

Collisions and Langmuir waves in Nonideal Plasmas

I. Morozov, G. Norman

*Institute for High Energy Densities of Russian Academy of Sciences,
IHED-IVTAN, Izhorskaya, 13/19, Moscow 125412, Russia*

The Langmuir waves in nonideal plasmas have been found by molecular-dynamic (MD) simulations [1–6] and considered theoretically [3,7–9]. Nevertheless their dispersion and damping have not been studied sufficiently. In this report the two-component fully ionized plasma is considered with the nonideality parameter $\Gamma = e^2(4\pi n_e/3)^{1/3}/(k_B T) = 0.1 - 4$, where n_e is the electron number density, T — temperature.

Dispersion and damping of Langmuir waves. The dispersion and damping decrement of the Langmuir waves are obtained from the dynamical structure factors (DSF)

$$S(\omega, k) = -\frac{k_B T}{4\pi^2 e^2} \frac{k^2}{\omega} \text{Im} \frac{1}{\varepsilon(\omega, k)}, \quad (1)$$

where $\varepsilon(\omega, k)$ is the dielectric function. The DSF can be calculated by equilibrium MD simulations using the corrected Kelbg interaction potential [10]. The details of simulation technique can be found in [2].

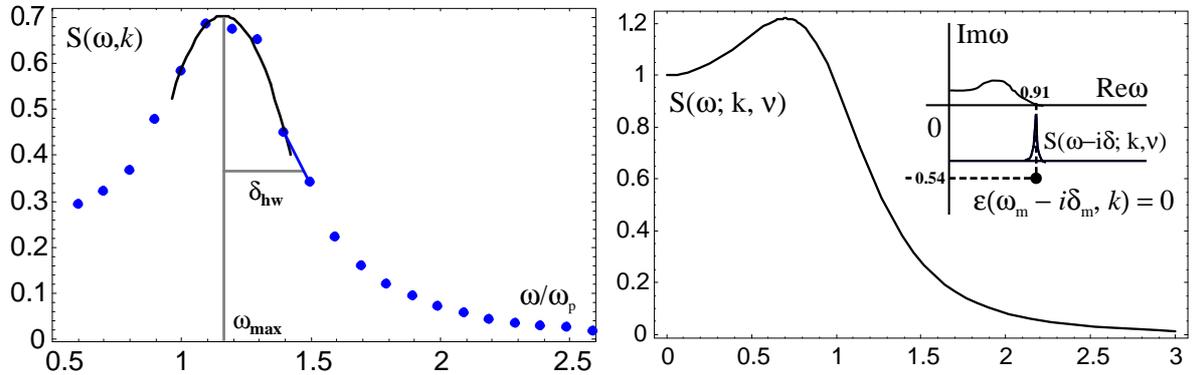


Fig. 1: Dynamical structure factor. Left figure shows interpolation procedure to obtain the position ω_{\max} and the halfwidth δ_{hw} of DSF peak. Right figure illustrates the difference between ω_{\max} and the solution of the dispersion equation due to strong damping $\nu/\omega_p = 0.8$, $kr_D = 0.2$.

The frequency ω of the Langmuir waves can be found from the position of correspondent peak of DSF ω_{\max} while the damping decrement δ corresponds to the halfwidth δ_{hw} of this peak (Fig. 1). In the long wavelength limit only the collisional damping $\delta_c = \delta(k \rightarrow 0)$ exists while for greater k the collisionless Landau damping δ_L becomes predominant, ω_p is the Langmuir plasma frequency, r_D is the Debye radius.

The dielectric function for an ideal plasma reads [11]

$$\varepsilon(\omega, k) = 1 + \frac{1}{k^2} \frac{1 - J_+\left(\frac{\omega+i\nu}{k}\right)}{1 - \frac{i\nu}{\omega+i\nu} J_+\left(\frac{\omega+i\nu}{k}\right)}, \quad J_+(x) = xe^{-\frac{x^2}{2}} \int_{i\infty}^x e^{\frac{\tau^2}{2}} d\tau. \quad (2)$$

