Effects of magnetic fields and nonlocal electron transport in laser-produced plasmas

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I. Introduction

It is well known that in laser-produced plasmas strong magnetic fields can be generated by various mechanisms such as the Weibel instability, the $\vec{\nabla}n \times \vec{\nabla}T$ sources [1], etc… The electron thermal transport is thus inhibited by such dc fields and simultaneously the nonlocal effects due to steep temperature gradients produce the same reduction effects. In this work, we study the electron thermal transport in laser-produced plasmas by taking into account the combined effects of the *B*-field and nonlocal features of the thermal transport. An external dc magnetic field $\vec{B}$ with arbitrary strength is considered. We restrict our analysis to small perturbations. For practical purposes, we have proposed accurate numerical fits of the thermal conductivities with respect to the number atomic Z, the Hall parameter $\Omega_e = \omega_c \lambda_{ei}/v_t$ where $\omega_c = eB/m$ is the electron cyclotron frequency, λ_{ei} is the electron-ion mean free path, v_t is the electron thermal velocity and the collisionality parameter $k\lambda_{ei}$ where k^{-1} is the inhomogeneity scale length.

II. Basic equations

The basic transport equation used in this work is the steady-state electron Fokker-Planck Equation (FPE). Assuming a plasma inhomogeneity along the x-axis, the expression of the FPE rewritten on the frame of the plasma reads,

$$(v_x + V)\cdot\frac{\partial f}{\partial x} - \frac{e}{m}\left(\vec{E} + \vec{v}\times\vec{B}\right)\cdot\frac{\partial f}{\partial \vec{v}} - (v_x + V)\frac{\partial V}{\partial x}\cdot\frac{\partial f}{\partial v} = C_{ei}(f) + C_{ee}(f) \quad , \qquad (1)$$

where $f(x, \vec{v}, t)$ is the electron distribution function (DF), $\vec{E}$ is the electric field, $\vec{B} = B\hat{z}$ is the magnetic field, $\vec{V} = V\hat{x}$ is the fluid velocity, and in the right-hand side of (1), the two terms correspond respectively to the Landau electron-ion and to the electron-electron collision operators.

We develop a nonlocal transport theory which describes the plasma response to small perturbations of the DF: δf, the density δn, the temperature δT, the electric field $\delta \vec{E}$ and the fluid velocity $\delta \vec{V}$ with respect to a global equilibrium defined by the Maxwellian F_M.

In the Fourier space, ($x \Leftrightarrow k$), Eq. (1) becomes

$$ik v_x \, \delta f - \frac{e}{m} \delta E_x \frac{\partial F_M}{\partial v_x} - \frac{e}{m} \delta E_y \frac{\partial F_M}{\partial v_y} - \frac{e}{m} B \left(v_y \frac{\partial f}{\partial v_x} - v_x \frac{\partial f}{\partial v_y} \right) - ik v_x \delta V \frac{\partial F_M}{\partial v_x} =$$
$$C_{ei}(\delta f) + C_{ee}(\delta f) \quad .(2)$$

To solve Eq. (2), it is convenient to expand δf on the spherical harmonic basis

$$\delta f = \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} f_l^m(y) Y_l^m(\theta,\varphi) \quad \text{where } y = v / \sqrt{2} v_t \ .$$

We limit our analysis to the diffusive approximation wich corresponds to moderate collisionality i.e., $k\lambda_{ei} < 1$. Thus, truncating δf at the second anisotropy ($l = 2$) and keeping only the lower order of the nonlocal terms in (2), we obtain the following equations

$$C_{ee}(f_0^0) - (ik v_t / \sqrt{3}) y F_1^- = \frac{4\sqrt{\pi}}{3} y^2 F_M(y) ik \delta V \tag{3}$$

$$C_{ee}(F_1^+) + C_{ei}(F_1^+) + \omega_c F_1^- + \sqrt{2}(ik v_t / \sqrt{5}) y F_2^- = \frac{4\sqrt{\pi}}{\sqrt{3}} y F_M(y) \frac{e\delta E_y}{m v_t} \tag{4}$$

