Driven magnetic reconnection about an X-point in the limit of no guide field

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The present work investigates the linear phase of driven magnetic reconnection about a magnetic X-point, in the absence of a guide field. Analytical results are presented for the long-time asymptotic reconnection rate.

Mathematical Model. The starting point of our study is the two-dimensional, two-fluid model, consisting of the following equations: quasi-neutrality condition, continuity equation, ion momentum balance $m_i \left[\partial \mathbf{V}_i / \partial t + (\mathbf{V}_i \cdot \nabla) \mathbf{V}_i \right] = e\mathbf{E} + e/c \mathbf{V}_i \times \mathbf{B} - e\eta\mathbf{J}$ and electron momentum balance $\mathbf{E} + 1/c \mathbf{V}_e \times \mathbf{B} = -m_e/e \left[\partial \mathbf{V}_e / \partial t + (\mathbf{V}_e \cdot \nabla) \mathbf{V}_e \right] + \eta\mathbf{J} - \nabla p/(en)$, where $\mathbf{J} = en(\mathbf{V}_i - \mathbf{V}_e)$. These equations are coupled with Maxwell's equations: $\nabla \cdot \mathbf{B} = 0$, $c\nabla \times \mathbf{E} = -\partial\mathbf{B}/\partial t$ and $\nabla \times \mathbf{B} = 4\pi/c\mathbf{J}$. The unknowns are for the particle density, n, the ion flow velocity, $\mathbf{V}_i$, the electron flow velocity, $\mathbf{V}_e$, the current density, $\mathbf{J}$, the magnetic field, $\mathbf{B}$, the pressure, p, and the electric field, $\mathbf{E}$. We have neglected the displacement current in Ampere's law and terms of the order of $\varepsilon = m_e/m_i$ when compared to unity. In order to close our system of equations, we consider the pressure to be isotropic and assume constant electron temperature and cold ions: $p = nT_e$, $T_i = 0$. We have chosen a cartesian coordinate system, where all the unknowns depend on x, y and t, while z is the ignorable coordinate.

The following steps are applied in order to simplify the model equations.

1. Representation of the vector fields as $\mathbf{B} = \nabla\psi(x, y, t) \times \mathbf{e}_z + B_z(x, y, t)\mathbf{e}_z$ and $\mathbf{V}_i = \mathbf{e}_z \times \nabla\varphi(x, y, t) + \nabla\chi(x, y, t) + V_z(x, y, t)\mathbf{e}_z$.

2. Linearization with respect to the current free equilibrium with no guide field defined by: $\mathbf{V}_{\mathrm{eq}} = 0$, $\mathbf{J}_{\mathrm{eq}} = 0$, $\mathbf{B}_{\mathrm{eq}} = \nabla\psi_{\mathrm{eq}} \times \mathbf{e}_z$, $n_{\mathrm{eq}} = \mathrm{const}$ and $\psi_{\mathrm{eq}} = B'_\perp xy$.

3. Choice of the following normalization: $t \longrightarrow t/\tau_A$; $x, y \longrightarrow x/d_i, y/d_i$; $n \longrightarrow n/n_{\mathrm{eq}}$; $(\varphi, \chi) \longrightarrow (\tau_A/d_i^2)(\varphi, \chi)$; $V_z \longrightarrow V_z/V_A$; $\psi \longrightarrow \psi/(d_i^2 B'_\perp)$; $B_z \longrightarrow B_z/(B'_\perp d_i)$; the Alfven velocity is $V_A = B'_\perp d_i/\sqrt{4\pi m_i n_{\mathrm{eq}}}$, the Alfven time is $\tau_A = \sqrt{4\pi m_i n_{\mathrm{eq}}}/B'_\perp$, the ion and electron skin depths are $d_i = \sqrt{c^2 m_i/(4\pi n_{\mathrm{eq}} e^2)}$ and $d_e = \sqrt{\varepsilon} d_i$ and the resistive collisional time is defined by $\tau_\eta = 4\pi d_e^2/(\eta c^2)$. Furthermore, we define $\varepsilon_\eta = d_e^2 \tau_A/(\tau_\eta d_i^2)$ and $\beta = 4\pi n_{\mathrm{eq}} T_e/(B'_\perp d_i)^2$.

