

PLASMA SPIN-UP DRIVEN BY A MARFE

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I. Introduction

The presence of MARFE's (Multifaceted Asymmetric Radiation from the Edge) in tokamaks has revealed an interesting radiative phenomenon in the outer plasma edge, present at relatively high densities. It works as a kind of plasma pump, where an initial density increase produces an increment of radiation which cools down the plasma. The resulting pressure drop produces a particle flow along the field lines that brings in more particles, from the low field side (LFS) of the magnetic surface, thus accumulating more particles and radiating more. MARFEs appear preferentially at the high field side (HFS) where the radial heat transport has a minimum. Although a MARFE is normally a transient structure, showing up before a density disruption, under certain circumstances, it has been possible to produce stationary MARFEs [1].

Here we show that the strong density asymmetry, intrinsic to these structures, can produce an important plasma spin-up. The whole picture of the process is the following. The MARFE is triggered near the HFS of the plasma by the radiative condensation instability. By feedback controlled gas-puffing, the radiative region can be kept stationary. Now, the fact that the density on the flux surface is poloidally asymmetric makes it susceptible to the Stringer spin-up instability [2], which is driven by magnetic curvature. This drives a poloidal plasma rotation. In this work, we study the spin-up process including the important poloidal dependence of the density. As we will show, this produces an enhanced spin-up, as compared with the usual spin-up due to anisotropic diffusion. We show that the drive is dominant over viscous damping (known as magnetic pumping). As a result of the spin-up there will be a sheared poloidal rotation at the edge, which could produce a transport barrier. This might explain the reported improved confinement observed in the HT-7 tokamak [3].

II. Plasma spin-up

The physics of the spin-up in presence of a MARFE is the same as the one resulting from anisotropic diffusion [4]. For the derivation we use the usual axisymmetric representation for the magnetic field: $\mathbf{B} = \nabla\psi \times \nabla\zeta + I(\psi)\nabla\zeta$, in terms of the flux function ψ . We define the poloidally asymmetric part of any quantity as $\delta S = S - \langle S \rangle$, where the angular brackets denote flux-surface average.

To study the dynamical behavior we consider the fluid equations in the superdiamagnetic and subsonic limit. The mass and momentum equations are,

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}_{\perp}) + \mathbf{B} \cdot \nabla \left(n \frac{u_{\parallel}}{B} \right) = S \quad (1)$$

$$m_i \left(\frac{\partial}{\partial t} (n\mathbf{u}) + \nabla \cdot (n\mathbf{u}\mathbf{u}) \right) = -\nabla p - \nabla \cdot \mathbf{\Pi} + \frac{1}{c} \mathbf{J} \times \mathbf{B} \quad (2)$$

$$-\nabla \phi + \frac{1}{c} \mathbf{u} \times \mathbf{B} = \mathbf{R}_{ei} \quad (3)$$

where the particle source is S (i.e. gas puffing). In the electron momentum equation (3), the term \mathbf{R}_e represents a general electron-ion momentum exchange (collisional and anomalous), which will be responsible for the diffusive radial transport. In the total momentum balance equation (2), the stress tensor $\mathbf{\Pi}$ is responsible for damping by magnetic pumping. We are interested in studying the case with an poloidally dependent equilibrium density, for which we take,

$$n(r, \theta) \approx n_0(r)(1 - \alpha(r) \cos \theta) \quad (4)$$

with the function $\alpha(r)$ determining the radial extent of the MARFE, while $n_0(r)$ corresponds to the equilibrium density profile. Note that the function $\alpha(r)$ is such that $|\alpha(r)| < 1$, it peaks at $r = r_M$, the MARFE's radial position, and vanishes in the plasma core.

We outline the derivation of an equation for the poloidal velocity, which was obtained in [5]. Ohm's law (3) implies that ϕ is a flux function and the poloidal velocity can be written as: $u_p = (B_p/B)(u_{\parallel} - cI\phi'/B)$. We split this velocity in a flux surface part and a θ -dependent part, by defining the variable: $\frac{u_p}{B_p} = \lambda(\psi) + \delta\lambda$. In order to find the evolution equation for $\lambda(\psi)$, we use the toroidal and parallel components of Eq.(2) and take the flux surface average,

$$m_i \frac{\partial}{\partial t} \langle nu_{\zeta} \rangle = -\frac{\partial}{\partial \psi} \langle nu_{\zeta} u^{\psi} \rangle \quad (5)$$

$$\frac{\partial}{\partial t} \langle n\mathbf{B} \cdot \mathbf{u} \rangle = \langle \mathbf{B} \cdot \mathbf{u} (\partial n / \partial t - S) \rangle + \langle \mathbf{R}_{ei} \cdot \nabla \times n\mathbf{u} \rangle + \langle (\mathbf{B} \cdot \mathbf{u}\mathbf{u} - \frac{u^2}{2} \mathbf{B}) \cdot \nabla n \rangle - \frac{1}{m_i} \langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi} \rangle, \quad (6)$$

