*31st EPS Conference on Plasma Phys. London, 28 June - 2 July 2004 ECA Vol.**28G**, P-2.120 (2004)*

Ion temperature gradient modes in the presence of a sheared ion flow

B. Coppi [1,2], E. Lazzaro [2], M. Lontano [2], C. Marchetto [2,3]

[1] *M.I.T., Cambridge, USA*
[2] *Istituto di Fisica del Plasma, C.N.R., EURATOM-ENEA-CNR Ass., Milan, Italy*
[3] *INFM, Parallel Computing Initiative, Italy*

Abstract

Recently, a theory of spontaneous momentum generation due to the toroidally asymmetric excitation of ion temperature gradient (ITG) modes in the presence of a radial profile of toroidal ion velocity has been formulated [1]. We present a systematic study of the dispersion equation relevant to the electrostatic slab modes driven by an ITG perpendicular to a uniform magnetic field, in the presence of an inhomogeneous equilibrium fluid velocity of the ions.

Introduction

Sheared electric drift poloidal rotation in magnetically confined plasmas seems to play a major role in the suppression of radial energy leakage, thus inducing enhanced confinement regimes of operation [2]. The pattern of the electric drift flow is strictly related to the general problem of spontaneous momentum generation and of its transport within a magnetized plasma in axisymmetric toroidal plasmas, which is also in a close relationship with the physics of accretion disks. Toroidal plasma rotation is frequently characterizing auxiliary heated plasmas [3], while poloidal flows can be produced by IBW [4].

Recently a theory of spontaneous momentum generation due to the toroidally asymmetric excitation of radially localized ion temperature gradient (ITG) modes in the presence of a radial profile of toroidal ion velocity has been formulated [1]. The resulting radial transport of the angular momentum during external plasma heating is in qualitative agreement with the plasma dynamics observed in Alcator C-MOD [3].

In this work we present a detailed investigation of the relevant dispersion relation derived in the frame of the slab two-fluid drift model [5], where equilibrium inhomogeneous ion velocities are introduced, $U_{\parallel}(x)\hat{\mathbf{z}}$ parallel to the ambient magnetic field, $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$, and $U_{\perp}(x)\hat{\mathbf{y}}$ perpendicular to both $\mathbf{B}_0$ and to the unperturbed density and temperature gradients, $\nabla \parallel \hat{\mathbf{x}}$. The objective is to investigate the role of the asymmetry, introduced by the sheared flows, in the excitation of waves with opposite values of $k_{\parallel}/k_{\perp}$, capable of affecting the anomalous transport of angular momentum.

The electrostatic dispersion relation

The linearized two-fluid equations in the drift approximation, in a plasma slab non uniform in the *x* direction, where the electrostatic potential perturbation is assumed to be localized around $x=x_0$, are written as:

$$\varpi\tilde{n}_k = nk_{\parallel}\tilde{v}_{\parallel,k} - \frac{c}{B_0}n'k_{\perp}\tilde{\varphi}_k \tag{1}$$

$$Mn\varpi\tilde{v}_{\parallel,k} = q_i nk_{\parallel}\left(1-\frac{k_{\perp}}{k_{\parallel}}\frac{U_{\parallel}'}{\Omega_c}\right)\tilde{\varphi}_k + k_{\parallel}\tilde{p}_k \tag{2}$$

$$\varpi\tilde{p}_k = -\frac{c}{B_0}p_i'k_{\perp}\tilde{\varphi}_k + \gamma p_i k_{\parallel}\tilde{v}_{\parallel,k} \tag{3}$$

$$\frac{\tilde{n}_k}{n} = \frac{e\tilde{\varphi}_k}{T_e} \tag{4}$$

$$n = n_i = \frac{n_e}{Z} \tag{5}$$

Here, the 2D space Fourier transform

$$\tilde{A}(x,y,z,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}dk_{\perp}\int_{-\infty}^{+\infty}dk_{\parallel}\tilde{A}_k(x,t)e^{i(k_{\perp}y+k_{\parallel}z)} \tag{6}$$