4. Choice of the following orderings for the wave vector, $k_\perp \sim k_\parallel \sim 1/\widehat{\delta}$, for the frequency, $\omega \lesssim k_\parallel V_A(\widehat{\delta}) \lesssim 1$, and for the ratio of the kinetic to the magnetic pressure, $\beta \gg \widehat{\delta}^2$. Note that $\widehat{\delta}(\lesssim 1)$ is a length scale characteristic of the reconnective process. With these assumptions, one can show that the compressible part of the ion flow, χ, can be neglected and the model reduces to three partial differential equations for ψ, φ and B_z, all the other unknowns enslaved to these three.

5. Choice of a class of solutions of the form [1]: $\psi(x, y, t) = \psi(x, t) + \psi(y, t)$, $\varphi(x, y, t) = \varphi(x, t) - \varphi(y, t)$ and $B_z(x, y, t) = B_z(x, t) + B_z(y, t)$.

Finally, two coupled partial differential equations are obtained for the current density and magnetic field perturbations along z (we have dropped the subscript z):

$$\begin{cases} \dfrac{\partial^2 J}{\partial t^2} - \varepsilon \dfrac{\partial^4 J}{\partial t^2 \partial x^2} = x^2 \dfrac{\partial^2 J}{\partial x^2} + 3x \dfrac{\partial J}{\partial x} - \dfrac{\partial^3}{\partial t \partial x^2} \left(x \dfrac{\partial B}{\partial x} \right) + \varepsilon_\eta \dfrac{\partial^3 J}{\partial t \partial x^2} \\[4mm] \dfrac{\partial^2 B}{\partial t^2} - \varepsilon \dfrac{\partial^4 B}{\partial t^2 \partial x^2} - x \dfrac{\partial}{\partial x} \left(x \dfrac{\partial B}{\partial x} \right) = x \dfrac{\partial^2 J}{\partial t \partial x} + \varepsilon_\eta \dfrac{\partial^3 B}{\partial t \partial x^2}. \end{cases} \quad (1)$$

The following initial condition is chosen to represent an initial $\mathbf{E} \times \mathbf{B}$ drift flow, driven by an externally applied electric field E_∞ [1]

$$\frac{\partial}{\partial t} \left(J - \varepsilon \frac{\partial^2 J}{\partial x^2} \right) \bigg|_{t=0} = \frac{E_\infty}{2} \frac{d^2}{dx^2} \left(\frac{x^2}{x^2 + \delta^2} \right), \quad (2)$$

where δ is a length-scale characteristic of the initial velocity profile that should be of the order or greater than the electron skin depth d_e (or $\sqrt{\varepsilon}$, in normalized units). The other initial conditions are: $J(x,0) = 0$, $B(x,0) = 0$ and $\partial B / \partial t|_{t=0} = 0$. Model (1) is completed by the the following boundary conditions: $\partial B / \partial x|_{x=0} = 0$, $\partial J / \partial x|_{x=0} = 0$, $\partial^3 B / \partial x^3|_{x=0} = 0$, $J(L) = 0$ and $B(L) = 0$, L being the external boundary.

By looking at Eqs. (1), it is interesting to note that if one sets to zero the axial perturbation of the magnetic field, model (1) recovers the model investigated in Ref. [1] (with strong guide field) in the limit of no sound Larmor radius effects, namely $\rho_s = 0$. Furthermore, the particular orderings we have chosen allow us to retain the Hall term, thus coupling the dynamics of J to a new equation for B.

