Edge Kink/Ballooning Mode Stability Scalings and ELM Triggering in Plasma with Separatrix

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Ideal MHD external kink modes driven by large current density and pressure gradient values in the pedestal region of the tokamak plasma are widely believed to trigger the edge localized modes (ELM). The key element in the existing models of ELM relaxation dynamics is the stability boundary in the pedestal edge current density and pressure gradient parameter space. The understanding of the ELM triggering experiments and design of ELM control schemes require an integration of the plasma evolution models with the stability codes.

1 The $w \times n \times q_{95} =$ const and $J_{||}/ < J >$ scalings for edge instabilities The scaling for the most unstable toroidal mode number n for edge instabilities was proposed in [1]. The value $w \times n$ is approximately constant with w being the pedestal width. In fact the scaling can be generalized to $w \times n \times q_{95} =$ const which is typical in case of high nq values. Besides the predictions for the most unstable mode number an important parameter is the critical value of the current density in the pedestal region. Results of the calculations from the KINX code that includes plasma up to the separatrix show that the ratio of the maximal parallel current density in the pedestal region $J_{||} = \max\limits_{pedestal} <jB>_V/<|B|>_V$

(where $<>_V$ is averaging over the volume between magnetic surfaces) to the averaged plasma current density $< J >= I_p/S_p$ (I_p and S_p being the plasma current and toroidal cross-section) provides a suitable measure for that value. The critical value of the normalized pedestal current density does not significantly depend on the width of the pedestal and just is set by the most unstable mode with the wave number according to the scaling.

The value of $J_{||}/ < J > \sim 1$ is a good estimate for the limit against current driven (not peeling) modes. Let us note that the value of the parallel current density at the plasma vacuum interface $J_{||edge}/ < J >$ (not the maximal parallel current density in the pedestal region $J_{||}$) was proposed as an analogous critical parameter in [3] (the ratio $J_{||_{edge}}/J_{||}$ was just slightly varied around 0.1). However there is a remarkable coincidence between the current limit with separatrix at the boundary and in the circular plasma [2]: $J_{||} = (m - nq) < J >$ giving $J_{||}/ < J >= 1$ when the resonant surface is far from the plasma boundary $m - nq \to 1$. Another important feature of the plasma with the separatrix is the "flat" current stability limit, which is almost independent on the pressure gradient up to the ballooning limit. This fact can be crucial for reproducing ELM sequences in the empirical models based on the current driven stability limits [1]. The coupling to ballooning modes reduces the stability threshold in terms of $J_{||}/ < J >$.

Critical values of $J_{||}/ < J >$ vary moderately under quite strong variations of the edge current density. On the other hand, the ratio $J_{||_{edge}}/J_{||}$ is an important parameter for the edge stability. The value of the constant in the scaling $w \times n \times q_{95} =$ const depends on the ratio. Lower values of the edge current density are stabilizing and lead to a shift in most unstable toroidal wave numbers to lower values of n that is connected with second stability access for lower-n modes [1].

The edge stability diagrams for the TCV equilibria (modifications of the profiles in the pedestal region are shown in Fig.1) in the plane ($p'/p'_c,J_{||}/ < J >$) are presented in Fig.2. Here p' is the maximal value of the pressure gradient in the pedestal, p'_c is the ballooning limit at the plasma edge. The comparison of the diagrams for the TCV cases with different q profiles provides an evidence of a good scaling not only for the current driven mode limit at low p' but also for the stability limits against coupled kink/ballooning modes in the chosen parametric plane. That is despite some difference between the curves

corresponding to the ballooning limit and shear reversal in the pedestal due to almost the same p'_c but lower averaged current density $< J >$ for high-q case, that affects also the bootstrap current density curve.