has been introduced, where the time dependence of each Fourier coefficient is in the form of $\tilde{A}_k(x,t) = \tilde{A}_k(t)\exp(-i\omega_k t)$, with the complex frequency $\omega_k = \omega_k^R + i\gamma_k$, $\omega_{-k} = \omega_k^{c.c.}$, and $\tilde{A}_{-k}(x) = \tilde{A}_k^{c.c.}$. Moreover, in Eqs.(1-4) the *x*-dependent frequency $\varpi(x) \equiv \omega_k - k_{\perp}U_{\perp}(x) - k_{\parallel}U_{\parallel}(x)$, the perpendicular $k_{\perp} = k_y$ and the parallel $k_{\parallel} = k_z$ components of the wave-vector, have been defined. Finally, $n_a(x)$, $T_a(x)$, $p_a(x)=n_a(x)T_a(x)$ are the density, temperature and pressure of the *a*-species, respectively, and γ is the adiabatic index of the ions. The apex indicates the total differentiation with respect to *x*. From Eqs.(1-5) the dispersion relation follows [1]

$$1 = \frac{k_{\perp}v_{*ne}}{\varpi} - D_B\frac{k_{\perp}k_{\parallel}}{\varpi^2}\left(U_{\parallel}' + \frac{k_{\parallel}}{\varpi\rho_i}p_i' - \frac{k_{\parallel}}{k_{\perp}}\Omega_c\right)\Bigg/\left(1-\gamma\frac{k_{\parallel}^2 v_{ti}^2}{\varpi^2}\right) \tag{7}$$

where $v_{*ne}(x) = -D_B n'$, $D_B = cT_e/eB_0$, $\rho_i(x) = n(x)M$, $\Omega_c = q_i B_0/Mc$, $v_{ti} = (T_i/M)^{1/2}$, q_i = Ze. Eq.(7) gives a cubic equation in ϖ, which can be put in the approximate form

$$\varpi^3 - \gamma\tau k_{\parallel}^2 c_s^2\left(1-\frac{Z}{\gamma\tau}\frac{k_{\perp}}{k_{\parallel}}\frac{U_{\parallel}'}{\Omega_c}\right)\varpi + Zk_{\parallel}^2 c_s^2\omega_{*Ti} = 0 \tag{8}$$

for $\eta_i = (\ln T_i)'/(\ln n_i)' > 1$, $\tau = T_i/T_e \geq 1$, $\frac{\omega_{*ne}}{\varpi} << 1$. In Eq.(8), $\omega_{*Ti}(x) = k_{\perp}cT_i'/(ZeB_0)$. By neglecting the $U_{\parallel}'$-term in Eq.(8), we recover the dispersion equation considered by Coppi *et* *al.* in 1967 [5]. From inspection of Eqs.(7,8) it is seen that a non-zero value of $k_{\parallel}$ introduces the effects of the inhomogeneous ion population, through its pressure and parallel velocity profiles, which can stabilize or destabilize the system. Moreover, and most important to our scopes, the $U_{\parallel}'$-term introduces an odd dependence on $k_{\parallel}$. According to the standard notations [6], there is an unstable solution of Eq.(8) if $\Delta = q^3 + r^2 = \frac{Z^2\tau^2}{4}\left(k_{\parallel}^2 k_{\perp} c_s^3\right)^2 \rho_s^2\left[\kappa_{Ti}^2 - \left(\kappa_{Ti}^{cr}\right)^2\right] > 0$, where $\kappa_{Ti} = -\left(\ln T_i\right)'$, and $\kappa_{Ti}^{cr} = \kappa_{Ti}^{cr,0}\left(1 - \frac{Z}{\gamma\tau}\frac{k_{\perp}}{k_{\parallel}}\frac{U_{\parallel}'}{\Omega_c}\right)^{\frac{3}{2}}$ is a "critical" ion temperature gradient; $c_s = \left(\frac{T_e}{M}\right)^{1/2}$, $\rho_s = c_s/\Omega_c$ and $\kappa_{Ti}^{cr.0} = \frac{2}{3^{3/2}}\frac{\gamma^{3/2}\tau^{1/2}}{Z}\left|\frac{k_{\parallel}}{k_{\perp}}\right|\frac{1}{\rho_s}$ refers to the case where no parallel velocity shear is present. Note that κ_{Ti}^{cr} can be made equal to zero with a suitable choice of $U_{\parallel}'$. For $\left|1 - \left(\kappa_{Ti}^{cr}/\kappa_{Ti}\right)^2\right| << 1$, the unstable mode has $\varpi_k^R \cong \left(\frac{Z}{2}\right)^{1/3}\left(k_{\parallel}c_s\right)^{2/3}\omega_{*Ti}^{1/3} \cong -\left(\frac{\gamma}{3}\right)^{1/2}\mathrm{sign}\left(k_{\perp}\right)\left|k_{\parallel}\right| v_{ti}\left(1 - \frac{Z}{\gamma\tau}\frac{k_{\perp}}{k_{\parallel}}\frac{U_{\parallel}'}{\Omega_c}\right)^{1/2}$ and $\gamma_k \cong \frac{Z^{1/3}2^{1/6}}{3^{1/2}}\left(k_{\parallel}c_s\right)^{2/3}\omega_{*Ti}^{1/3}\left(\frac{\left|\kappa_{Ti}\right|}{\left|\kappa_{Ti}^{cr}\right|} - 1\right)^{1/2}$.

