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Spin Stability of Asymmetrically Charged Plasma Dust

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Recently it has been reported that, under some circumstances, dust particles suspended in the sheath edge are observed to spin [1, 2, 3]. The present paper shows that there is a natural electrostatic mechanism that should cause even perfectly spherical particles in a perfectly irrotational, magnetic-field-free flowing plasma to spin. The stability criterion and the final spin state are obtained[4].

When the particle is made of an insulating material, it can support potential differences around its surface. In the limit of zero conductivity, and ignoring all charging effects other than electron or ion collection, the surface charge density accumulates in such a way as to bring the local electric current density to zero.

If the electron distribution is thermal, the electron flux density is given [5] by $\Gamma_e = n_\infty \sqrt{T_e/2\pi m_e} \exp(e\phi/T_e)$, where T_e, m_e, and n_∞ are the electron temperature, mass, and density far from the particle, and ϕ is the surface potential relative to infinity.

The ion flux to the spherical surface is asymmetric in a flowing plasma. The intuitive direction of asymmetry, which does not always occur [6], is that there should be greater ion flux density to the upstream side. In that case, as illustrated in Figure 1, the upstream side will acquire a less negative potential, and a less negative surface charge than the downstream side, causing an electric dipole moment, P that is opposite the direction of the flow, which we take to be the z-direction. Since the particle is on average negatively charged, and stationary within a plasma flowing past and giving rise to an ion drag force, it requires a compensating electric field E pointing in the direction of the flow. An electric dipole pointing opposite to an external electric field is unstable with respect to rotation. This is the underlying mechanism discussed here.

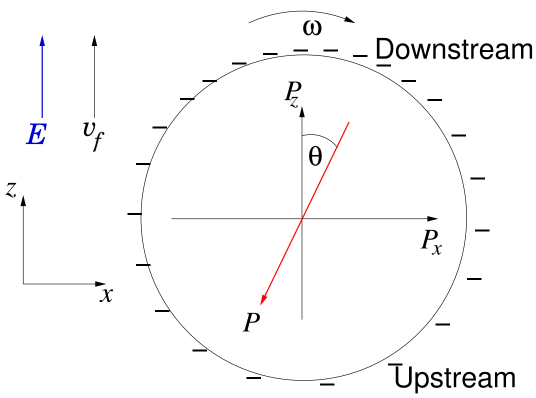

Figure 1: Schematic illustration of the charged sphere and dipole moments.

This spherical particle is not a dipole of fixed magnitude, however. Its surface charge is determined dynamically. If it starts to spin, its surface charge will no longer be in current-balance, so the dipole moments will evolve with time. If we suppose the y-axis to be chosen along the axis of rotation, then the important dipole components are in the z- and x- directions (P_z, P_x), and can be adequately described by the following coupled equations.

$$\frac{dP_x}{dt} + \frac{P_x}{\tau_c} = \omega P_z \,, \qquad \frac{dP_z}{dt} + \frac{P_z - P_o}{\tau_c} = -\omega P_x \,, \tag{1}$$

where ω is the instantaneous angular spin rate, P_0 is the natural dipole moment for a non-spinning sphere, negative in value when the upstream side is more positive, and τ_c is the characteristic charging time of the dipolar component of the charge on the sphere.

If the particle experiences a drag torque from ion or atom impacts that gives rise to a spin damping time-constant τ_d, then its equation of motion is

$$\frac{d\omega}{dt} + \frac{\omega}{\tau_d} = -\frac{EP_x}{I} \tag{2}$$

where I is the moment of inertia.

The stability of this system may be determined by noting that the steady state ($\omega =$ constant, $d/dt = 0$) solution of eqs (1) is

$$P_z = P_0/(1+\omega^2\tau_c^2) \ , \qquad P_x = P_0\omega\tau_c/(1+\omega^2\tau_c^2) \tag{3}$$

showing that the linear (low spin velocity) approximation for P_x is $P_x \approx P_0\omega\tau_c$. Substituting into eq(2) we obtain

$$\frac{d\omega}{dt} = \left[\frac{\tau_c}{\tau_r^2} - \frac{1}{\tau_d}\right]\omega \ , \tag{4}$$

