*31st EPS Conference on Plasma Phys. London, 28 June - 2 July 2004 ECA Vol.**28G**, P-1.132 (2004)*

Enhanced acceleration of charged particles by crossing RF waves, Resonant Moments Method (RMM)

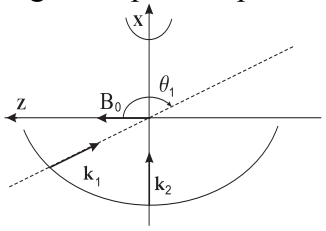
Maxim Ponomarjov and Daniele Carati,

Statistical and Plasma Physics, CP 231,
Campus Plaine, Universite Libre de Bruxelles, 1050 Brussels, Belgium

A mechanism for enhanced acceleration of charged particles using crossing radio frequency (RF) waves propagating at different angles with respect to an external magnetic field is investigated. The idea is to launch low amplitude secondary waves in order to improve the parallel momentum transfer from the high amplitude primary wave to the charged particles. The use of two parallel counter-propagating waves has recently been considered and numerical tests have shown that the two-wave scheme may lead to higher averaged parallel velocity [1,2]. On the other hand in Refs. [3,4] it has been concluded that it is more effective to accelerate electrons when the waves propagate obliquely to the external magnetic field. The majority of the existing works are based on the description of a single particle dynamics in one (or more but under condition of equality of their parallel refractive indexes) plane monochromatic radio frequency waves. In this paper, a mechanism is discussed for the acceleration of electron populations resulting from the effect of crossing electromagnetic waves propagating in a dispersive medium according to the geometry represented in Fig.1. To analyze this mechanism, the resonance moments (RM) of the distribution, i.e. velocity moments computed inside the RL only, are evaluated. The first order RM suggests that a peculiar angle of the primary wave launching θ_1 results in a maximal averaged parallel flux. Although the RM approach has to be considered as an approximation, this prediction is reasonably confirmed by direct statistical simulations.

The electromagnetic configuration that is considered (Fig.1) is the combination of a strong magnetic field (assumed to be along the OZ direction), a primary wave propagating obliquely with respect to the magnetic field and a secondary wave propagating perpendicularly to the magnetic field. Both the primary and the secondary waves are assumed to be in the plane (XOZ).

Fig.1: Proposed Optimal two-wave plane Configuration



$\overrightarrow{k_1} = k_1(\sin\theta_1, 0, \cos\theta_1)$ - for primary (high amplitude) wave

$\overrightarrow{k_2} = k_2(1,0,0)$ - for secondary (low amplitude) wave

$\overrightarrow{B} = B_0(0,0,1)$

Qualitative Physical Description, Resonance Layers, Resonant Moments Method (RMM)

Let us consider a large population of charged particles and only the average effect of the waves on this population. The fraction of the particles close to the Doppler resonance condition, which define resonance layers $(RL)_N$ in the velocity space, becomes then as important as the time these particles remain resonant. Primary wave tends to push resonant particles out of the resonant layers (RL). The low amplitude secondary wave is introduced tending to re-fill RL maintaining a pseudo-equilibrium velocity distribution, that continuously interact with the primary one. This secondary wave does not carry any parallel momentum and cannot induce any net parallel motion of the particles. These assumptions generate a basis for the applications of a Resonance Moments Method (RMM) in which moments of the velocity distribution are computed by using an average over the resonant layers only instead of a complete phase-space average. The flow parameters can be effectively approximated by the Resonance Moments (RM) defined by:

$I^g = \int_{\vec{v}\in(RL)} d^3v f(\vec{v}) g(\vec{v})$. The RL_N correspond to ellipses in the velocity space and the integral I* can be evaluated analytically using elliptical coordinates. For instance, assuming a Maxwellian distribution, the evaluation of I for $g=v_{||}$, which will be denoted I* can be done explicitly. This quantity corresponds to the averaged parallel flux of the particles that belongs to the RL. It is related to the net averaged parallel current produced by the particle-wave interactions tending to remove the particles from the RL while the pseudo-thermalization of the particles due to the combined effect of the two waves is to refill constantly the layer. The pseudo-thermalization is thus expected to add a net averaged parallel velocity proportional to I*. The exact expression even for I* is quite long. It is thus more illustrative to present the Fig.2 in which I* is a function of θ_1, the angle of the primary wave.

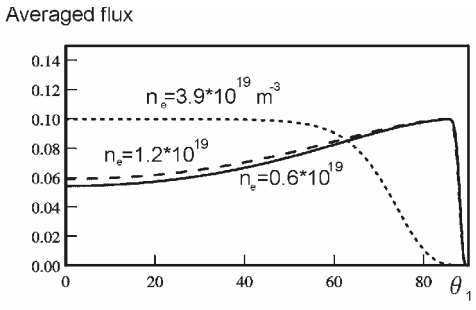


Fig.2 Averaged parallel flux I* (in arbitrary units) of the particles in the RL as a function of the angle of the primary wave, θ_1 (in two-wave plane Configuration – Fig.1) for different particle densities. We see a clear maximum of the particle flux at non-zero propagation angle of the primary wave as expected due to asymmetry of the resonance layer (RL) in velocity space and pseudo-stochastization of the particle population by the secondary wave.