Results and conclusions. The dynamics of system (1) can be obtained numerically by a suitable discretization in time and space. However, it is possible to perform analytical calculations by considering, for example, the long-time asymptotic limit. In order to do so, we apply the Laplace transform to Eqs. (1) and use asymptotic matching techniques to solve the transformed equations in the limit of $s \longrightarrow 0$ (s being the Laplace variable). The current density perturbation at the X-point, $J_0(s)$, is then antitransformed in order to obtain the asymptotic reconnection rate, defined by Ohm's law at the X-line: $E_z(0,0,t) = 2\varepsilon \partial J(0,t) / \partial t + 2\varepsilon_\eta J(0,t)$. For brevity, we report here only the results of the analytical calculations. In the case without a guide field, the long-time asymptotic value of the current density perturbation at the X-line, in the collisionless ($\varepsilon_\eta = 0$) and resistive (labelled with the subscript 'res') cases are

$$J(0,t) \longrightarrow \frac{E_\infty}{2\delta^2 t} \qquad J_{\text{res}}(0,t) \longrightarrow \frac{E_\infty}{4\delta^2 t}. \quad (3)$$

These analytical predictions are in good agreement with numerical simulations, as shown in Fig. 1. Furthermore, the long-time asymptotic values of the magnetic field perturbation at the X-line are

$$\frac{B(0,t)}{E_\infty} \longrightarrow \frac{\gamma - 1 + \log(2L) + \log t}{2\delta^2 t^2} \qquad \frac{B_{\text{res}}(0,t)}{E_\infty} \sim \frac{\gamma - 2 + 2\log\left(\dfrac{2L}{\sqrt{\varepsilon_\eta}}\right) + \log t}{8\delta^2 t^2}, \quad (4)$$

where γ is Euler's constant. We have also investigated the long-time asymptotic behaviour for the model of Ref. [1] in the case where ion sound Larmor radius effects are

neglected, which is formally equivalent to the first of Eqs. (1) where B is artificially set to zero. The long-time asymptotic values of the current density perturbation at the X-line, in the collisionless and resistive limits, are

$$J\left(0,t\right) \longrightarrow \frac{E_\infty}{2\delta^2 t} \qquad J_{\text{res}}\left(0,t\right) \longrightarrow \frac{E_\infty}{4\delta^2 t}. \tag{5}$$

The comparison among relations (3), (4) and (5) provides some conclusions.

- The presence of finite resistivity does not change the long-time asymptotic behaviour (proportional to $1/t$ for J and $\log t/t^2$ for B) of the fields, but only changes the coefficients in front of it. It is interesting to note that only $B_{\text{res}}\left(0,t\right)$ shows a weak dependence upon resistivity.

- The long-time asymptotic value of the B depends on the position of the external boundary L, while the same does not hold for J. This is due to the fact that the mathematical solution for $B(x,t)$ contains logarithmic contributions with a characteristic length scale of the order of L. Thus, the spatial structure of B is not localized within a certain length scale but extends up to the external boundary. Consequently, a change in L changes the macroscopic structure of B.

- The long-time behaviours of the current density perturbation of the full system (1), expressed by relations (3), are exactly equal to the ones obtained with the model of Ref. [1] with $\rho_s = 0$. This is the proof that, for the particular case under examination, the magnetic field perturbation (that is due to the presence of the Hall term in the model equations) does not influence the long-time asymptotic reconnection rate. This result can be understood by looking at relations (4): at large times, the magnetic field perturbation decays much faster ($\propto \log t/t^2$) than the decayment of J and can not affect the asymptotic reconnection rate.

- According to expressions (3) and (5), the total reconnected flux is

$$\psi(0,0,t) \sim -\frac{E_\infty c\tau_A^2 d_e^2}{2\delta^2}\left(\frac{1}{t} + \frac{1}{\tau_\eta}\log\frac{t}{\tau_A}\right). \tag{6}$$

  It is interesting to note from relation (6) that there can be a net amount of total reconnected flux only if resistivity is present. This is different from what found in Ref. [1], where in the collisionless case the presence of finite ρ_s leads to a net amount of reconnected.