The tables below illustrates the $w \times n \times q_{95} = $ const scaling under both variations of the pedestal width and the ratio of the edge current density to the maximum in the pedestal. Series of the TCV equilibria with self-consistent bootstrap current were used to demonstrate that. Marginal values of edge pressure gradient p'/p'_c are given for different values of toroidal mode number n for $p'_{edge}/p' = 0.42$ and the two values of the pedestal width parameter w. Let us note that the marginal values of $p'/p'_c > 1$ and the limit is set by coupled edge kink/balllooning modes.

n	$p'/p'_c : x_0 = 0.98, w = 0.02$	$J_{||}/ < J >$	n	$p'/p'_c : x_0 = 0.99, w = 0.01$	$J_{||}/ < J >$
10	1.8056	0.91	20	1.7959	0.91
15	1.7413	0.88	30	1.7448	0.89
20	1.7393	0.88	40	1.7778	0.90

The next table corresponds to $p'_{edge}/p' = 0.18$ and shows that the range of most unstable modes shifts to lower values of n with pedestal moving deeper into plasma (in other words the second stability access takes place for lower values of n).

n	$p'/p'_c : x_0 = 0.97, w = 0.02$	$J_{||}/ < J >$	n	$p'/p'_c : x_0 = 0.985, w = 0.01$	$J_{||}/ < J >$
5	2.4111	1.17	10	2.3320	1.13
10	2.1725	1.07	20	2.2222	1.09
15	2.2222	1.09	30	2.3597	1.15

It is worthwhile to note that the stability thresholds change very flatly when n deviates from the most unstable toroidal wave number. Another interesting point is a simple rule of thumb relating the unstable band of n with the pedestal shape under fixed pedestal width w: two times lower p'_{edge}/p' (or $J_{||edge}/J_{||}$) corresponds to a factor of $2/3$ for the unstable toroidal wave numbers.

2 Higher triangularity. ITER AT case To investigate the influence of the plasma geometry on the edge mode stability and check the scaling robustness under the profile variation an ITER related equilibrium with reversed shear [4] was chosen. The original equilibrium with $\beta_N = 2.6$ is unstable against global $n = 1$ external kink mode. In order to make the edge stability picture clearer the β value was lowered by a factor of 2 keeping the current density profile approximately fixed.

The main difference between the stability diagrams for TCV (Fig.2) and ITER (Fig.3) cases is the stability boundary in normalized pressure gradient p'/p'_c. The coupling between current driven kink and ballooning modes sets the limit to the value of p'/p'_c preventing an access to the second stability region. However it happens at larger values of p'/p'_c in the ITER case with higher triangularity of the plasma boundary: $p'/p'_c \sim 2.7$ (triangularity $\delta = 0.5$ for the ITER case) versus $p'/p'_c \sim 1.7$ (triangularity $\delta = 0.3$ for the TCV case) for $J_{||edge}/J_{||} = 0.42$.

3 Edge current induction modelling Experiments on the TCV tokamak have shown that rapid vertical movement of diverted ELMy H-mode plasmas can affect the time sequence of ELMs [5]. The effect is attributed to the induction of an edge current during the movement of the plasma column in the spatially inhomogeneous vacuum field of a single null configuration. In TCV the fast vertical movement is provoked by the positional control G-coils inside the vacuum vessel. The goal of the modelling with the quasi-equilibrium evolution PET code was to get quantitative estimates of the value of the induced current density and the profiles. Two plasma models were incorporated into the two corresponding versions of the PET code. The first one is the quasi-equilibrium plasma evolution under condition of flux conservation and with self-consistent surface current (FCSC) modelling ideally conducting plasma [6]. The recent version of the PET code implements the quasi-equilibrium plasma evolution with self-consistent magnetic field diffusion (MFD).

The influence of the edge temperature on the current perturbations was analyzed using the MFD model. The temperature profile was kept fixed to preserve the quasi

stationary current density profile (inversely proportional to the conductivity) for TCV plasma. In the series with the ratio of the axis temperature to the edge temperature $T_a/T_b = 10$ the perturbed value $\delta J_{\parallel edge}$ was found to be proportional to the temperature (but not to conductivity $\sim T^{3/2}$). Even a small perturbation of the current density in the pedestal can affect the ELM dynamics provided that the perturbations are well phased with the ELM crashes when plasma reaches marginal stability. For the highest edge temperature in the series $T_b = 1keV$ the values of the total current in the pedestal current perturbations approaches the magnitudes of the surface current perturbations in the FCSC model ($\delta I_{edge}/I_p \sim 1\%$), thus providing an independent verification of the models and the numerics used. The corresponding values of perturbed current density $\delta J_{\parallel edge}/ < J > \sim 0.5$ would be sufficient to trigger the current-driven modes. The perturbed profile is highly localized at the plasma boundary (the half-width is $\delta\psi \sim 0.01$) that makes necessary high grid resolution near the edge in the plasma evolution computations.

