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Nonlinear interactions of multiple tearing modes with resistive wall and external helical magnetic perturbation

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Recent experiments, either in Tokamaks or in Reversed Field Pinch's (RFP), strongly suggest that avoidance of wall (external resistive shell) locked modes is essential to the further progress towards obtaining nuclear reactor grade plasmas. In fact, the enhanced plasma-wall interaction associated with the wall-locked dynamo(tearing) modes is a major limiting factor in all RFP experiments[1].

In this work, we use a fully time-dependent momentum equation to describe the dynamics of the dynamo modes under the effects of (1) mode coupling torque induced by the nonlinear interaction of the multiple (three) modes, (2) the braking torques produced by the resistive wall, (3) the transient viscous torques, which act to restore the frequency of the mode to its natural value and (4) the torques produced by the externally applied resonant magnetic perturbation (fixed or rotating). All these types of torques act at the vicinity of the rational surfaces of the multiple modes.

I. Physical model

We consider a cylindrically symmetric plasma of minor radius r = a surrounded by a resistive shell (thin shell) with radius r = b and a perfect conducting shell with radius r=c . The well-know $\alpha_0 - \Theta_0$ equilibrium model[2] for the RFP plasma is adopted. The perturbed radial magnetic field associated with the (m,n) tearing mode can be written as $b_r(\mathbf{r},t) = b_r^{m,n}(r,t)\exp[i(m\theta - n\phi)]$; For convenience, we introduce the linearized perturbed magnetic flux function $\Psi_{m,n}(r,t) = ir b_r^{m,n}(r,t)$. Furthermore, the "no slip" constraint[3] is used, according to which each non-linearly developed (m,n) tearing mode rotates with the plasma at its own rational surface. For the sake of simplicity, in this work we assume that the plasma rotates in the toroidal direction only.

By taking account two m=1 modes which possess the toroidal mode number n and n+1, and one m=0 mode with the toroidal mode number n=1, and assuming that these modes are intrinsically tearing unstable, we obtain the normalized toroidal component of the equation of motion of the plasma, written as

$$\rho r\frac{\partial \Delta\Omega}{\partial t}-\frac{\partial}{\partial r}\left(r\mu\frac{\partial \Delta\Omega}{\partial r}\right)\frac{1}{\tau_\mu}=\sum_{m,n}T_w^{m,n}\delta(r-r_{m,n})+\sum_{m,n}T_{coup}^{m,n}\delta(r-r_{m,n})+\sum_{m,n}T_{ex}^{m,n}\delta(r-r_{m,n})$$

Where the summation of m and n is over the three modes:(1,n),(1,n+1),(0,1), and $\Delta\Omega=\Omega_\phi(r,t)-\Omega_{\phi 0}(r)$, $\Omega_{\phi 0}$ is the natural toroidal rotation frequency of the plasma. $\Omega_\phi(r,t)$ denotes the time variation of the plasma toroidal rotation profile. $T_W^{m,n}$ represents the E-M braking torque induced by the resistive shell[4], and $T_{coup}^{m,n}$ is the non-linear interaction torque between the unstable modes[5]. $T_{ex}^{m,n}$ is the torque acting on the (m,n) mode, produced by the externally applied magnetic resonant perturbation[6]. These equations are solved numerically by using the finite element method. It is easily shown that these equations are reducible to the equations of torque balance in the steady state theory.

II. Results

By using the above equations we have studied the formation of the "slinky mode", and the interaction of the multiple modes with the resistive wall. It is found that in comparison to a single mode, multiple modes enhance the braking effects of the resistive wall. In Fig. 1 we plot the rotation frequency of (1,8) mode as a function of its amplitude for both of the single-mode model and the three -mode model (in the presence of (1,9) and (0,1) modes, $\psi^{0,1}=\psi^{1,9}=0.001$). The amplitude increase with time linearly as $d\psi/dt=10^{-4}$. The parameters used are for previous RFX calculation[4]. It is easy to demonstrate the change in mode frequency (or the plasma rotation frequency near the each mode rational surface) of each mode due to the EM torques of the resistive shell is not only produced by the resistive torque which it received as the single mode, but also has contribution derived from the resistive torques received by other modes. In other words, in the case of multiple modes, the slowing down in rotation of any single mode is influenced by the slowing down of the other modes through the viscous diffusion. The torque acting on the vicinity of each mode is transferred to other mode singular surfaces by the viscous plasma.

During the interaction of the multiple modes with the resistive shell, two thresholds exist. 1) the "phase locking" threshold, above which the mode phase will be locked and the so-called "slinky mode" will be formed. This threshold is mainly determined by the non-linear interaction of the modes. 2) the "wall-locking" threshold for each mode, beyond which, the mode frequency will jump downward to the low frequency branch. Fig.2 shows the variation of the relation between the two thresholds by change the penetration time scale of the resistive wall. In Fig4 (a) the RFX parameter of $\tau_{bRFX}^{m,n} \approx 3.3$ms is used, it shows the formation of the "slinky" occurs (ϕ_3=const., $\phi_3 = \phi^{1,n+1} - \phi^{1,n} - \phi^{0,1} - \phi_3(t=0)$) after all three modes are locked to the wall; (b) $\tau_b^{m,n}=10\cdot\tau_{bRFX}^{m,n}$, shows that the "slinky mode" is formed after the m=0 n=1 mode has locked to the wall, but the other two m=1 modes are still in rotation. c) $\tau_b^{m,n}=100\cdot\tau_{bRFX}^{m,n}$, shows the formation of the "slinky mode" which occurs before wall-locking of any of the three modes. We conclude that for an RFP having a shell with a high resistivity, there is a high possibility that the wall-locking occurs earlier than the phase locking.