$$C_{ee}(F_1^-) + C_{ei}(F_1^-) - \omega_c F_1^+ + 2(ik v_t / \sqrt{15}) y f_2^0 - \sqrt{2}(ik v_t / \sqrt{5}) y F_2^+ = \frac{4\sqrt{\pi}}{\sqrt{3}} y F_M(y) \frac{e\delta E_x}{m v_t} +$$
$$2(ik v_t / \sqrt{3}) y f_0^0 \tag{5}$$

$$C_{ee}(F_2^+) + C_{ei}(F_2^+) + 2\omega_c F_2^- - \sqrt{2}(ik v_t / \sqrt{5}) y F_1^- = \frac{8\sqrt{\pi}}{\sqrt{30}} y^2 F_M(y) ik \delta V \tag{6}$$

$$C_{ee}(F_2^-) + C_{ei}(F_2^-) - 2\omega_c F_2^+ + \sqrt{2}(ik v_t / \sqrt{5}) y F_1^+ = 0 \tag{7}$$

$$C_{ee}(f_2^0) + C_{ei}(f_2^0) + (ik v_t / \sqrt{15}) y F_1^- = -\frac{4\sqrt{\pi}}{\sqrt{45}} y^2 F_M(y) ik \delta V \ . \tag{8}$$

In Eqs. (3)-(8), , instead of complex anisotropic components $f_l^{m \neq 0}$, we use real functions :

$$F_1^+ = -i \left(f_1^{+1} + f_1^{-1} \right), F_1^- = f_1^{-1} - f_1^{+1}, \ F_2^+ = \left(f_2^{-2} + f_2^{+2} \right), \ F_2^- = i \left(f_2^{-2} - f_2^{+2} \right).$$

III. Numerical results and discussion

To solve the coupled integrodifferential equations (3)-(8), we use the numerical approach reported in Ref. 2. For this purpose, the FPE is reduced to a simple set of ordinary differential equations, which can be easily solved iteratively, with the use of standard numerical methods. We have computed very accurately the 6 components of the DF as functions of $k\lambda_{ei}$, Ω_e and Z and deduced the thermal conductivities using the standard definition of Braginskii in free current plasmas [3],

$$\delta\vec{q} = -\kappa_\perp \frac{d\delta T}{dx}\hat{x} - \kappa_\wedge \frac{d\delta T}{dx}\hat{y} \qquad , \qquad (9)$$

where $\kappa_\perp$ is the transverse thermal conductivity and $\kappa_\wedge$ is the oblique thermal conductivity with respect to the direction of the *B*-field.

For practical purposes, we propose numerical fits with Lorenzian shape for the conductivities normalized to the local conductivities given by Braginskii (*Br*) [3].

For the transverse conductivity, we have found

$$\frac{\kappa_\perp}{\kappa_{\perp Br}} = \frac{1}{1+\gamma_\perp^2 (k\lambda_{ei})^2} \qquad , \qquad (10)$$

where $\gamma_\perp = \gamma_1 f(\Omega_e, Z)$ with $\gamma_1 = 28.\sqrt{Z\beta(Z)}$ and $\beta(Z) = \frac{Z+0.24}{Z+4.2}$ is the numerical fit which accounts for the electron-electron collision contribution, $f(\Omega_e, Z) = \frac{1}{1+a(Z)(\Omega_e)^2}$

and $a(Z) = \frac{3.45Z^{3.71}+6.60}{0.124Z^{3.157}+1.845}$.

For the oblique conductivity, the following expression is derived

$$\frac{\kappa_\wedge}{\kappa_{\wedge Br}} = \frac{1}{1+\gamma_\wedge^2 (k\lambda_{ei})^2} \qquad , \qquad (11)$$

where $\gamma_\wedge = \gamma_2 g(\Omega_e, Z)$ with $\gamma_2 = 66.16\sqrt{Z\alpha(Z)}$ and $\alpha(Z) = \frac{Z+1}{Z+5.5}$

$g(\Omega_e, Z) = \frac{1}{1+b(Z)(\Omega_e)^2}$ and $b(Z) = \frac{4.14Z^{2.16}+13.86}{0.0157Z^{2.29}+1.648}$.