where the rotational time constant is defined by $\tau_r^2 \equiv -I/EP_0$. Equation (4) shows immediately that the state $\omega = 0$ is unstable and spontaneous spin up will occur if

$$\tau_c/\tau_r^2 > 1/\tau_d. \tag{5}$$

The spin velocity will saturate when the equation of motion eq (2) is satisfied with $d\omega/dt = 0$. Substituting for P_x from eqs (3), immediately yields the final state:

$$\omega = \frac{1}{\tau_c}\sqrt{\frac{\tau_d\tau_c}{\tau_r^2} - 1}. \tag{6}$$

To relate the *charging time-constant* to plasma parameters, we write the time derivative of the surface potential as being the current density divided by the capacitance per unit area:

$$\frac{d\phi}{dt} \ = \ \frac{-en_\infty}{C_d}\left(\frac{T_e}{2\pi m_e}\right)^{1/2}\left[\exp\left(\frac{e(\phi-\phi_f)}{T_e}\right) - 1\right]\exp\left(\frac{e\phi_f}{T_e}\right), \tag{7}$$

where ϕ_f is the (constant) equilibrium floating potential for this surface element. The capacitance per unit area, $C_d = \sigma/\phi$, to be precise, is the capacitance attributable to a dipolar (first spherical harmonic) distribution of surface charge density σ over the sphere. For a sphere of radius a in vacuum, this capacitance may be shown to be $3\epsilon_0/a$, while for a sphere shielded by a short Debye length plasma ($\lambda_D \ll a$) it is ϵ_0/λ_D. Therefore a reasonable approximation is to take $C_d = \epsilon_0(1/\lambda_D + 3/a) \equiv \epsilon_0/l_c$.

Expanding the exponential in the vicinity of $\phi - \phi_f = 0$, then shows

$$\frac{1}{\tau_c} \ \equiv \ \frac{d\ln(\phi-\phi_f)}{dt} = \frac{1}{\sqrt{2\pi}}\frac{l_c v_{te}}{\lambda_{De}^2}\exp\left(\frac{e\phi_f}{T_e}\right), \tag{8}$$

The *drag time-constant*, τ_d, may be approximated by assuming that ions or atoms that collide with the particle are reemitted with angular velocity on average equal to that of the sphere surface. Therefore the rate of loss of angular momentum is equal to mass flux to the sphere, F, times $(2/3)a^2\omega$. (The surface averaging factor 2/3 is appropriate for a uniform flux, but may be slightly altered for non-unform.) Therefore

$$\frac{1}{\tau_d} = \frac{F2a^2/3}{I} \ . \tag{9}$$

The *electric dipole rotation time-constant*, τ_r, is determined by the magnitude of the electric field in which the particle resides, as well as the dipole moment.

The electric field must provide just sufficient force to balance the drag of the plasma particles (ions and neutrals mostly) upon the sphere. But, in calculating the dipole's stability we must use an electric field that provides a force equal to only the momentum collection rate of ions and neutrals. Therefore, writing the monopolar capacitance per unit area of the sphere as C_m (approximately $\epsilon_o[1/\lambda_D + 1/a]$), we find that the local electric field acting acting on the particle is required to be

$$E = -\frac{Fv_f}{\phi_f C_m 4\pi a^2} \ , \tag{10}$$

where $\phi_f C_m 4\pi a^2$ is the total particle charge.

It has been shown [6] that one can write the ratio of the upstream to downstream flux as $\exp(KM)$, where, $M \equiv v_f/\sqrt{T_e/m_i}$ is the Mach number, and K is a calibration factor of order unity (but whose precise value depends on plasma conditions). To compensate such a flux variation requires a cosine potential variation of amplitude $\phi_1 = \frac{T_e}{e}\frac{KM}{2}$, leading to an electric dipole moment

$$P_0 = -C_d \frac{T_e}{e}\frac{KM}{2}\frac{4}{3}\pi a^3 \ . \tag{11}$$

Substituting eqs (10) and (11) into the definition of τ_r yields

$$\frac{\tau_r^2}{\tau_d} = \left(\frac{4e|\phi_f|}{T_e}\frac{C_m}{C_d}\right)\frac{1}{KM^2}\frac{a}{\sqrt{T_e/m_i}} \ . \tag{12}$$

Notice that the sphere inertia has cancelled out of this expression. It only serves to determine the overall timescale of the problem. But notice, more importantly, that the mass flux to the sphere has also cancelled out, because the local driving electric field is proportional to drag, as well as the spin damping, and both are proportional to mass flux. We denote the coefficient in parentheses by ζ, which is typically ~ 4.