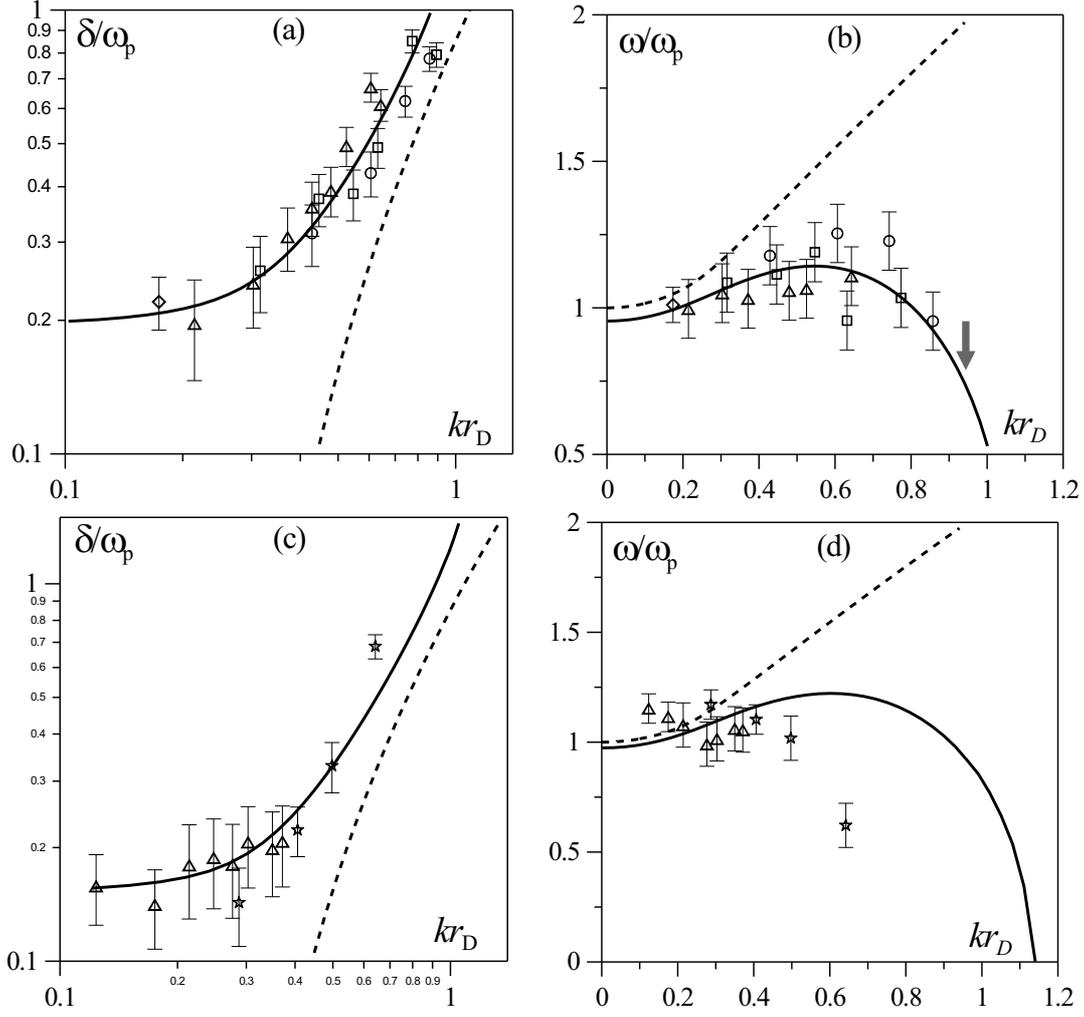


Fig. 2: Damping decrement of the Langmuir waves (a), (c) and dispersion of the DSF maximum (b), (d). MD results for various number of particles: $N = 1500$ — rhombus, $N = 800$ — triangles, $N = 250$ — squares, $N = 100$ — circles. Landau theory with a given collision frequency: $\nu = 0$ — dashed curve, $\nu = 0.42$ — solid curve. The arrow points to the wave number where the DSF peak disappears. (a),(b) $\Gamma = 1.28$, (c),(d) $\Gamma = 3.84$.

Here and below all quantities are given in the dimensionless units $\omega/\omega_p \rightarrow \omega$, $\nu/\omega_p \rightarrow \nu$, $\sigma/\omega_p \rightarrow \sigma$, $\delta/\omega_p \rightarrow \delta$, $kr_D \rightarrow k$. The collisional damping δ_c can be obtained by fitting MD results for $\delta_{hw}(k)$ with Eq. (2) via effective collision rate $\nu = 2\delta_c$ (Figs. 2a and Figs. 2c). Dependence of ν on plasma nonideality (Fig. 3) has a maximum at $\Gamma \approx 2$. It stays below $\nu < \omega_p$ contrary to the extrapolation of the Landau collision rate [14] which obviously fails for the SCP.