In Figs. 1-4, we present normalized transverse and oblique conductivities as functions of $k\lambda_{ei}$ for different values of Ω_e for Z=3 and Z=20. We can see a strong dependence of the conductivities on the *B*-field for $k\lambda_{ei} > 0.01$. For $\Omega_e < 0.01$, the B-effects are negligible and we retrieve the results of the literature in unmagnetized plasmas. In this case, the

nonlocal effects reduce drastically the conductivities typically for $k\lambda_{ei} > 10^{-3}$. For strong magnetic fields $\Omega_e > 1$, the nonlocal effects disappear and transport is governed by the Braginskii's transport coefficients which scale as $\kappa_\perp \sim 1/B^2$ and $\kappa_\wedge \sim 1/B$. In the intermediate region, $0.01 < \Omega_e < 1$, both magnetic fields and nonlocal effects are important and they should be taken into account for thermal transport across magnetic field. We can also observe that the reduction of the nonlocal effects by the *B*-fields are more efficient in plasmas with lower atomic numbers.
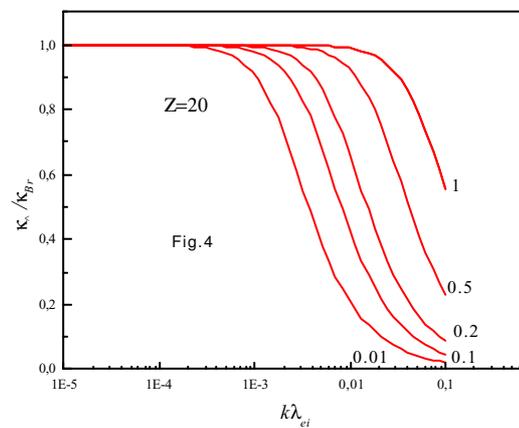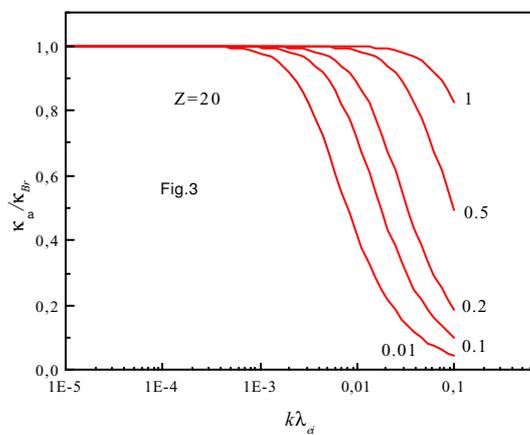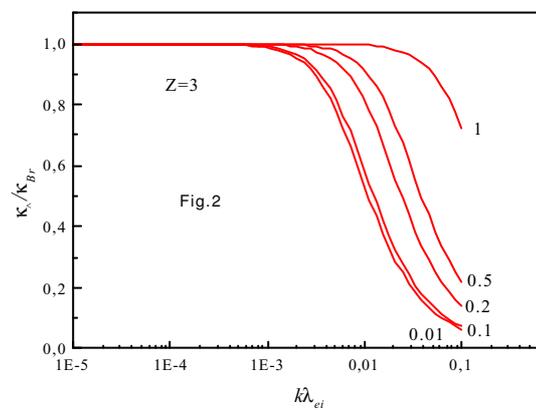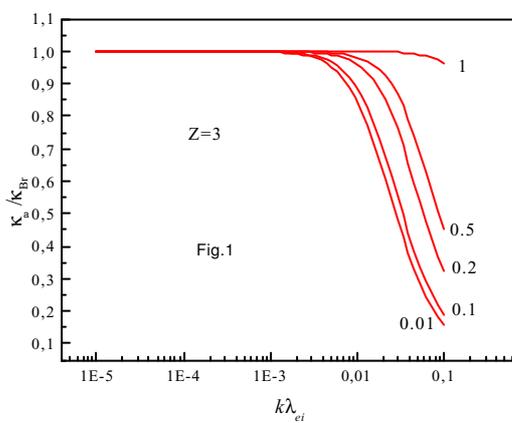
IV. References

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