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Generalised evolving background f_0 for plasma δf Particle-in-cell simulations

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1. Introduction

The noise in electrostatic PIC simulation is the error inherent in calculating the charge density with a Monte Carlo (statistical) technique using a finite number of sampling points (markers).

The δf scheme aims to reduce the noise in the PIC method by splitting the full distribution function as $f = f_0 + \delta f$, with f_0 referred to as the background. The charge density is calculated analytically from f_0 and statistically from δf . If δf is small compared to f_0 the error in the resultant charge density is smaller than for a full-f scheme [1], *i.e.* the noise is reduced. Such a procedure requires the specification of an f_0 close to the true value of the distribution function f. It is through this additional information that the simulation wins.

The importance of a good choice of the background has been demonstrated both theoretically [2][1] and practically [3]. However, it is clear that if f_0 is fixed and f evolves far from its initial value, δf is not going to remain small. Hence one seeks methods by which f_0 may be evolved in time in order to avoid this.

For collisional dominated simulations the obvious choice of f_0 is a Maxwellian distribution, f_M. Brunner *et.al.* [4] showed the practicality of such a scheme which evolved the three parameters of a Maxwellian f_0 with fluid equations, closed using information from the markers. Valeo [5] also considered an evolving Maxwellian background for modelling of electron plasma waves.

When the system is not collision dominated a Maxwellian f_0 is not necessarily appropriate. We seek therefore to develop methods which use a more general representation of the evolving background.

Our current project is to examine the properties of one such generalised scheme which represents f_0 on a fixed phase space grid thus allowing for an arbitrary number of free parameters. We investigate the feasibility of this approach for solving the Vlasov-Poisson system in a 2 dimensional (x,v) phase space.

2. The model

We solve the Vlasov equation,

$$\partial_t \, f_s + v \, \partial_x \, f_s + \frac{q_s E}{m_s} \, \partial_v \, f_s = 0. \tag{1}$$

Where $f_s(x, v)$ may be the electron or ion distribution function. In the former case $s = e$ the eclectic field E is calculated using Poisson's equation with the ions assumed to provide a fixed neutralising background.

$$\partial_x \, E = -\frac{e}{\epsilon_0} \int f_e \, dv + \frac{eN_0}{\epsilon_0} \tag{2}$$

here

$$N_0 L \stackrel{\text{def}}{=} \int f \, dx \, dv \qquad\qquad L \stackrel{\text{def}}{=} \int dx \tag{3}$$

. In the latter case $s = i$ the electric field is calculated using the quasineutrality condition $N_i = N_e$ together with an adiabatic electron response,

$$\int f_i \, dv = N_0 \, e^{\frac{e\phi}{T_e}} \tag{4}$$

$$\frac{e\phi}{T_e} = \log \frac{N_i}{N_0} \tag{5}$$

. In the PIC method the markers, distributed as $p(x, v)$, represent f as

$$\sum_i^N f_i \delta(x - x_i)\delta(v - v_i) \tag{6}$$

and solution of discretised forms of Eqs (2) or (4) will require evaluation of

$$\int f(x, v)\Lambda_j(x) \, dx \, dv \tag{7}$$

where the Λ_j are the functions used to discretise the density. The δf method corresponds to calculating this as

$$\int f_0(x, v)\Lambda_j(x) \, dx \, dv + \frac{1}{N} \sum_i^N \frac{\delta f_i \Lambda_j(x_i)}{p_i} \tag{8}$$

.

3. Adaptive f_0 for the electron plasma wave

The Landau Damping of an electron plasma wave was chosen as the test scenario for the scheme. Two adaptation algorithms have been looked at

$$(i) \quad f_0(t) \simeq f_0(t - \Delta t) \qquad (ii) \quad f_0(t) \simeq \langle f \rangle_{x,t}$$

. In the first case

$$f_0(x, v) = f_M(v) + \alpha_{ij}(t) S_{ij}(x, v) \tag{9}$$

With the S_{ij} cubic splines in x (corresponding to the functions used to represent the density) and piecewise constant in v. The coefficients α_{ij} were calculated using a marker deposition in phase space at the previous time step and a mass lumping approximation for the spline inversion problem. Under this scheme the background may contribute to the charge density.

In the second case the adaptive part of the background was taken to be a time and space-averaged departure from the initial Maxwellian, represented by piecewise constant functions.

$$f_0(v) = f_M(v) + \alpha_i(t) C_i(v) \tag{10}$$

Here a running average in time was used,

$$\alpha_i(t^{j+1}) = \beta \frac{1}{L} \int f(x, v, t^{j+1}) C_i(v) \, dv \, dx + (1 - \beta)\alpha_i(t^j) \tag{11}$$

representing a decaying time average weighted as $\exp -\frac{\beta}{\Delta t} t$, in the limit of small β. For this scheme, f_0 does not contribute directly to the charge density.