Having the values of ν the dispersion of DSF peak can be evaluated from [11] and compared with MD results (Fig. 2b). It shows a good agreement which extends to $\Gamma \approx 3$. This agreement could only be obtained if the difference between the position of DSF maximum ω_{hw} and the solutions of the dispersion equation $\varepsilon(\omega + i\delta, k) = 0$ is taken into account via Eq. (1). The stronger the damping δ the more essential shift of ω_{hw} to low frequencies is observed (Fig. 1). This effect could be misinterpreted as a

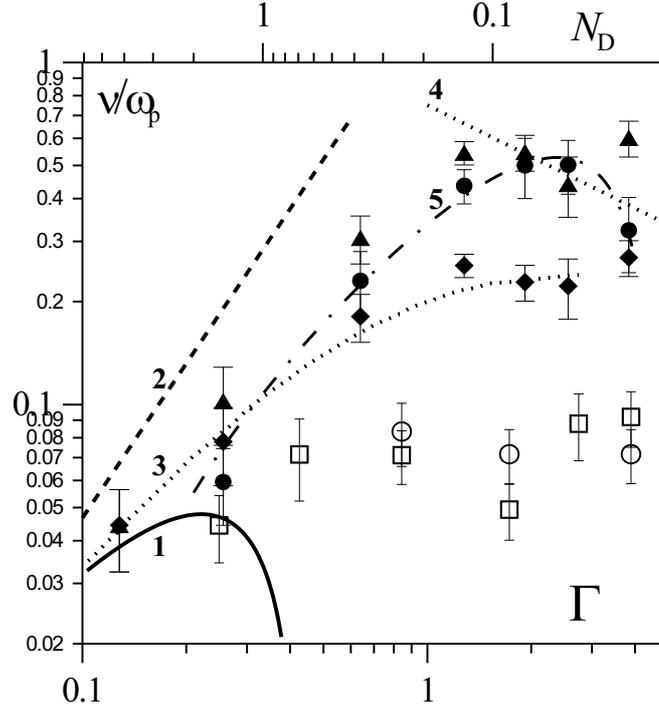


Fig. 3: Collisional frequency depending on the nonideality parameter: filled rhombus are obtained from the current ACF at $\omega = 0$, filled triangles are the same for $\omega = \omega_p$, filled circles and curve 5 are the collisional damping of the Langmuir waves $\nu = 2\delta_c$, open circles is the energy relaxation rate ν_e , open squares are the same for the initial state with crystal-like ion distribution. Curves: 1 is the ideal plasma theory [14], 2 is the same with fixed Coulomb logarithm $L_e = 3.2$, 3 is the theory [8], 4 is an asymptote [7].

negative dispersion. For $\Gamma > 3$ such an approach based on the ideal plasma model [11] fails (Fig. 2c).

Effective collision frequency. The dynamic collision frequency $\nu(\omega)$ is calculated by an alternative approach from the current autocorrelation function. This approach is based on the generalized Drude formula for the conductivity or dielectric function [12]

$$\sigma(\omega) = \frac{\omega_p^2}{4\pi} \frac{1}{\nu(\omega) - i\omega}, \quad \varepsilon(\omega) = 1 + \frac{4\pi i\sigma(\omega)}{\omega} = 1 - \frac{\omega_p^2}{\omega(\omega + i\nu(\omega))}. \quad (3)$$

The linear response theory relates $\sigma(\omega)$ to equilibrium current autocorrelation function

$$\sigma(\omega) = \frac{\Omega_0}{k_B T} \langle J^z; J^z \rangle_{\omega+i\eta}, \quad \langle \mathbf{J}; \mathbf{J} \rangle_{\omega+i\eta} = \int_0^\infty e^{i(\omega+i\eta)t} \langle \mathbf{J}(t)\mathbf{J}(0) \rangle dt, \quad (4)$$

where Ω_0 is the volume of the system.

The values of $\nu(0)$ agree with the analytical estimations of the static conductivity $\sigma(0) = 4\pi/\nu(0)$ [8] while the values of $\nu(\omega)$ agree with the collisional damping δ_c mentioned above. The dependence of ν on ω has a maximum which becomes more pronounced and shifts to small frequencies with the increase of Γ . These results are supplied with a qualitative theoretical study.

Energy relaxation rate. The collision frequency ν being the momentum relaxation time can be compared with the energy relaxation time obtained from simulations of relaxation in two-temperature nonideal plasmas in [13]. As energy relaxation time essentially depends on the ion-electron mass ratio M/m the following extrapolation is to be done for the comparison

$$\tau_{\epsilon}(\Gamma, M) = \tau_{\epsilon}^1(\Gamma) \left(\frac{M}{m} \right)^{\alpha(\Gamma)}, \quad \alpha = 1 - 0.15\Gamma + 0.035\Gamma^2, \quad \Gamma < 4. \quad (5)$$

The values of the relaxation times $\tau_{\epsilon}(\Gamma, M)$ were obtained in [13] as an exponential fit of the decay of ion-electron temperature difference $\Delta T \sim e^{-t/\tau_{\epsilon}}$. The value of τ_{ϵ}^1 can be compared with ν using the relation derived from the ideal plasma theory [14] $\nu_{\epsilon} = M/(8\pi m\tau_{\epsilon}^1)$. These values are shown in Fig. 3 by open circles and squares.

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