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2-D Vlasov Simulations and Theory of Sheared Electron Instabilities in Strongly Magnetized Plasmas

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1. INTRODUCTION

Shear is often present in the velocities of particle streams in space and laboratory plasmas (Gavrishchaka, et al, PRL, **85**, 4285, 2000).

In the first part of this paper we present 2-D Vlasov simulations together with a supporting linear analytic theory for the kinetic instability of equal and opposite electron streams parallel and antiparallel to a strong background magnetic field, *B*. The magnitude of the initially equal and opposite electron stream velocities is taken to vary sinusoidally with position across magnetic field lines. The neutralizing ions are static and the plasma is current-free. We shall refer to this as a *sheared symmetric electron 2-stream instability*. A linear stability analysis is derived based on the eigenvalues and eigenfunctions of Poisson's eqn for the electrostatic potential. Poisson's eqn is shown to be a Mathieu eqn in the limit of weak shear. Potential eigenfunctions and temporal growth rate eigenvalues obtained from the Mathieu eqn are compared with numerical results from the simulations. The fastest growth rates and associated potential eigenfunctions compare well in the limit of weak shear. The simulation reveals significant differences between the nonlinear evolution of the sheared and unsheared 2-stream instabilities. (These results can also be found in Goldman et al, to be published in *Transport Theory and Statistical Physics*)

In the second part of this presentation we study the effects of shear in an *asymmetric (Buneman) instability with sheared electrons.* Here, a sheared stream of hot electrons drifts in the parallel direction relative to cold unmagnetized unsheared dynamic ions. The drift varies sinusoidally with position across magnetic field lines as in the sheared symmetric electron 2-stream instability. However in the Buneman case a current is present. Once again, this system can be modeled theoretically by reducing Poisson's equation for the electrostatic potential to a Mathieu equation in the limit of weak shear. The growth rate and parallel wavenumber of the fastest growing eigenmode (ground state) and eigenfunction are evaluated and are found to compare favorably with preliminary 2-D Vlasov simulations featuring strongly magnetized electrons and unmagnetized ions.

2. THE APPROXIMATION OF STRONGLY MAGNETIZED ELECTRONS

In theory and simulation of both instabilities a central approximation made in this paper is that the electrons are taken as strongly magnetized. In this limit the electron gyroradius is assumed small compared to the Debye length, or, equivalently the electron cyclotron frequency is assumed large compared to the plasma frequency. This ordering is well satisfied in the auroral ionosphere, for example, where the electron cyclotron can be between 5 and 15 times the electron plasma frequency. In the 2-D strongly magnetized approximation the electron Vlasov eqn depends *explicitly* only on the parallel coordinate and velocity, z and v_z, but depends *implicitly* on the perpendicular coordinate, y through the electrostatic potential, ϕ. Poisson's eqn is fully two-dimensional, depending on both z and y.

The strongly magnetized electron approximation has been used successfully to model *unsheared* electron 2-stream instabilities measured on the FAST satellite in the auroral ionosphere (Newman, et al, *Phys. Rev. Lett.*, 87:255001:1–4 (2001); Oppenheim, et al, *Phys. Rev. Lett.*, 83:2344–2347, (1999)).

3. SHEARED SYMMETRIC ELECTRON 2-STREAM INSTABILITY

3.1 Simulations of the sheared symmetric electron 2-stream instability

In 2-D periodic Vlasov simulations the ions are taken as static and two initial electron beams, are taken to be parallel and antiparallel to **B**, each with sheared velocity, $\pm v_b(y)/2 = \pm(v_o/2)[1-\varepsilon\cdot\cos(2\pi y/L_y)]$. L_y is the length of the simulation box in the y-direction (across **B**), and $\varepsilon = 0.33$ is the so-called shear parameter. Each electron beam is modeled by a κ distribution, with $\kappa = 1$ and v_e is the beams' common thermal velocity = $v_0/7.5$ and $n_\pm$ the (same) density of each beam. The κ distributions are of the following form

$$(1) \qquad f_\pm(v) = \frac{2n_\pm}{\pi v_e}\left[1+\left(\frac{v \pm v_0(y)/2}{v_e}\right)^2\right]^{-2}$$

