

Modelling of Equilibrium and Stability in Tokamak with Reversed Current Density

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The tokamak regimes with nearly zero toroidal current in the central region (the "current hole") attracted much attention due to a robust formation of the internal transport barrier and good confinement properties. Nonlinear MHD modelling [1, 2] of the current hole formation showed that the $n = 0$ reconnection prevents the current density from becoming negative in the plasma core. However the equilibrium structure in the current hole is still an open field of research. There is some experimental evidence of the negative current density in the current hole and no clear indications of the $n = 0$ reconnection events predicted by the theoretical models.

The negative current density in the core precludes the existence of the nested flux surface equilibria [3]. However the tokamak equilibria with $n = 0$ islands do exist. The properties of such equilibria were studied in detail using model current density profiles [4]. A variety of the negative current density equilibrium configurations with different magnetic surface topology and more realistic profiles is computed including the cases with finite β . It provides a whole range of novel equilibrium configurations suitable for ideal and resistive MHD modelling.

A traditional approach to the MHD equilibrium and stability code development is the use of magnetic coordinates. In the presence of magnetic islands, there is no a monotone magnetic surface label in the whole plasma volume. One of the possibilities is to separately treat the subdomains with nested flux surfaces [5, 6]. The use of moving adaptive grids to compute the equilibria prescribing different flux functions in several subdomains is quite efficient when the magnetic surface topology is fixed like in doublet configurations. However more flexible approach is desirable in a general case.

An anisotropy of the MHD equations makes the stability modelling a challenging task even in linear axisymmetric case especially if computational grids fixed in space are employed. The use of unstructured adaptive grids presents an attractive possibility of an efficient and versatile approach to the MHD computations with variable magnetic surface topology.

1 Current hole equilibria For the Grad-Shafranov equation

$$-R^2 \nabla \cdot \left(\frac{\nabla \psi}{R^2} \right) = R j_\phi \quad (1)$$

two flux functions $p(\psi)$ and $f(\psi)$ should be prescribed to define the equilibrium toroidal current density

$$j_\phi = R p' + f f' / R, \quad \text{where } p' = dp/d\psi, \quad f' = df/d\psi.$$

The main complication of the equilibrium problem with current density reversal is different specification of the flux functions to prescribe the negative current density region in the core and the positive current density region outside it. The two regions are delimited by some closed magnetic surface. It is assumed that both flux functions p' and $f f'$ vanish on that surface. Some index line of the grid can be chosen to adapt to the delimiting surface. To be able to specify profiles at the plasma periphery it is convenient to define one more grid index line lying close to the magnetic island x-point. In contrast to the delimiting surface, this index line (referred below as "island line") does not correspond to any magnetic surface, it is not adapted and is chosen on each iteration as the index line closest to the x-point.

Besides the profiles, we prescribe the diameter (horizontal size) of the delimiting surface $2a_d$ and the current ratio I_{in}/I_{out} as in [4]. The following steps of the numerical procedures are performed:

- the approximation of the delimiting surface by tracing the flux surface through some reference node of the computational grid using current values of the function ψ ;
- the adaptation of the chosen index line to the delimiting surface;
- the Picard iteration for the equation (1);
- the shift of the delimiting surface position between the iterations to preserve its diameter and using the change of the solution as a feedback to stabilize the iterations.

For profile specification the combinations of the monotone functions $F(s)$ and the "hollow" distribution $H(s)$ are used:

$$F(s) = 1 - (1 - s)^4, \quad 0 \leq s \leq 1, \quad F(s) = 0, \quad s < 0, \quad F(s) = 1, \quad 1 < s$$

$H(s) = (1 - \tanh^2(\frac{0.5-s}{0.15}))$, $0 \leq s \leq 1$, $H(s) = 0$, $s < 0$, $1 < s$. The profiles ff' and p' are chosen as follows:

- $ff' = h_{in}F(s)$, $p' = 0$, $s = (\psi - \psi_{ds})/(\psi_{ax} - \psi_{ds})$, $p' = 0$ inside the delimiting surface,
- $ff' = F(s) - 5c_iH(s)$, $p' = c_iH(s)$, $s = (\psi - \psi_{ds})/(\psi_{xp} - \psi_{ds})$ between the delimiting surface and "island line",
- $ff' = 1$, $p' = c_eH(s)$, $s = (\psi - \psi_{xp})/(\psi_b - \psi_{xp})$ in the "island line" exterior.

