Electron Bernstein Waves in Spherical Tori

A. K. Ram and A. Bers

Plasma Science & Fusion Center, M.I.T, Cambridge, MA 02139, U.S.A.

C. N. Lashmore-Davies

Culham Science Centre, UKAEA, Abingdon, Oxon OX14 3DB, U.K.

ABSTRACT

This paper summarizes the theoretical and numerical results we have obtained for the excitation and propagation of electron Bernstein waves in spherical tokamak plasmas.

INTRODUCTION

In the electron cyclotron range of frequencies the high-β NSTX and MAST plasmas are overdense to the traditional extraordinary X mode and/or the ordinary O mode. For low harmonics of the electron cyclotron frequency, the X and O modes are cutoff at the edge and for high harmonics the plasma is essentially optically thin to these modes. However, for low harmonics, the X mode and the O mode can mode convert to the electron Bernstein waves (EBW) which have no density cutoffs and can propagate into the core of the plasma. From ray tracing analysis we find that EBWs are locally and strongly absorbed at the Doppler shifted electron cyclotron resonance or its harmonics [1]. For spherical tokamak plasmas, the location of the EBW energy deposition is significantly different for excitations on or away from the equatorial plane [1].

EXCITATION AND PROPAGATION OF EBWs

The excitation of EBWs is studied in a slab geometry model where the x coordinate is in the direction of the inhomogeneity, y is along the poloidal direction, and z along the toroidal direction. Assuming that the magnetic field is along z, the propagation of waves in an inhomogeneous cold and collisionless plasma is given by

$$K_\perp \frac{d^2 E_y}{dx^2} + \frac{\omega^2}{c^2}\left(K_\perp^2 - K_X^2 - \frac{c^2}{\omega^2}k_\parallel^2 K_\perp\right) E_y = k_\parallel K_X F \tag{1}$$

$$\frac{d}{dx}\left[\frac{1}{K_\parallel}\frac{dF}{dx}\right] + \frac{\omega^2}{c^2}\left(1 - \frac{c^2}{\omega^2}\frac{k_\parallel^2}{K_\perp}\right) F = \frac{\omega^2}{c^2}\frac{K_X}{K_\perp}k_\parallel E_y \tag{2}$$

where $F = ik_\parallel E_x - dE_z/dx$, $dF/dx = (\omega^2/c^2)\,K_\parallel E_z$, c is the speed of light, ω is the wave frequency, $k_\parallel$ is the component of the wave vector $\vec{k}$ along the magnetic field $\vec{B} = B_0(x)\hat{z}$,

$$K_\perp = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2}, \quad K_X = -\frac{\omega_{ce}}{\omega}\left(\frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2}\right), \quad K_\parallel = 1 - \frac{\omega_{pe}^2}{\omega^2} \tag{3}$$

ω_{pe} and ω_{ce} are the electron plasma and cyclotron angular frequencies, respectively.

For the case when $k_\parallel = 0$, Eqs. (1) and (2) are uncoupled and describe the propagation of the X mode and the O mode, respectively, across the magnetic field. The X mode equation (1) has a resonance at $K_\perp = 0$ corresponding to the upper hybrid resonance (UHR). Eq. (1) also has two cutoffs corresponding to the zeros of $K_\perp^2 - K_X^2$. The resonance power absorption of the X mode at the UHR corresponds to the power mode converted to the EBWs [1]. Since the O mode is completely decoupled from the X mode, if an O mode is excited at the edge its power will be reflected back out at the O mode cutoff located where $K_\parallel = 0$. In this analysis we have ignored the effect of temperature on the propagation of waves and assumed that there is no electron cyclotron resonance between the edge of the plasma and the cutoffs of the X and O modes. Thus, the only absorption is at the UHR.