The term involving the friction, R_{ei} , will be responsible for the poloidal spin-up. In deriving (6) use was made of the continuity equation (1) for the case of a stationary MARFE: $\nabla \cdot (n\mathbf{u}) - \langle \nabla \cdot nu_{\perp} \rangle = 0$. Expressing $\lambda(\psi)$ in terms of the averaged toroidal and parallel components of the flow and using the last two equations, we can write the equation for λ (neglecting terms of order $\sim B_p/B_T$),

$$\begin{aligned} \frac{\partial \lambda}{\partial t} &= \frac{1}{\Theta \langle nB^2 \rangle} \left[I \left\langle \frac{u^{\psi}}{R^2} \frac{\partial}{\partial \psi} n I \lambda \Delta_R \right\rangle - \frac{I \langle n \rangle}{\langle nR^2 \rangle} \frac{\partial}{\partial \psi} I \lambda \langle nu^{\psi} \Delta_R \rangle + \lambda \left\langle \left(B^2 - \frac{nI^2}{\langle nR^2 \rangle} \right) u^{\psi} \frac{\partial n}{\partial \psi} \right\rangle \right. \\ &- \lambda \left\langle \left(B^2 - \frac{\langle n \rangle I^2}{\langle nR^2 \rangle} \right) \frac{\partial}{\partial \psi} \langle nu^{\psi} \rangle - I \left\langle \frac{u^{\psi}}{R^2} \frac{\partial}{\partial \psi} \left(\frac{nR^2}{\langle nR^2 \rangle} \langle nu_{\zeta} \rangle \right) \right\rangle \right. \\ &- \frac{I \langle n \rangle}{\langle nR^2 \rangle} \frac{\partial}{\partial \psi} \left(\frac{\langle nR^2 u^{\psi} \rangle}{\langle nR^2 \rangle} \langle nu_{\zeta} \rangle \right) - \frac{I \langle nu_{\zeta} \rangle}{\langle nR^2 \rangle} \left(\left\langle u^{\psi} \frac{\partial n}{\partial \psi} \right\rangle - \frac{\partial}{\partial \psi} \langle nu^{\psi} \rangle \right) \\ &\left. + F(\delta\lambda) - \frac{\langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi} \rangle}{m_i} \right], \quad (7) \end{aligned}$$

where $\Theta = 1 - \frac{I^2 \langle n \rangle^2}{\langle n B^2 \rangle \langle n R^2 \rangle}$, $\Delta_R = (n R^2 / \langle n R^2 \rangle) - 1$ and $F(\delta\lambda)$ is a function containing all terms dependent on $\delta\lambda$ but unimportant here. The contravariant and covariant vector components, $u^\gamma = u \cdot \nabla \gamma$ and $u_\gamma = |\nabla \gamma|^2 u^\gamma$ (where $\gamma = \psi, \theta, \zeta$) were used. Toroidal and poloidal components are $u_T = |u_\zeta \nabla \zeta|$ and $u_p = |u_\theta \nabla \theta|$, respectively. The radial velocity $u^\psi = \mathbf{u} \cdot \nabla \psi = R^2 R_{ei}^\zeta$, is proportional to the toroidal friction force. The dependence of Eq.(7) on $\langle n u_\zeta \rangle$ is of lower order in ϵ , so that the RHS is just a function of λ . The viscous stress term, giving the magnetic pumping, is also proportional to λ , and we write as: $\langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi} \rangle = \mu_0 \lambda + \kappa$, with $\mu_0 = \eta_0 \langle (\hat{\mathbf{b}} \cdot \nabla B)^2 \rangle$, $\kappa = \eta_0 \langle (\hat{\mathbf{b}} \cdot \nabla B)^2 \delta \lambda \rangle$ and $\eta_0 = 4.095 p_i / \nu_{ii}$.

We reduce Eq.(7) using the circular flux surface limit ($R = R_0 + r \cos \theta$, $\mathbf{B} = (B_0/R)(\hat{\zeta} + \epsilon \hat{\theta})$) and the density (4). We replace $\partial/\partial \psi \rightarrow (1/r)(\partial/\partial r)$ and $u^\psi \rightarrow r u_r$. Since $\langle n u_\zeta \rangle / \lambda n_0 I \sim \epsilon$, the lowest order equation in ϵ is,

$$\frac{\partial \lambda}{\partial t} = \Gamma \lambda + H \quad (8)$$

$$\Gamma = \frac{-1}{\sigma} \left[\left(1 + \frac{\alpha}{2}(1 - \alpha)\right) \frac{1}{r} \frac{\partial}{\partial r} r u_{r1} + \frac{u_{r1}}{n_0} \frac{\partial n_0}{\partial r} - \frac{1}{n_0 r} \frac{\partial}{\partial r} (r n_0 \alpha [u_{r0} + u_{r2}]) + \frac{\eta_0 \epsilon}{2 m_i n_0 q^2 R^2} \right]$$