To confirm our analytical predictions we perform direct numerical simulations for the two wave scheme (Fig.1) with the primary wave that is near the 2^{nd} harmonic (X2) of the EC frequency and the secondary wave – to the 3^{rd} harmonic (X3), which are used for instance in the TCV tokamak experiments [5], and compared results with ones for launching the one (X2) wave only. The constant magnetic field B_0=1.42 T, dimensionless wave amplitudes ($\overline{A_i} = A_i\Omega / cB_0$) $\overline{A_1} = 0.1$ for primary wave, $\overline{A_2} = 0.01$ for secondary wave. The waves supposed to propagate according to the linear dispersion relation for a cold, homogeneous plasma in external uniform magnetic field B_0 (Stix, 1962). The trajectories of the electrons are calculated using Hamilton equations. According to initial conditions at t=0 all the electrons are confined inside the small box placed in the center of the resonance layer in the configuration (coordinate) space. The box dimensions in the coordinate space are all about 10^{-3}h (h – the width of the resonance layer in the coordinate space); in the velocity space initially we assume relativistic Maxwellian velocity distribution of representative electrons. The Hamiltonian equations are solved using a 4th order Runge-Kutta method with variable time step. The Hamiltonian is not an invariant in this problem because of the explicit time dependency of the vector potential. However, an evolution equation is derived for H and has been solved using the same method with variable time step. This solution has then been compared with the analytical form of the Hamiltonian expressed in terms of the numerical solutions of equations. In the simulations presented hereafter the relative deviation between these two expressions of the Hamiltonian is always smaller than 10^{-5} for all time and all the electrons. A population of $5*10^4$ electrons is used in the simulations. The results from the simulations confirm that the angle dependency is not trivial and that parallel propagation (θ_1=0) for the primary wave is not always optimal. Figs.3-4 show a significant increase of the average parallel velocity for low densities due to the secondary wave. Moreover, the angle dependency of the average parallel velocity appears to be maximal around $\theta_1 = 10^o$ - 60^o . This value thus appears to reasonably differ from the privileged value of the RM I* (Fig.2). This is not too surprising since the RM have been computed assuming a Maxwellian distribution with zero mean. Thus, although the global picture from the RM description is reasonable, a more precise order RMM approximations including iterative approach, taking into account the averaged velocity suggested by the RM would be required for more accurate predictions.

Fig.3 For low density runs $n_e \approx 0.6 \times 10^{19} m^{-3}$, averaged electron velocity versus the propagation angle of the primary wave θ_1 (in degrees) at the normalized times: t=3000 (left picture), t=7000 (right picture) for two wave scheme (blue line), and one wave (green)

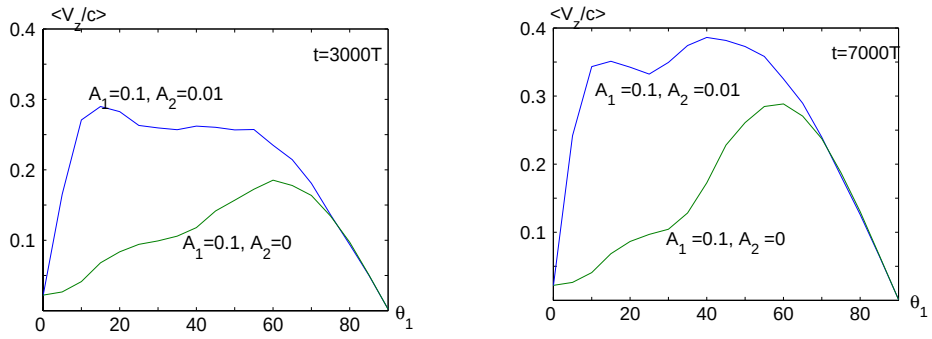
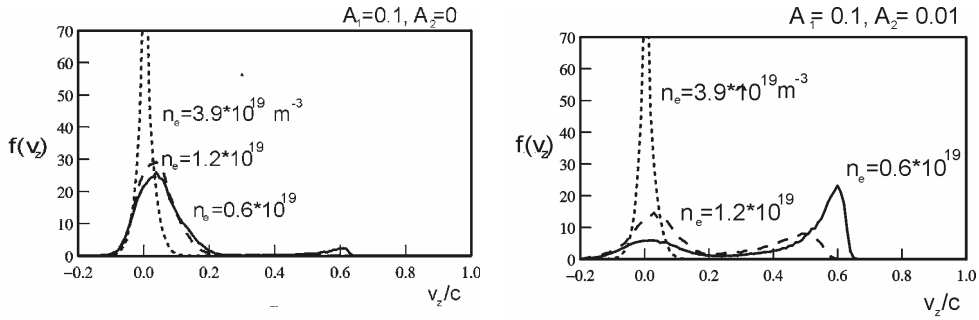


Fig.4. Probability Distribution of the parallel velocity for $\theta_1 = 45^o$



Our simulations confirm that the secondary perpendicular wave is the most effective for a stochastization of the particle trajectories and, consequently, to maintain a pseudo-equilibrium. The stochastization effect of the secondary wave yields promising increase of the average parallel velocity of the particles. It is energetically profitable to obtain this significant increase with addition just the secondary wave of ten times lower amplitude than the one of the primary wave.

The authors are grateful to Dr. B. Weyssow and Prof. R. Balescu for useful discussions. D.C. is researcher of the Fonds National pour la Recherche Scientifique (Belgium). This work has been supported by the contract of association EURATOM - Belgian state. The content of the publication is the sole responsibility of the authors and it does not necessarily represent the views of the Commission or its services.

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