<u>Consequences of</u> $k_{\parallel}/k_{\perp}$ <u>symmetry breaking</u>

Let us consider a plasma region around $x = x_0$ where $U_{\parallel}' > 0$. Let's choose two modes with the same $k_{\perp} > 0$ and opposite $k_{\parallel}$, that is, $k_{\parallel 1} = -\bar{k}_{\parallel} < 0$ and $k_{\parallel 2} = \bar{k}_{\parallel} > 0$. Assume that at $x = x_0$ mode 1 is marginally stable, that is $\gamma_{k1} = 0$, due to the fact that locally $\kappa_{Ti} \equiv \left(\kappa_{Ti}^{cr}\right)_1$. We can use $\left(\kappa_{Ti}^{cr}\right)_1$ to calculate Δ_2 relevant to mode 2; it takes the form $\Delta_2 = \frac{2Z}{3^3}k_{\perp}\bar{k}_{\parallel}^5 c_s^6\frac{U_{\parallel}'}{\Omega_c}\left[3\gamma^2\tau^2 + Z^2\left(\frac{k_{\perp}}{\bar{k}_{\parallel}}\right)^2\left(\frac{U_{\parallel}'}{\Omega_c}\right)^2\right] > 0$, corresponding to an unstable solution. For small values of the sheared parallel ion velocity, that is $\frac{k_{\perp}}{\bar{k}_{\parallel}}\frac{U_{\parallel}'}{\Omega_c} << \frac{\gamma\tau}{Z}$, we get $\varpi_{k2}^R \cong -\left(\frac{\gamma}{3}\right)^{1/2}\bar{k}_{\parallel}v_{ti} \cong \left(\frac{Z}{2}\right)^{1/3}\left(\bar{k}_{\parallel}c_s\right)^{2/3}\omega_{*Ti}^{1/3}$ and $\gamma_{k2} \cong \left(\frac{2Z}{3}\right)^{1/2}\left(\bar{k}_{\parallel}k_{\perp}c_s^2\right)^{1/2}\left(\frac{U_{\parallel}'}{\Omega_c}\right)^{1/2}$, thus being $\frac{\gamma_{k2}}{\varpi_{k2}^R} << 1$. In the opposite limit, for $\frac{k_{\perp}}{\bar{k}_{\parallel}}\frac{U_{\parallel}'}{\Omega_c} >> \frac{\gamma\tau}{Z}$, putting $\sigma_{\pm} \equiv \left[\left(2^{1/2}+1\right)^{1/3} \pm \left(2^{1/2}-1\right)^{1/3}\right]/2$, we get $\omega^R_{k2} \cong -\left(\frac{Z}{3}\right)^{1/2} \sigma_-\left(\bar{k}_\parallel k_\perp c_s^2\right)^{1/2}\left(\frac{U'_\parallel}{\Omega_c}\right)^{1/2} \cong \quad \cong \left(\frac{Z}{2}\right)^{1/3} \sigma_-\left(\bar{k}_\parallel c_s\right)^{2/3} \omega_{*Ti}^{1/3}$ and

$\gamma_{k2} \cong Z^{1/2}\sigma_+\left(\bar{k}_\parallel k_\perp c_s^2\right)^{1/2}\left(\frac{U'_\parallel}{\Omega_c}\right)^{1/2}$, being $\gamma_{k2} > \omega^R_{k2}$.

