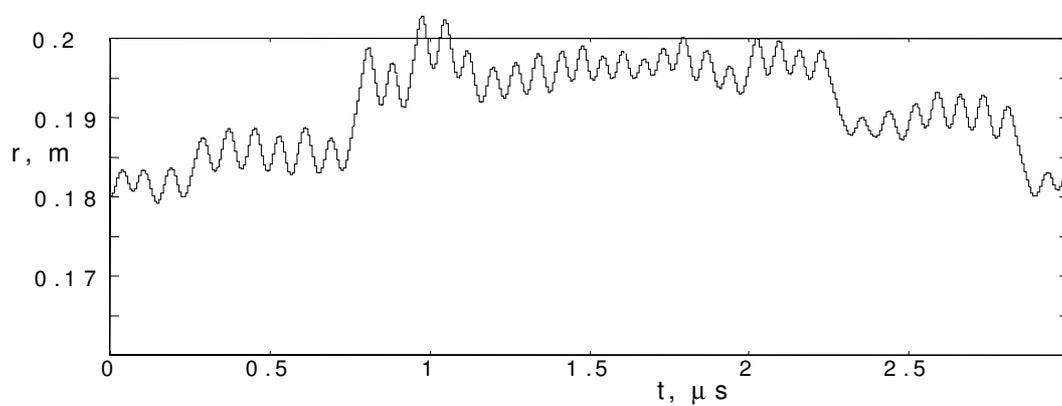

Fig. 2. Radial coordinate of the particle (proton) under the action of LHD-waves.
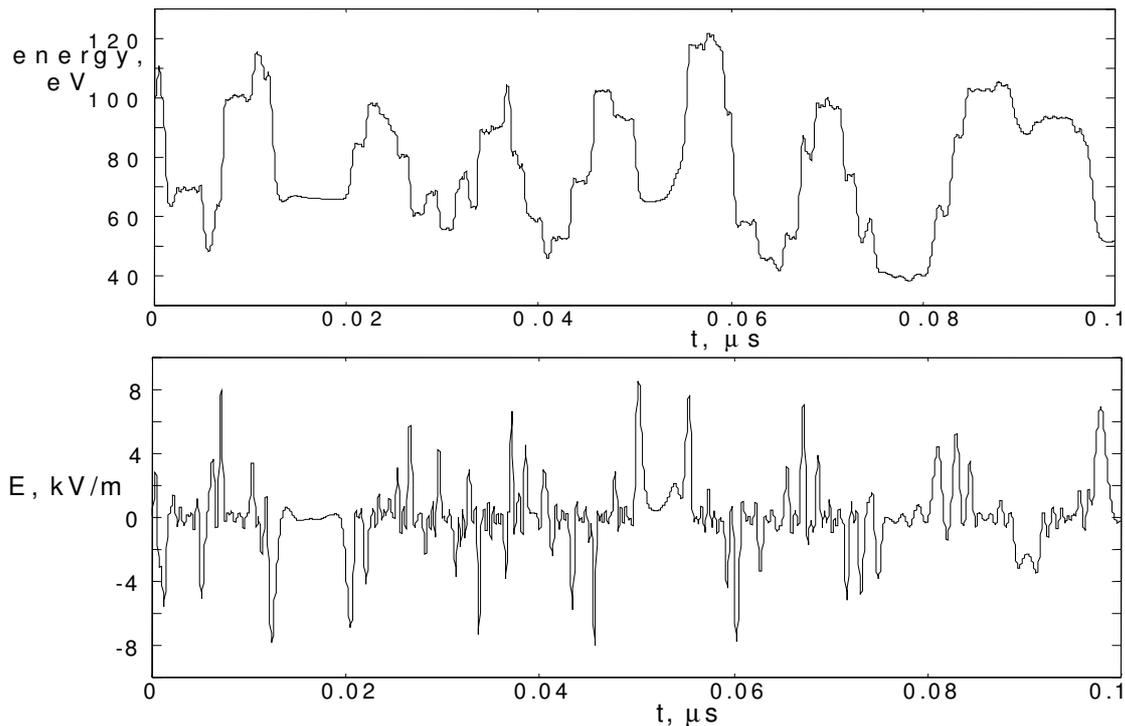B_0=1 T, a=0.2 m, dB/dr=12 T/m, ε=0.1

Fig. 3. Time dependencies of the particle energy, and the azimuthal component of the wave electric field acting on the particle during one gyroperiod for the condition of Fig. 2

Acknowledgements

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