4. Preliminary results and conclusions

A population of 128,000 markers with $p \propto f$ was used. f_0 was gridded with 23 points in x

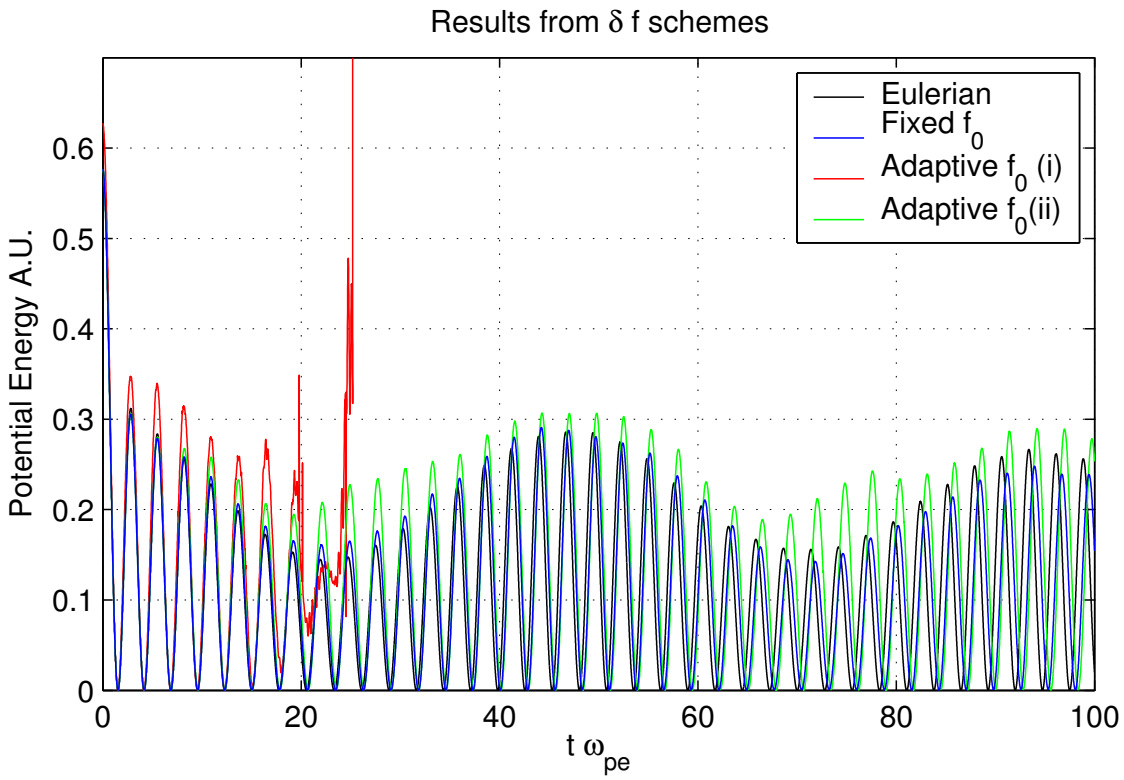

Figure 1: A comparison of the total electrostatic potential energy as a function of time for the Eulerian, fixed f_0 PIC and the two adaptive f_0 PIC variant codes. Algorithm (i) becomes unstable around $t\omega_{pe} \simeq 23$.

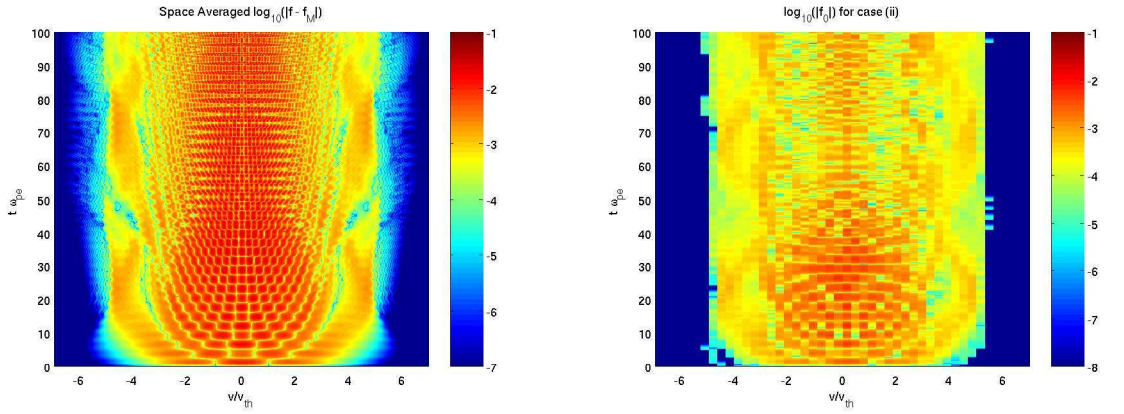

Figure 2: Left: The time evolution of the departure of the space averaged distribution function from the Maxwellian (Eulerian code). Right: The time evolution of f_0 for algorithm (ii). The adaptation captures some of the behaviour in the trapping region $v_\phi \simeq 3.8 v_{th}$.