Here $\psi_{ax}, \psi_{ds}, \psi_{xp}, \psi_b$ are the values of the poloidal flux function at the main magnetic axis, at the delimiting surface, at the x-point and at the boundary respectively. In the case of negative core current, $h_{in} < 0$, the following inequalities take place: $\psi_{ax} < \psi_{ds} < \psi_{xp}, \psi_b < \psi_{xp}$. The coefficient "5" in the formula for ff' is used to approximate the bootstrap aligned current density in the vicinity of the current hole with $a_{hole}/R \sim 10$. The value of the coefficient h_{in} is adjusted to satisfy the prescribed total current ratio I_{in}/I_{out} .

The performed experiments showed that use of continuous profiles makes the numerical procedure less robust compared to the cases with piece-wise constant profiles: the reliable converged equilibria were obtained for relatively flat core current density (power "4" in the formula for $F(s)$) and for sufficiently negative core total current ($I_{in}/I_{out} \lesssim -0.3$ with the diameter of the delimiting surface $a_d = 0.2$).

The shape of the magnetic island is sensitive to plasma pressure. It is especially noticeable in the case of circular cross-section and nonzero pressure in the region between the delimiting surface and the island ($c_i \neq 0, c_e = 0$), see Fig.1. The gradual increase of the pressure shifts the negative current region outwards changing the island topology: the island magnetic axis splits ("m=2" island structure arises); "m=1" island recovers for higher pressure gradient with the x-point at the low magnetic field side.

In the case of elongated plasma (Fig.2) the magnetic island structure has two separatrices, one inside another. Again due to the Shafranov shift of the magnetic surfaces finite pressure can change the island boundary: the separatrix with an x-point at the low magnetic field side becomes external when the pressure is sufficiently large.

Fig.3 shows the examples of equilibria with nonzero pressure at the plasma periphery ($c_i = 0, c_e \neq 0$). The increase of the pressure with fixed total current ratio I_{in}/I_{out} makes the negative current density more negative (Fig.3,b). It is an explanation of visible inward shift of the core region. If the total current I_{out} is calculated not taking pressure into account (Fig.3,c), the core position does not significantly deviate from the corresponding force-free configuration (Fig.3,a).

All the Figures 1,2,3 show the magnetic surfaces and current density profiles; different colors correspond to the level lines of the poloidal flux function ψ , the parameters of the negative current region are $a_d = 0.2$, $I_{in}/I_{out} = -0.3$; aspect ratio $R/a = 3$.

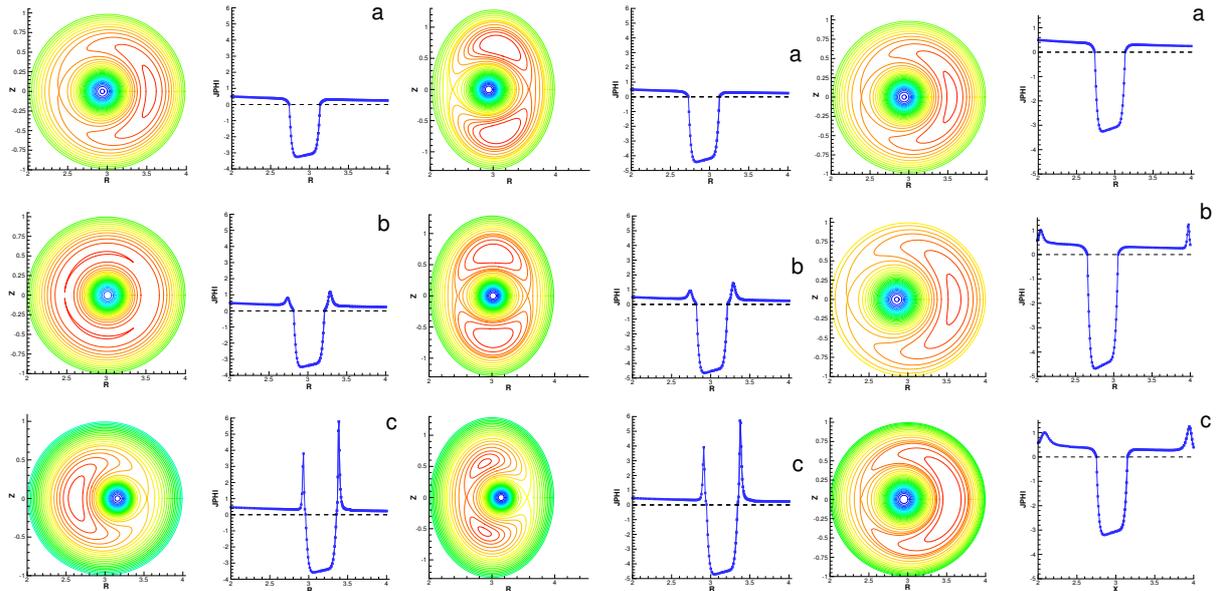


Fig.1 Circular cross-section, increasing pressure in the region between the delimiting surface and island separatrix: $c_i = 0$, $c_e = 0$, a) $c_i = 0$, b) $c_i = 0.5$, c) $c_i = 3$.
 Fig.2 Elongated cross-section ($E = 1.3$), increasing pressure in the region between the delimiting surface and island separatrix: $c_e = 0$, a) $c_i = 0$, b) $c_i = 0.65$, c) $c_i = 3$.
 Fig.3 Circular cross-section, increasing pressure at the plasma periphery: $c_i = 0$, a) $c_e = 0$, b) $c_e = 0.25$, c) $c_e = 0.25$, ignoring pressure in the current evaluation.