   In the stability studies the induced current density was modelled by a skin profile with a maximum at the plasma edge similar to the perturbations obtained in the quasi-equilibrium modelling (Fig.4a). The conclusions from the KINX stability calculation are consistent with the general picture of current driven edge instabilities and, in particular, the fact that the current-driven instabilities with low mode numbers are triggered when $p' < p'c$. In accordance to that, the modes with lower toroidal mode numbers are driven unstable with the perturbed current density than in the series with the pedestal current density defined by bootstrap current peaking in the middle of pedestal $\sqrt{\psi} = x_0$ (Fig.4b). The stabilizing influence of the current perturbation with the reversed sign also follows the same line: only low-n modes are affected. The corresponding increase of the shear lead to the loss of the second ballooning mode stability access at the plasma edge and destabilization of higher-n modes (Fig.4c). It means that, the most unstable wave numbers can be insensitive to or even destabilized by the negative edge current perturbations. Transport calculation would be needed to assess the details of the pressure gradient evolution once $n = \infty$ ballooning modes are unstable.

4 Conclusions For fixed plasma boundary and the pedestal shape (described by the ratio of the edge current density to the maximal in the pedestal $J_{\parallel edge}/J_\parallel$) the edge stability diagrams are close to each other in the parametric plane ($p'/p'_c, J_\parallel/ < J >$) for the modes with toroidal mode numbers n following the scaling $w \times n \times q_{95} = $ const under variations of the pedestal width w and the value of q_{95}. The value of $J_\parallel/ < J > \sim 1$ is a good approximation for the limit against the current driven modes. The most unstable toroidal mode number n decreases and the corresponding limit in $J_\parallel/ < J >$ increases for lower values of the ratio $J_{\parallel edge}/J_\parallel$. The limit in pressure gradient p'/p'_c set by coupled current driven/ballooning modes is sensitive to the shape of plasma boundary.

   The induced edge current in the ELM triggering experiments was found to be proportional to the edge temperature. The corresponding values of the perturbed current density $\delta J_{\parallel edge}/ < J >$ reaches the current driven mode stability threshold $J_{edge}/ < J > \sim 1$ for edge temperature more than 1keV. The edge current perturbation of both signs are likely to effect only the modes with the toroidal modes numbers lower than in the corresponding unperturbed ELM sequence. An integrated modelling of the ELM triggering including the free boundary equilibrium evolution with magnetic field diffusion, transport and consistent stability calculations is desirable.

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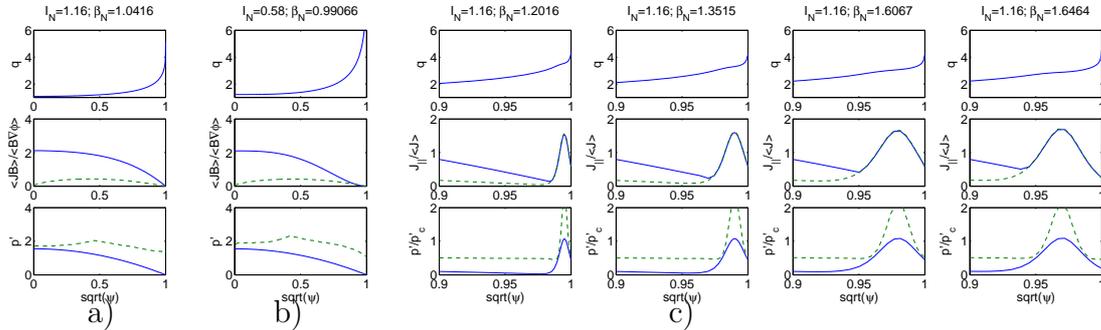

