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Hydrodynamic instabilities in cylindrical geometry Self-similar models and numerical simulations

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Hydrodynamic instabilities play an important role in the target compression for inertial confinement fusion (ICF). We present the analytical solution of a perturbated isentropic implosion. We compare the analytical solution to the results obtained with perturbation and 2D Lagrangian hydrodynamic codes.

1 Introduction

Performance of direct drive high yield targets may be severely limited by hydrodynamic instabilities. Implosion process is generally achieved with a sequence of shocks emerging from the ablator to the DT in the target. It is known that in order to optimize the compression of hot spot at the end of implosion process, a specific sequence of shocks is required, which must be as close as possible to an isentropic compression. The quality hot spot is also limited by the growth of Rayleigh-Taylor instabilities around the ablator interface. In order to understand and modelize such instabilities, two complementary approaches are used in this paper. The first one is the modelization of both base and perturbed flows in the linear regime by a self-similar approach, the second one is the validation of numerical methods devoted to ICF applications on such flows. We will present analytical solutions for base flow and for the perturbations in a cylindrical geometry. These solutions will allow us to understand the effects of aspect ratio and compressibility during implosion. Then we will focuse on two numerical methods. They are both based on the Lagrangian formalism for hydrodynamic equations. The first one is dedicated to the study of linear stability of unsteady complex flows. The code calculates the basic one-dimensional solution and first order three-dimensional perturbations. It is a so-called perturbation code, named PERLE. The second one is a two-dimensional Lagrangian hydro-code, named CHIC, based on a new cell-centered scheme with a Godunov type solver.

2 An analytical model for linear perturbations of Kidder's self-similar solution

We briefly recall the main features of Kidder's isentropic implosion of perfect gaz in cylindrical geometry [1] which is a special case of selfsimilar cumulative flows in converges geometries. This 1D solution will be used as our base flow. Let $R(r,t)$ be the radius of a fluid particle at time t which was initially at r. We build the self-similar solution using $R(r,t) = h(t)\ r$ and $P = s\ \rho^\gamma$ where s is a constant and $h(t)$ a function to determine. Finally after variable separation one obtains two ODE whose solution are $h(t) = \sqrt{1 - \left(\frac{t}{\tau}\right)^2}$ and $\tau = \sqrt{\frac{\gamma-1}{2}\frac{r_2^2-r_1^2}{c_2^2-c_1^2}}$, where τ is the collapse time, r_1 and r_2 are the internal and external radii, c_1 and c_2 are the isentropic sound speeds located at r_1 and r_2. The initial density profile are given by (for $r \in [r_1, r_2]$):

$$\rho^0(r) = \left(\frac{r_2^2 - r^2}{r_2^2 - r_1^2}\rho_1^{\gamma-1} + \frac{r^2 - r_1^2}{r_2^2 - r_1^2}\rho_2^{\gamma-1}\right)^{\frac{1}{\gamma-1}}.$$

The isentropic compression is obtained by imposing the following pressure laws at the internal and external interfaces of the domain :

$$\begin{cases} P(R(r_1,t),t) = P_1 h(t)^{-\frac{2\gamma}{\gamma-1}}, \\ P(R(r_2,t),t) = P_2 h(t)^{-\frac{2\gamma}{\gamma-1}}. \end{cases}$$

This fully analytical solution is obtained under the assumption that $\gamma = 1 + 2/\nu$ where $\nu = 1, 2, 3$ whether we have a planar, cylindrical or spherical symmetry. Here we choose $\gamma = 2$. We are now going to perturb the internal and external faces of the shell in order to study hydrodynamical instabilities [2,3,4]. Let $\overrightarrow{\xi}(\overrightarrow{r}, t)$ be a small displacement around the unperturbed eulerian trajectory $\overrightarrow{R}(\overrightarrow{r}, t)$. By linearising Euler equations around the base motion, one obtains to first order in $\overrightarrow{\xi}(\overrightarrow{r}, t)$:

$$(\mathcal{L}) \begin{cases} h^4(t)\tau^2 \dfrac{d^2\overrightarrow{\xi}}{dt^2} + \overrightarrow{r}\,\mathrm{div_r}\overrightarrow{\xi} - \dfrac{\gamma\tau^2}{\rho^0}\mathrm{grad_r}\left(P^0 \mathrm{div_r}\overrightarrow{\xi}\right) \\ -\,\mathrm{grad_r}\overrightarrow{\xi}\cdot\overrightarrow{r} - \overrightarrow{r}\times\mathrm{rot_r}\overrightarrow{\xi} = \overrightarrow{0}, \end{cases}$$

where $\overrightarrow{r}$ denotes the lagrangian position vector, ρ^0 and P^0 are density and pressure of the base motion. In order to solve $(\mathcal{L})$, let us introduce $\overrightarrow{\xi}(\overrightarrow{r}, t) = \overrightarrow{X}(\overrightarrow{r})G(t)$. After some rearrangement, we find for $t \in [0, \tau[$ and $r \in [r_1, r_2]$:

$$(\mathcal{T})\ \ h^4(t)\tau^2\frac{d^2G}{dt^2} + \mu G(t) = 0,$$

$$(\mathcal{S})\ \ (\mu - 1)\overrightarrow{X} + (\gamma - 1)\sigma\overrightarrow{r} + \frac{\gamma\tau^2 P}{\rho}\mathrm{grad_r}\sigma + \mathrm{grad_r}(\overrightarrow{r}\cdot\overrightarrow{X}) = \overrightarrow{0},$$

where $\sigma = \mathrm{div_r}\overrightarrow{X}$ and μ is a separation constant. We impose the initial values $G(0) = 1$ and $\frac{dG}{dt}(0) = 0$, then $(\mathcal{S})$ is solved under incompressibility assumption, i.e. $\sigma = 0$, therefore thanks to $(\mathcal{S})$, $\mathrm{rot}_r\overrightarrow{X} = \overrightarrow{0}$, hence $\overrightarrow{X} = \mathrm{grad}_r\Phi$ where $\Phi(r,\theta)$ satisfies the Laplace equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2} = 0,$$

which admits the following two independant solutions : $\Phi_1 = r^{-n}\cos(n\theta)$ and $\Phi_2 = r^n\cos(n\theta)$ with $n \in \mathbb{N}$. Then we define the two fundamental solutions of $(\mathcal{S})$: $\overrightarrow{X}_1 = \mathrm{grad}_r\Phi_1$ with $\mu_1 = n+1$ and $\overrightarrow{X}_2 = \mathrm{grad}_r\Phi_2$ with $\mu_2 = -n+1$. After time scaling $(t \to t/\tau)$, we solve the two evolution equations for G_1 with μ_1 and for G_2 with μ_2 :

$$\begin{cases} G_1(t) = \sqrt{1-t^2}\cos\left[\dfrac{\sqrt{n}}{2}\log\left(\dfrac{1-t}{1+t}\right)\right], \\ G_2(t) = \dfrac{1}{2}\sqrt{1-t^2}\left[\left(\dfrac{1-t}{1+t}\right)^{\frac{\sqrt{n}}{2}} + \left(\dfrac{1+t}{1-t}\right)^{\frac{\sqrt{n}}{2}}\right]. \end{cases}$$

We notice that $\lim_{t\to1^-} G_1(t) = 0$ and for $n > 1$ $\lim_{t\to1^-} G_2(t) = +\infty$ with $G_2(t) \sim 2^{\frac{\sqrt{n}-1}{2}}(1-t)^{\frac{1-\sqrt{n}}{2}}$ for $t \to 1^-$. The time evolution of amplification for the point with initial radius r and $\theta = 0$ is given by

$$\xi_r(r,t) = n\left(-A_1 r^{-n-1}G_1(t) + A_2 r^{n-1}G_2(t)\right),$$

where the constants A_1, A_2 are determined by the initial levels of perturbations ξ_1 and ξ_2 at r_1 and r_2, following

$$\begin{cases} -\,n r_1^{-n-1}A_1 + n r_1^{n-1}A_2 = \xi_1, \\ -\,n r_2^{-n-1}A_1 + n r_2^{n-1}A_2 = \xi_2. \end{cases}$$