- The total reconnected flux being zero in the collisionless case can be understood by looking at the evolution of the system. At $t = 0$, we apply an electric field E_z that moves the plasma towards the X-line: magnetic field lines are pushed together in one direction while pulled out perpendicularly and reconnection is achieved by means of electron inertia. We observe the formation of vortices moving towards the X-line in the diffusion region, while the external region is reminiscent of the initial flow configuration but with the direction of the flow that reverses periodically in time. At later times, new vortices form but phase mixing reduces the global intensity of the flow. Thus, reconnection becomes very inefficient as the flow is advected around circular paths and the transport of mass
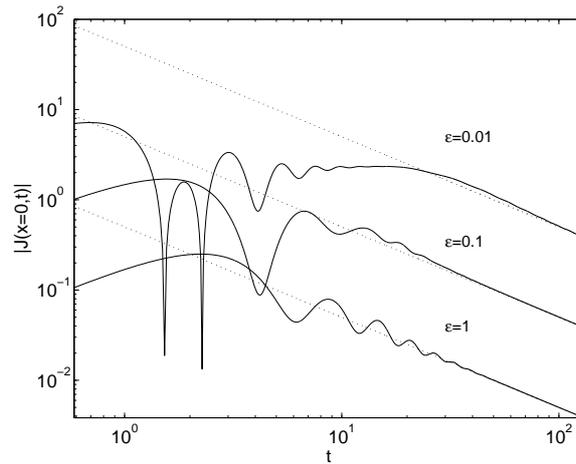
Figure 1: Time behaviour of $|J(0,t)|$ for different ε, obtained by solving numerically system (1) in the collisionless case ($\varepsilon_\eta = 0$, $\delta = \sqrt{\varepsilon}$). The dotted lines correspond to the analytical prediction given by expression (3). The log-log scale is used.
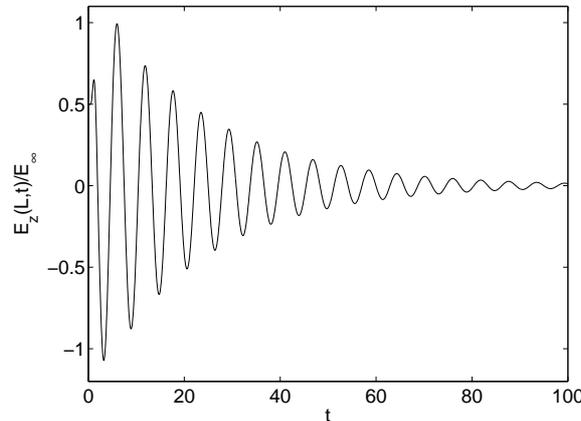

Figure 2: Behaviour of $E_z(L,t)$ for a simulation with $\varepsilon_\eta = 0$, $\varepsilon = 0.1$, $\delta = \sqrt{\varepsilon}$ and $L = 5$ in the collisionless case.

is slow. We can understand why the total reconnected flux is zero: it is due to an initial phase of alfvenic reconnection, characterized by large transport towards the X-point, followed by a slower deconnection process characterized by vortex formation and inversion of the flow. Note that this picture is consistent with the evolution of the axial electric field at the boundary, which oscillates in time and goes asymptotically to zero, as can be seen from Fig. 2. This is different from Ref. [1], where the axial electric field is sustained at infinity: $E_z(x \longrightarrow \infty) = E_\infty$. In addition, the solution in Ref. [1] relies on finite ρ_s, which represents a singular perturbation to our problem. Thus, our solution can not be recovered from that in Ref. [1] in the limit $\rho_s \longrightarrow 0$.

References

[1] J. J. Ramos, F. Porcelli, R. Verastegui, *Phys. Rev. Lett.* **89** (5), 055002 (2002).