If we assume that, for small $n_\parallel = c k_\parallel / \omega$, only the X mode is excited at the plasma boundary, then the equation for X mode propagation is approximately given by

$$\left(K_\perp - n_\parallel^2\right) \frac{d^2 E_y}{dx^2} + \frac{\omega^2}{c^2} \left[\left(K_\perp - n_\parallel^2\right)^2 - K_X^2\right] E_y = 0 \ . \tag{4}$$

This equation is obtained from (1) and (2) by assuming that $E_z \approx 0$. In this approximation the X mode resonance is no longer at $K_\perp = 0$ but at $K_\perp - n_\parallel^2 = 0$ which, for small $n_\parallel$, is a small correction. From an analogous analysis carried out in [2], we find that the fraction of the power on the X mode that is converted to EBWs is given by

$$C_{XB} = 4 e^{-\pi \eta} \left(1 - e^{-\pi \eta}\right) \cos^2(\phi) \tag{5}$$

where

$$\phi = \frac{\pi}{2} + \frac{1}{2} \cos^{-1}\left(\frac{N_R^2 - N_I^2}{N_R^2 + N_I^2}\right) \tag{6}$$

$N = N_R + i N_I$,

$$\begin{aligned} N &= \ln\left(\frac{i\eta}{\eta_1}\right) + \pi \left[i \coth\left(\frac{\pi}{2}\eta\right) + \cot\left(\frac{\pi}{2}\eta_1\right)\right] - \frac{1}{\eta_1} - \frac{i}{\eta} \\ &\quad + \psi\left(1 + \frac{\eta_1}{2}\right) - \psi\left(1 + \frac{i\eta}{2}\right) \end{aligned} \tag{7}$$

ψ is the Psi function,

$$\eta = \left(1 - n_\parallel^2\right)^{3/2} \left[\frac{\omega_{ce} L_n}{c\alpha}\left(\sqrt{1 + \alpha^2} - 1\right)^2\right]_{UHR} \tag{8}$$

$$\eta_1 = \left(1 - n_\parallel^2\right)^{3/2} \left[\frac{\omega_{ce} L_n}{c\alpha}\left(\sqrt{1 + \alpha^2} + 1\right)^2\right]_{UHR} . \tag{9}$$

In Eqs. (8) and (9) the right-hand sides are evaluated at the UHR ($K_\perp = 0$), L_n is the density scalelength, and $\alpha = \omega_{pe}/\omega_{ce}$. If ϕ is zero or an integer multiple of π, then from

(5) we note that $C_{XB} \geq 0.5$ if $0.05 \lesssim \eta \lesssim 0.6$. For $n_\parallel = 0$, from (8) we find that the necessary condition for the X-B mode conversion to be better than 50% requires

$$0.013 \text{ T cm} \lesssim |BL_n|_{UHR} \lesssim 0.16 \text{ T cm} \qquad (10)$$

This shows that the X-B mode conversion process is efficient for sharp density gradients, i.e., short density scalelengths. As $n_\parallel$ is increased from zero, the fraction of the X mode power that is coupled to EBWs decreases.

From Eq. (2) it is clear that for $k_\parallel = 0$ the O mode does not couple power to the EBW. For the O mode power to couple to EBW it is necessary to have oblique propagation of the O mode, i.e., $k_\parallel \neq 0$. Then the O mode power can couple to the X mode which in turn can undergo resonance absorption. The necessary condition for optimum coupling of the O mode power to the X mode is [3]

$$n_{\parallel,c} = \frac{ck_\parallel}{\omega} = \left[\left(1 + \frac{\omega}{\omega_{ce}} \right)^{-1/2} \right]_c \qquad (11)$$

where the right-hand side is to be evaluated at the spatial location where $\omega = \omega_{pe}$. However, in order for the O mode power not to get reflected back out on the X mode requires that $\eta > 1$ [1, 4]. From (8) we find that this condition is satisfied when the density scalelength is long.

From the above discussion we conclude that the X-B and the O-X-B mode conversion processes are optimized in different regions of the two-dimensional parameter space spanned by wave frequency and parallel wavelength. The X-B mode conversion process is optimized for $k_\parallel \approx 0$ and wave frequencies which place the UHR in the short density scalelength region. The O-X-B mode conversion process is optimized for $k_\parallel$ given in (11) and for wave frequencies which place the UHR in a longer density scalelength region of the plasma. The optimum X-B process occurs closer to the edge of the plasma than the optimum O-X-B conversion. A more detailed analysis based on a kinetic treatment of the EBW in sheared magnetic fields confirms the basic results of the analysis given above [1, 4].