Fig.1 Profiles of the TCV shot #20333 reconstructed equilibria (a) and the high-q case with the same plasma boundary (b). Modifications of the pedestal profiles using $1 - \tanh^2\left(\frac{x_0 - \sqrt{\psi}}{w}\right)$ function for pressure gradient and bootstrap aligned current density (c): $w = 0.005, 0.01, 0.02, 0.02$, $x_0 = 0.995, 0.99, 0.98, 0.97$. The bootstrap current and marginal ballooning pressure gradient are shown by dashed lines
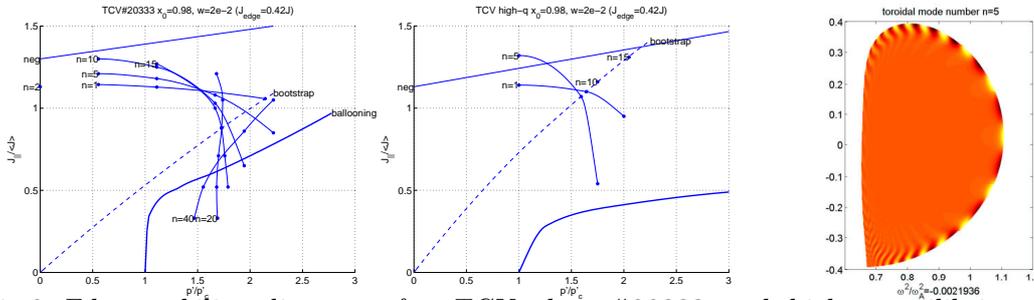

Fig.2 Edge stability diagrams for TCV shot #20333 and high-q equilibrium cases. $J_{||edge}/J_{||} = 0.42$. Ballooning mode stability boundary, bootstrap current density and shear reversal trajectories are shown.
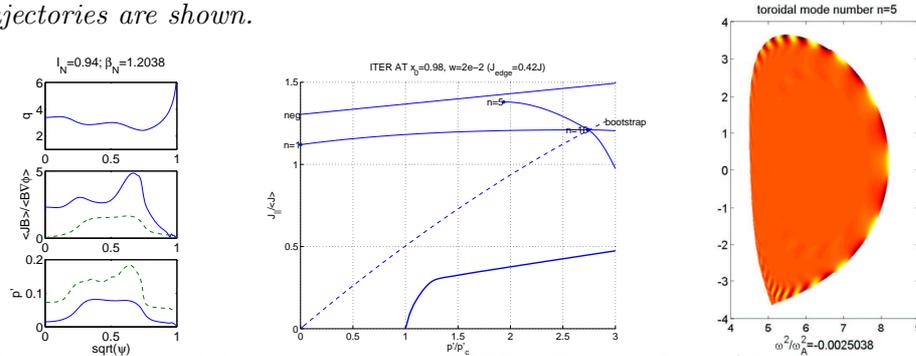

Fig.3 Profiles and edge stability diagrams for ITER AT case. $J_{||edge}/J_{||} = 0.42$.
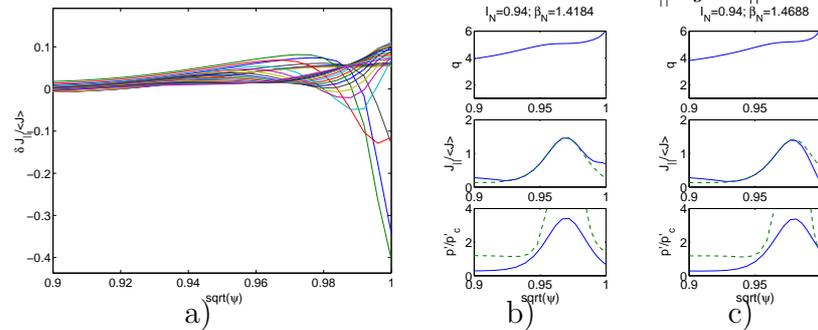

Fig.4 Evolution of the perturbed current density profile during the plasma vertical movement (a) and the bootstrap aligned profiles with added (b) and subtracted (c) skin current.