$H = F_0(\delta\lambda) + \kappa / m_1 n_0 \epsilon \sigma$, where, $\sigma = (2 - 9\alpha/4 + 3\alpha^2/2 + q^{-2})\epsilon$, $u_{rn} = \langle u_r \cos(n\theta) \rangle$ and $F_0(\delta\lambda)$ is the order- ϵ contribution to $F(\delta\lambda)$. The average u_{r1} measures the poloidally asymmetric component of the radial velocity. The parameter Γ includes both the spin-up drive and the damping by magnetic pumping, but the latter is an order ϵ smaller than the spin-up and it will be easy to overcome the damping; the instability condition is $\Gamma > 0$. We notice that when $\alpha = 0$, the condition reduces to the usual spin-up found in [6]: $(d/dr)(r n u_{r1}) < 0$. In our case, the MARFE produces the asymmetrical density (4), and the function $\alpha(r)$ modifies the stability criterium. For definitness we assume the radial flow is due to a diffusive and a convective component, so that $n u_r = -D dn/dr - n u_0(r)$, with $u_0(r)$ independent of θ , but the diffusion coefficient is allowed to be poloidally varying as $D = D_0(1 + \mu \cos \theta)$ in addition to $n(\theta)$. The general situation is difficult to analyze because of the different combinations of parameters. In Fig.1 we show the spin-up parameter Q , which is the Γ of Eq.(9) without the viscosity, as function of the MARFE strength. It is seen that the spin-up criterium ($Q > 0$) is modified by μ ($N \equiv \alpha n'_0 / n_0$, $A \equiv \alpha \alpha' / \alpha$). The radial dependence of $n_0(r)$ and $\alpha(r)$ are also important, as can be seen in Fig.2, where the effect of varying α' is shown. A limiting case may be illustrative:

When D is poloidally dependent but α is constant, the condition $\Gamma > 0$ gives,

$$|\Sigma| \equiv \left| (r n_0'' + n_0') (\mu - \alpha + \mu \frac{\alpha}{2} (1 - \alpha)) - \frac{n_0'^2}{n_0} r \mu \frac{\alpha}{2} (1 - \alpha) - \frac{(r \alpha n_0 u_0)'}{D_0} \right| > \frac{\eta_0 \epsilon r}{2 m_i D_0 q^2 R^2}, \quad (9)$$

where the prime denotes radial derivative, while the constraint on the density profile is $\Sigma > 0$. Condition for $\alpha = 0$ requires the density profile to be concave-up near the

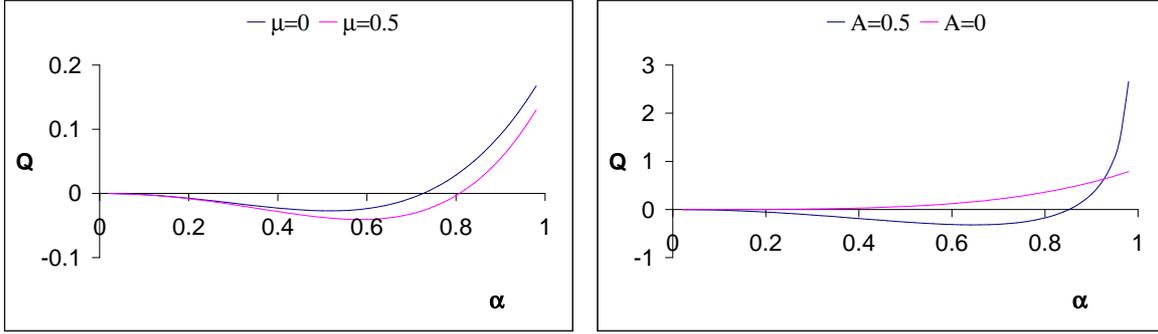


Figure 1: Spin-up parameter for $\epsilon = 0.1$, Figure 2: Spin-up parameter for $\epsilon = 0.1$, $A = 0$, $A' = 0.1$, $N = -1.1$ and $N' = -0.3$. $N = -1.1$, $N' = -0.3$, $A' = -0.1$, $\mu = 0$

edge. There is, however, another possibility for spin-up when the diffusion coefficient is symmetric ($\mu = 0$) and just n is poloidally dependent; for $\alpha = \text{constant}$ and $u_0 = 0$, the conditions for this are,

$$|rn_0'' + n_0'| > \frac{\eta_0 \epsilon r}{2m_i D_0 \alpha q^2 R^2}, \quad (10)$$

and $rn_0'' + n_0' < 0$. This constraint is the opposite to the case with $\alpha = 0$ and a poloidally dependent D , and it means that the density profile should be concave-down at the edge, which is the usual profile shape. Condition (10) can be satisfied for a sufficiently asymmetric density profile ($\alpha \approx 1$).

III. Conclusions

We have shown that when a stationary MARFE is produced in a tokamak, the flow pattern thus created gives rise to a poloidal spin-up in the MARFE region (the edge). Our treatment includes a poloidal dependence in density, and we find the spin-up is important for a large density anisotropy (strong MARFE). The rotation, having an intrinsic radial shear, would lead to the formation of a transport barrier. This could explain the observed improved confinement in HT-7 [3].

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