Figure 1 shows numerical results obtained from the kinetic theory for NSTX-type parameters [1, 4]. Figure 1(a) shows the case when both the X mode and the O mode are excited at the same frequency of 15 GHz. Figure 1(b) compares the case of optimum conversion from the X mode, for a wave frequency of 15 GHz, to the optimum conversion from the O mode for a wave frequency of 28 GHz. It is evident from these figures that the mode conversion to EBWs of the X mode and of the O mode are optimized for different ranges of the parameter space spanned by the wave frequency and parallel wave numbers.

We have also studied the propagation of EBW rays in model NSTX profiles. The ray trajectories are obtained in a fully kinetic (Maxwellian), non-relativistic, plasma. We
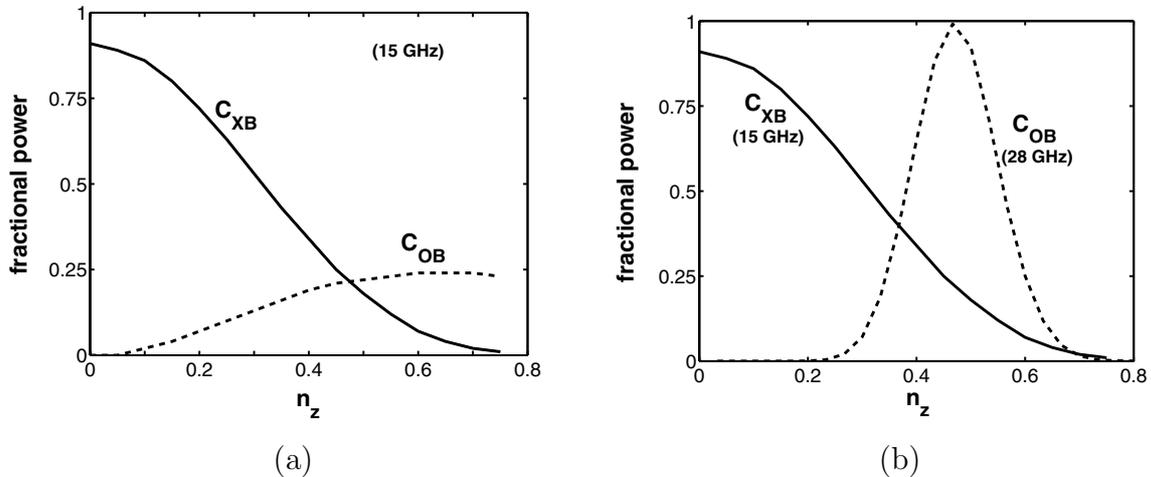
Figure 1: Fractional power mode converted to EBWs for an externally launched X mode, C_{XB}, or externally launched O mode, C_{OB}, as a function of n_z.

find that the EBW damps locally at the Doppler-shifted electron cyclotron resonance or its harmonics. The $k_\parallel$ along the ray trajectories vary over a large range if the rays are launched off the equatorial plane. This is due to the effect of a significant poloidal magnetic field [5]. The large $k_\parallel$ upshifts lead to an appreciable Doppler shift from the actual electron cyclotron resonance (or its harmonics) so that the damping could be far removed from the electron cyclotron resonance layer. In spite of the large increases in $k_\parallel$ the damping on electrons is still due to the Doppler-shifted cyclotron resonance rather than Landau damping. The plasma region in which the damping of the EBW is desired, either for heating or for driving plasma currents, can be controlled by an appropriate launching angle in the poloidal plane.

ACKNOWLEDGEMENTS

References

[1] A. K. Ram and S. D. Schultz, *Phys. Plasmas* **7**, 4084 (2000).

[2] A. K. Ram, A. Bers, S. D. Schultz, and V. Fuchs, *Phys. Plasmas* **3**, 1976 (1996).

[3] J. Preinhaelter and V. Kopecky, *J. Plas. Phys.* **10**, 1 (1973).

[4] A. K. Ram, A. Bers, and C. N. Lashmore-Davies, *Phys. Plasmas* **9**, 409 (2002).

[5] A. K. Ram and A. Bers, *Phys. Fluids B* **3**, 1059 (1991).