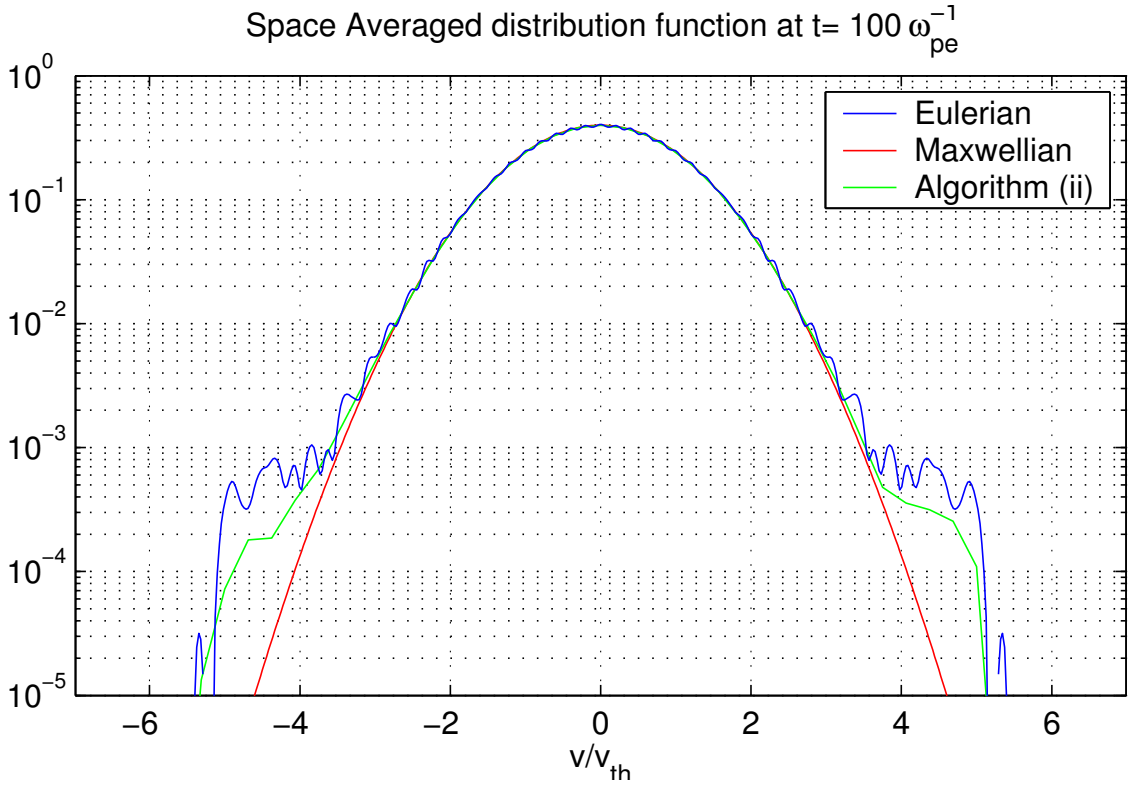

Figure 3: The spaced averaged distribution function at the end of the simulation $t\omega_{pe} = 100$, flattening in the trapping region is reflected in the adaption of f_0 (green line) for algorithm (ii)

and 64 points in v and a time step of $0.01/\omega_{pe}$. An Eulerian code was used to benchmark the PIC methods. Results are plotted in figure 1. A fixed $f_0 = f_M$ scheme was able to reasonably match the Eulerian result. Adaptation algorithm (i) was subject to an as yet unidentified numerical instability. A formal evaluation of the relative error in the density due to f_0 and that due to δf is required for this case. For algorithm (ii), $\beta = 0.05$, the background was able to capture some of the secular effects, the flattening in the trapping region (around $v_\phi = 3.8 v_{th}$), see figures 2 and 3. The representation of such secular terms, in particular profile flattening, is of great interest in *e.g.* gyrokinetic turbulence simulation.

[1] S. J. Allfrey and R. Hatzky. A revised delta-f algorithm for nonlinear pic simulation. *Comput. Phys. Commun.*, 154:98, 2003.

[2] A. Y. Aydemir. A unified monte carlo interpretation of particle simulations and applications to non-neutral plasmas. *Phys. Plasmas*, 1:822, 1994.

[3] R. Hatzky, T. M. Tran, A. Könies, R. Kleiber, and S. J. Allfrey. Energy conservation in a nonlinear gyrokinetic particle-in-cell code for ion-temperature-gradient-driven modes in θ-pinch geometry. *Phys. Plasmas*, 31:898, 2002.

[4] S. Brunner, E. Valeo, and J. A. Krommes. Collisional delta-f scheme with evolving background for transport times scale simulations. *Phys. Plasmas*, 6(12):4504, 1999.

[5] E. J. Valeo and S. Brunner. A kinetic δf model of plasma and acoustic waves which includes an evolving background distribution. APS DPP 2001.