The preliminary results about the effects of the externally applied rotating perturbation are also obtained. We add a steady state rotating magnetic perturbation with rotating frequency $\omega_{ex}^{m,n}$ and the amplitude $\psi_{ex}^{m,n}$ measured outside the resistive shell. To start with, the external magnetic perturbation has only the (0,1) harmonic. The numerical results show that the m=0, n=1 mode can be brought to rotate together with the external perturbation if the perturbation amplitude overcomes a threshold value of $\psi_{exc}^{0.1}$. This value depends on the rotating frequency and the mode amplitude. The higher frequency leads higher threshold, and the larger amplitude of the mode requires stronger external perturbation.

We found that if the three modes have all locked to the wall and formed as a "slinky mode" before the application of the external rotating field, the (0,1) mode can be induced to rotate with almost the same frequency of the external ((0,1) harmonic) rotating field; while the other two modes, (1,n) and (1,n+1), may still keep the phase locking with the (0,1) mode if the $\omega_{ex}^{0,1}$ is not too high. Fig. 3 shows the mode frequencies vary with time. In the beginning, the three modes have very slow rotation frequency and almost locked to the wall, in addition, they are also phase locked ($\omega_3 = \omega^{1,n+1} - \omega^{1,n} - \omega^{0,1} = 0$). For $t > t_{ex}$, the external (0,1) rotating field $\psi_{ex}^{0,1}$ is applied and increased linearly with time as $d\psi_{ex}/dt=2\cdot10^{-5}$. The amplitudes of the three modes are assumed to be unchanged. When the value of ψ_{ex} reaches the threshold value($\psi_{exc}^{0,1} \approx 0.002$), the (0,1) mode "attaches" to the external field and rotates with the same frequency as $\omega_{ex}^{0,1}$, having the phase a little delayed w.r.t. the external perturbation. The (1,9) and (1,8) modes remain in phase locking (slinky) with the m=0 mode (ω_3=0). Fig.3(1) and (2) demonstrate the phase relation between the three modes, in the time periods (1) and (2) marked in the Fig.3, corresponding to the time before the application of the external field and after the attachment of (0,1) mode to the external field.

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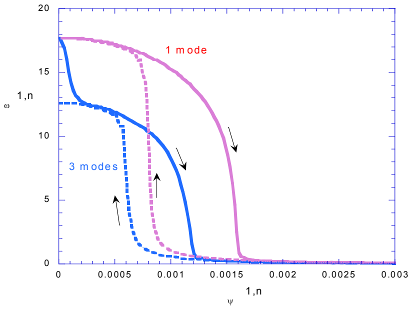


Fig.1

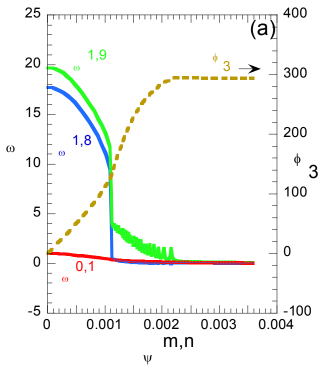
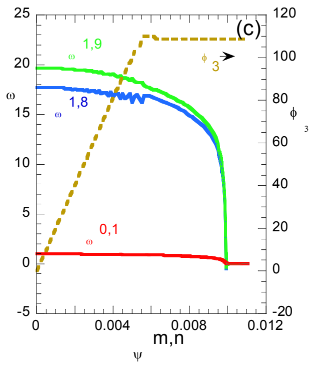


Fig.2(a)



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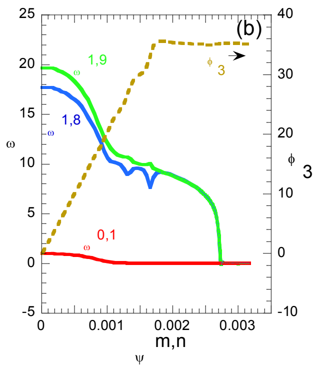

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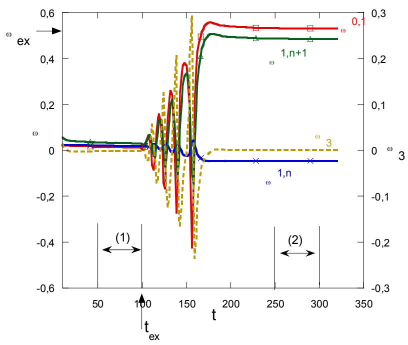

Fig.3

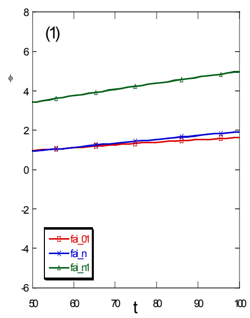

Fig.3(1)

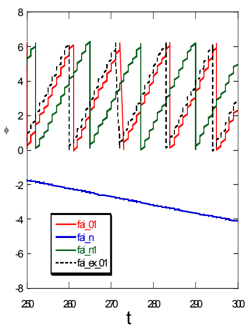

Fig.3(2)