A quasilinear flux of parallel ion velocity in the *x*-direction is associated with waves propagating in the (*y,z*) plane. For a spectrum of growing modes, *i.e.* with $\gamma_k > 0$ (as it is the case for wave 2), and in the limit $k_\parallel^2 v_{ti}^2 / |\varpi_k|^2 << 1$, the flux takes the form [1]

$$\Gamma_{x,v_{i\parallel}} = \rho_i \langle \tilde{v}_{Ex} \tilde{v}_{i\parallel} \rangle \cong -\frac{\rho_i}{2\pi} \frac{c^2}{B_0^2} \int_{-\infty}^{+\infty} dk_\perp \int_{-\infty}^{+\infty} dk_\parallel \frac{k_\perp^2}{|\varpi_k|^2} \gamma_k e^{2\gamma_k t} |\tilde{\varphi}_k|^2 \left(U'_\parallel + 2\frac{k_\parallel \varpi_k^R}{|\varpi_k|^2} \frac{p'_i}{\rho_i} - \frac{k_\parallel}{k_\perp}\Omega_c \right) \quad (9)$$

For $U'_\parallel = 0$, waves with positive and negative $k_\parallel$ are likely to be excited with the same growth rate. In this situation, Eq.(9) shows that there is a balance between quasilinear momentum fluxes, inward and outward, brought out by different components of the wave spectrum, and no net momentum transport in *x* takes place. On the contrary, if for example $U'_\parallel > 0$, the situation can occur in which only modes with $k_\parallel > 0$ are excited, (that is, waves of type 2). Then the integrand in Eq.(9) is no longer an odd function of $k_\parallel$, the radial (*x*) flux of parallel momentum associated with waves having $k_\parallel / k_\perp < 0$ (definitely inward for $p'_i < 0$ and $U'_\parallel = 0$) is suppressed, while that produced by waves with $k_\parallel / k_\perp > 0$ is non-zero. Momentum flux can be made inward for sufficiently large values of $U'_\parallel > 0$. Indeed, for $|\tilde{\varphi}_k|^2 = \delta(k_\perp - k_0)\left[I_1 \delta(k_\parallel + \bar{k}_\parallel) + I_2 \delta(k_\parallel - \bar{k}_\parallel)\right]$, with $\gamma_{-\bar{k}_\parallel, k_0} = 0$, Eq.(9) becomes

$$\Gamma_{x,v_{i\parallel}} = -\frac{cZen_i}{2\pi B_0} k_0^2 \frac{\gamma_k}{|\varpi_k|^2} e^{2\gamma_k t} I_2 \left[\frac{U'_\parallel}{\Omega_c} - \frac{4}{3^{3/2}} \frac{\gamma^{3/2}\tau^{3/2}}{Z} \frac{\bar{k}_\parallel^2 c_s}{k_0} \frac{\varpi_k^R}{|\varpi_k|^2} \left(1 + \frac{Z}{\gamma\tau} \frac{k_0}{\bar{k}_\parallel} \frac{U'_\parallel}{\Omega_c} \right)^{\frac{3}{2}} - \frac{\bar{k}_\parallel}{k_0} \right] \quad (10)$$

where the index "k" means $(\bar{k}_\parallel, k_0)$. Considering the minus sign and the square brackets only in Eq.(10), we obtain that for $(k_0/\bar{k}_\parallel)|U'_\parallel/\Omega_c| << \gamma\tau/Z$, $\Gamma_{x,v_{i\parallel}} \propto \left[4\gamma\tau/(3Z) - 1\right]\bar{k}_\parallel/k_0 > 0$, and for $(k_0/\bar{k}_\parallel)|U'_\parallel/\Omega_c| >> \gamma\tau/Z$, $\Gamma_{x,v_{i\parallel}} \propto -U'_\parallel/\Omega_i < 0$.

Part of this work has been supported by the "Short Term Mobility Programme" of C.N.R.

1 – B. Coppi, Nucl. Fus. **42**, 1 (2002); B. Coppi, Proc. *19th Fus. Energy Conf.*, 14-19 October 2002, Lyon, France, IAEA-CN-94, paper TH/P1-02.
2 – P.W. Terry, Rev. Mod. Phys. **72**, 109 (2000).
3 – J.E. Rice, *et al.*, Nucl. Fus. **43**, 781 (2003); Nucl. Fus. **44**, 379 (2004).
4 – R. Cesario, *et al.*, Phys. Plasmas **8**, 4721 (2001).
5 – B. Coppi, *et al.*, *Phys. Fluids* **10**, 582 (1967).
6 – M. Abramowitz, I.A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1972).