3 Perturbation code PERLE

This tool is dedicated to the study of linear stability. We consider the base one-dimensional solution and its first order 3D perturbation in the lagrangian formalism. After a Fourier decomposition versus transverse space variables, the 3D problem reduces to the resolution of a 1D system for each wavelength. Thus we directly calculate the 1D base flow and the perturbation amplitude. This concept has been known since the 70's [5]. Here we use recent methods developped at the end of 90's, based on Godunov type schemes (see for example [6]). No specific assumption on the base flow except symmetry allow us to consider unsteady complex dynamics.

4 Hydrodynamic code CHIC

The numerical method in CHIC is based on a new scheme. This scheme uses a cell centered dicretisation based on a total energy formulation. The vertices velocity and the fluxes are evaluated using a nodal solver. This solver has been built from conservation arguments and one entropy inequality. It can be viewed as a two-dimensional extension of the acoustic Godunov's solver. This method fully answers an important question conserning Lagrangian hydrodynamic : one can compute in a coherent way, the displacement of vertices as well as the face fluxes. This method is currently the subject of a publication [7].

5 Numerical Results

We now present results for the following cases : mode $n = 8$ with a perturbation of the external interface and, mode $n = 8$ with a perturbation of the internal and external surfaces. In all the cases we use $a_0 = 10^{-6}$, $r_1 = 0.9$, $r_2 = 1$, $P_1 = 0.1$, $P_2 = 10$ and $\rho_2 = 10^{-2}$. From this we get $\rho_1 = 10^{-3}$, $s = 10^5$ and $\tau = 7.265\ 10^{-3}$. The computational domain is defined using the symetries of the mode 8 i.e. $(r, \theta) \in [r_1, r_2] \times [0, \frac{\pi}{2}]$. We use equiangular meshes with $(n_x, n_y) = (25, 44)$, $(n_x, n_y) = (50, 88)$ and $(n_x, n_y) = (100, 176)$. For the perturbation code, we present converged calculations. Figures 1 and 2 show the results and we can see the good agreement beetween all the solutions.

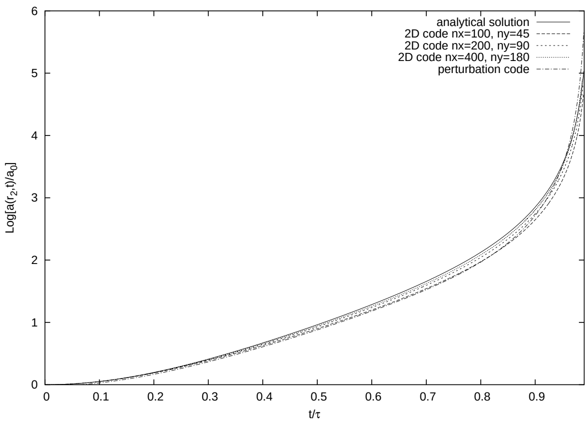

Figure 1: $\xi_1 = 0$, $\xi_2 = 1$

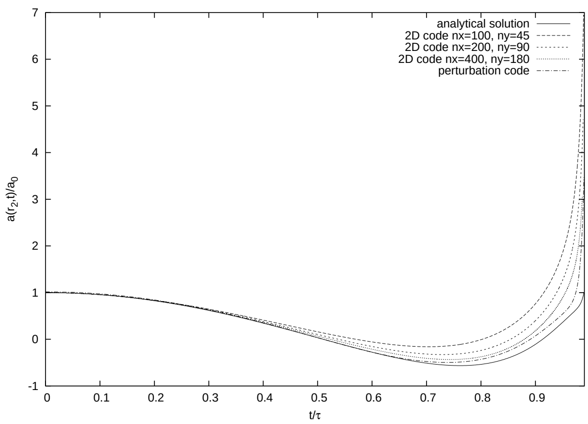

Figure 2: $\xi_1 = 2.565293$, $\xi_2 = 1$

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