2 Discussion on numerical methods for stability studies The most of linear MHD stability codes works for the conventional toroidal configurations with nested magnetic surfaces. The codes usually get benefits from using flux coordinates and displacement projections normal and tangential to magnetic surfaces. An extension of these traditional methods to cases without nested magnetic surfaces makes it necessary to explicitly decompose the plasma region into several subdomains like it was done for doublet configurations [6]. Much more general and promising approach would be to refuse from the use of special coordinates and magnetic surface projections. It would make stability calculation closely related to nonlinear MHD modelling [7, 8, 9, 10].

Several important questions arise on the model and discretization method which should still accurately approximate some basic properties: divergence-free magnetic field, singular operator $\mathbf{B}\nabla$ and so on. In particular, if it is sufficient to involve high order finite elements for that?

On the other hand MHD modelling typically demonstrates strong anisotropy of the solutions: magnetic reconnection, ideal MHD activity at resonant surfaces, ELMS, SOL etc. Even the eigenfunctions for ideal $n = 0$ modes can display anisotropic features when unconventional equilibrium configurations with internal separatrix are considered. In particular, in doublet configuration (Fig.4) the most unstable eigenfunction $n = 0$ demonstrates fast change of the displacement at the internal separatrix. Analogous behavior can be expected for the current hole configurations.

Optimal grids for such calculations should have highly stretched cells (anisotropic grids). To resolve a priori unknown solution features automatic grid adaptation looks a very attractive capability. Probably the most general and flexible approach is the use of unstructured grids with relatively simple and robust adaptation techniques (in particular, local refinement/derefinement).

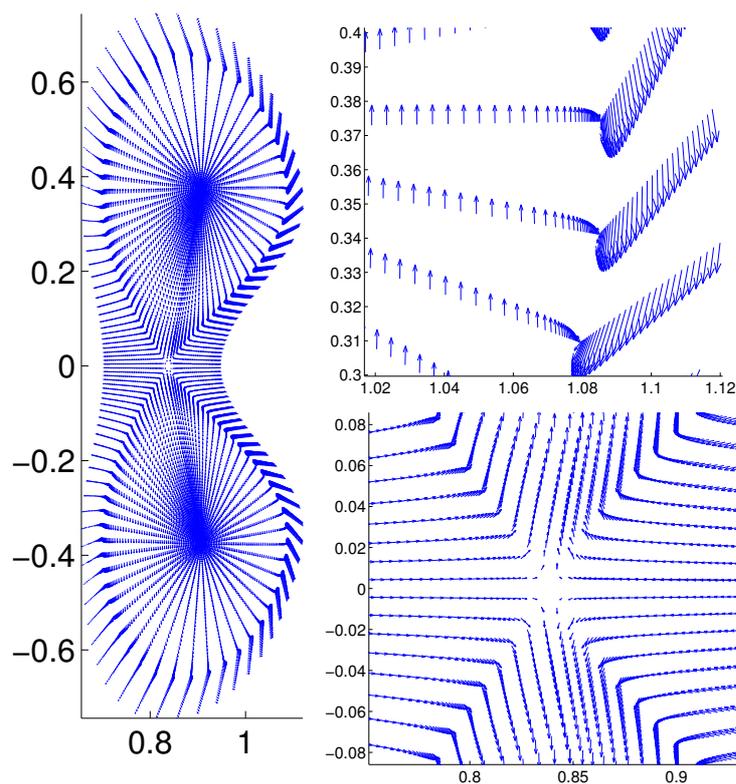


Fig.4 Ideal $n = 0$ mode eigenfunction for doublet configuration with conventional (positive) current density in whole the plasma domain. The zoomed fragments at the x -point and at upper part of the separatrix are shown.

As a conclusion it seems reasonable and promising to investigate possible approaches to MHD modelling on unstructured adaptive grids. First steps in that direction caaan be calculation of equilibrium and ideal MHD stability (starting, for instance, with $n = 0$ modes) for axisymmetric "current hole" configurations.

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