Using these expressions, we may write the instability criterion as

$$\frac{\lambda_{De}^2}{l_c a} > \frac{1}{M^2}\frac{\zeta}{K}\sqrt{\frac{m_i}{2\pi m_e}}\exp(e\phi_f/T_e) \ . \tag{13}$$

As is well known[5], the floating potential (here an average value over the sphere) is typically between 2 and 3 times T_e/e, assuming no electron emission. Spin instability usually requires the Debye length to be somewhat greater than the sphere radius. In that limit, we can approximate $l_c \approx a/3$, $C_m/C_d \approx 1/3$; arriving at a simplified approximate criterion

$$\frac{\lambda_{De}}{a} \gtrsim \frac{1}{M\sqrt{K}} \ . \tag{14}$$

If K were positive, this criterion would be satisfied by most experiments, since most use particulate sizes substantially smaller than the Debye length. However, recent detailed self-consistent collisionless calculations[6] have shown that if the ion to electron temperature ratio is small, as it often is in experiments, then contrary to intuition, the value of K is *negative* for a wide range of values of λ_{De}. Negative values of K are always stable, and no particle spin then occurs. To obtain positive K, according to [6], requires either $T_i \sim T_e$ or $\lambda_{De} \gtrsim 20a$ (at $T_i = 0.1T_e$).

Employing the same approximations, the steady state spin frequency in an unstable case is

$$\omega \approx \frac{a}{3\lambda_{De}} \, \omega_{pi} \sqrt{\frac{\lambda_{De}^2 K M^2}{a^2} - 1}. \tag{15}$$

In a typical low density laboratory experiment, for example in an argon plasma, if $T_e = 3$ eV, $n_e = 10^{14}$ m^{-3}, then $\lambda_{De} \approx 1$ mm, and $\omega_{pi} \approx 2 \times 10^6$ s^{-1}. The predicted rotation frequency for small particles when $K \approx 1$ is then approximately $6 \times 10^5 M$ s^{-1}, which for any appreciable value of the Mach number (M) is much larger than the values (10 to 30 Hz) that have been reported [3] as occurring in some experiments. It is therefore not possible to conclude that the mechanism in the form discussed here is responsible for those observations. One simple modification of the theory is to take the neutrals to have zero drift with respect to the sphere, then the effective value of the τ_r^2 would be increased by the ratio $(F_n + F_i)/F_i$ of the total (neutral plus ion) flux to the ion flux. That modification is equivalent, in the above formulas, to multiplying K by the factor $F_i/(F_n + F_i)$.

In summary, an isolated spherical insulating particulate suspended by an electric field in a flowing plasma is predicted to exhibit an instability that causes it to spin. The theoretical instability criterion derived is that the Debye length should exceed approximately the probe radius divided by the Mach number, when the ion flux asymmetry has its intuitive direction and size. The saturated state is then that the particle rotates at a fraction of the ion plasma frequency. However, for the conditions of many experiments, intuition is incorrect, and the direction of charge asymmetry is such as to yield stability. Therefore, one should regard the presence or absence of particulate spin as a diagnostic of the charging asymmetry, since that asymmetry seems likely to be the most uncertain factor in practice.

References

[1] B.T. Draine and A. Lazarian. *Ap. J.*, 508:157, 1998.

[2] O. Ishihara and Sato N. *IEEE Trans. Plasma Sci.*, 29:179, 2001.

[3] V.N. Tsytovich, N. Sato, and G.E. Morfill. *New J. Phys.*, 5:43, 2003.

[4] I.H. Hutchinson. *New J. Phys.*, 6:43, 2004.

[5] I.H. Hutchinson. *Principles of Plasma Diagnostics.* Cambridge University Press, Cambridge, 2nd edition, 2002.

[6] I.H. Hutchinson. *Plasma Phys. Control. Fusion*, 45:1477, 2003.