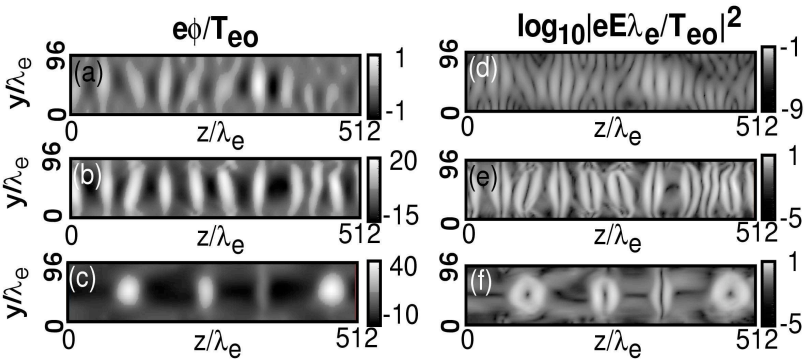

Figure 1: Snapshots of the electrostatic potential ϕ (a–c) and electric field intensity (d–f) at three times during the evolution of a simulation initialized with an *unsheared* two-stream distribution: (a) and (d) linear phase at $\omega_e t = 12$; (b) and (e) early nonlinear phase at $\omega_e t = 38$; (c) and (f) late nonlinear phase at $\omega_e t = 500$.

The results of the simulation are shown in Figure 1 at left. The potential and the electric field energy density are shown as functions of z and y. The earliest times show the linear phase of the two-stream instability. The potential structures fall away at the y-edges of the simulation cell. At later times the sausage-shaped tubes break up and perpendicular structures appear which are interpreted as lower hybrid waves. Also seen are potential holes — characteristic of phase space holes and bipolar structures. The periodicity in the parallel direction in (a) corresponds to dimensionless parallel wavenumber $K_z = k_z v_0/2\omega_e \approx 0.6$ and the growth rate associated with initial noise symmetric in y is $\gamma \approx 0.32\ \omega_e$.

3.2 Theory of the sheared symmetric electron 2-stream instability

A theoretical description of the sheared 2-stream instability can be constructed by linearizing Poisson's equation using the distribution functions in eqn. (1) and expanding in the shear parameter ε. The result is a *Mathieu eqn.* for the potential, $\phi(K_z,y)$:

$$(2) \qquad \left[\frac{d^2}{d\theta^2} + a - 2q\cos(2\theta)\right]\varphi(\theta) = 0, \qquad \theta \equiv 2\pi\left[y/L_y - 0.5\right]$$

The coefficients a and q are complex functions of ω (in units of the plasma frequency) and K_z (Goldman et al, to be published in *Transport Theory and Statistical Physics*). The solutions to eqn (2) with periodic boundary conditions (as in the simulation) are eigenvalues for the growth rates of purely growing eigenmodes labeled by K_z and an eigenvalue number "r" = 0, 2, 4, … which labels the y-dependence (analogous to k_y in the unsheared uniform 2-stream instability). The solution to the eigenvalue problem reveals that the fastest growing mode is the symmetric ground state, with r = 0, K_z = 0.5, and growth rate 0.315, in fairly good agreement with the Vlasov simulation.

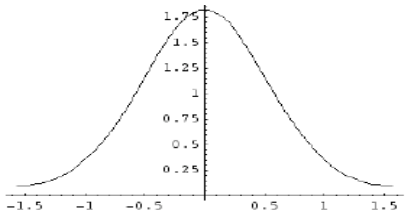

Fig. 2: Ground state potential eigenfunction as a function of θ = 2π(y/L_y-0.5) perpendicular to **B**

The ground state potential eigenfunction is shown in Fig. 2. It is peaked in the center and falls off in y towards the edges of the periodic cell as do most of the potential structures in Fig. 1, which can be interpreted as a superposition of the first few eigenfunctions of the Mathieu eqn.

4. ASYMMETRIC (BUNEMAN) INSTABILITY WITH SHEARED ELECTRONS

4.1 Simulations of the asymmetric (Buneman) instability with sheared electrons

Using a different Vlasov simulation code, in 2-D periodic simulations with dynamic ions and a strongly-magnetized Maxwellian distribution of electrons drifting with respect to the ions with sheared parallel drift velocity, $v_b(y) = v_o[1-\varepsilon\cdot\cos(2\pi y/L_y)]$, a class of asymmetric (Buneman-like) instabilities are studied. The electrons have thermal velocity $v_e = v_0/5$ and the shear parameter is ε = 0.2. The simulation is performed in the zero momentum frame in which ions have a small uniform drift. The ions have mass 400 times that of the electrons and are relatively cold (T_i/T_e = 0.016). The potential grows at a linear growth rate determined to be γ = 0.074 associated with a parallel wavenumber, $K_z \equiv k_z v_0/\omega_e = 1.22$. At time $t\omega_e \approx 70$ the ion density is seen to accumulate near the edges in y of the simulation where the sheared velocity is lowest. Fig. 3 shows the ion density and velocity as functions of z and y.

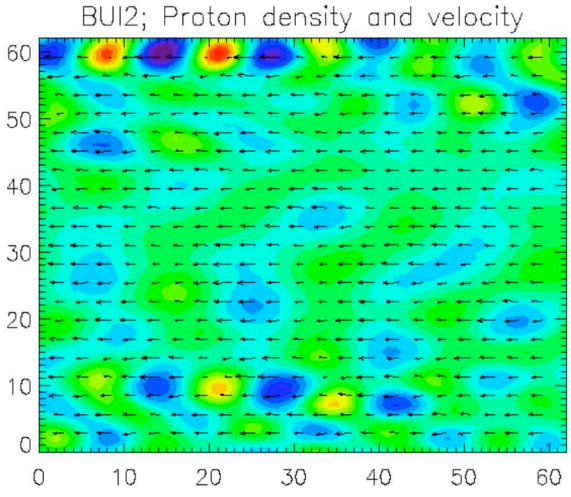

Fig. 3: Ion density contours (color) and ion mean velocity (arrows) as function of x/2 (horizontal) and y/4 (vertical) from simulation of sheared Buneman.

4.2 Theory of the asymmetric (Buneman) instability with sheared electrons

The sheared electrons are treated in a similar fashion as for the two-stream instability except now there is only electron component with velocity $v_0(y)$, and the electron distribution is given by a κ = 1 distribution equivalent to eqn. (1). However now ions are introduced into the theory as an unmagnetized undrifted cold fluid distribution in

addition to the strongly magnetized drifting hot electrons. The linearized Poisson eqn may then be written as,

$$(3) \qquad \partial_y^2\varphi(y) - k_z^2\left[1 + \frac{\omega^2}{\omega^2 - \omega_i^2}\chi_e(\omega, k_z, y)\right]\varphi(y) = 0$$

Where ω_i is the ion plasma frequency and χ_e is the linear electron susceptibility for a $\kappa = 1$ distribution. This susceptibility depends nonlinearly on the sheared velocity $v_0(y)$ and the shear parameter ε. Expanding eqn (3) in ε yields a Mathieu eqn of precisely the form of eqn. (2), however, with coefficients a and q now given by:

$$(4a) \qquad a(\omega, K_z) = -sK_z^2\left[1 + \frac{\omega^2\chi_e(K_z,\xi)}{\omega^2 - \alpha^2}\right] = -sK_z^2\left[1 - \frac{v^2\omega^2}{\omega^2 - \alpha^2}\frac{\xi + 3i}{K_z^2(\xi + i)^3}\right]$$

$$(4b) \qquad q(\omega, K_z) = -\frac{s\varepsilon v^3\omega^2}{\omega^2 - \alpha^2}\frac{\xi + 4i}{(\xi + i)^4}, \quad \xi \equiv v\left[\frac{\omega}{K_z} - 1\right]$$

Here, the dimensionless length parameter $s \equiv (L_y\omega_e/\pi v_0)^2 = 66$, and the mass ratio, $\alpha = m/M = 1/400$, as in the simulation. The solutions to eqn (2) and eqn (4) with periodic boundary conditions (as in the simulation) now give eigenvalues for the growth rates and *real* frequencies of eigenmodes labeled by K_z and an eigenvalue number "r" = 0, 2, 4, …. Solutions for the eigenfunctions reveals that the fastest growing mode is the symmetric ground state, with r = 0, $K_z = 1.2$, and growth rate 0.076, in good agreement with the 2-D Vlasov simulation of the sheared Buneman instability.

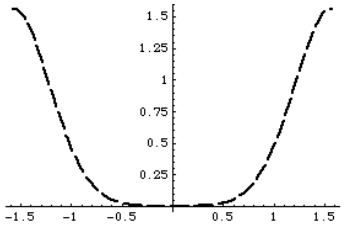

Fig. 4: Absolute value of ground state potential eigenfunction as a function of $\theta = 2\pi\,(y/L_y - 0.5)$ perpendicular to **B** for the sheared Buneman instability

The ground state potential eigenfunction is shown in Fig. 4. In contrast to Fig. (2) for the 2-stream instability, the ground state eigenfunction is now peaked at the *edges* and falls off with y towards the *center* of the periodic cell as do most of the density structures in Fig. 3.

5. CONCLUSION

The effects of shear on the electron drift velocity in two different kinds of basic plasma instabilties have been studied in strongly magnetized electron plasmas — the two-stream and Buneman instabilities. For both, in the limit of weak shear, the linearized Poisson eqn (for cold ions) becomes a Mathieu-eqn eigenvalue-problem which can be solved for the parallel wavenumber at which maximum growth occurs and the magnitude of that growth as well as the associated (ground state) potential eigenfunction. Theoretical results agree well with 2-D Vlasov simulations in the linear stages of evolution of these instabilities.

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