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Equilibrium and Stability for the ARIES Compact Stellerator Reactor

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Equilibrium and ideal magnetohydrodynamic (MHD) stability studies are reported for Compact Stellarator (ARIES-CS) reactor design equilibria based on a scaled three-period NCSX configuration and a two-period quasi-axisymmetric variant, the MHH2 stellarator. With a stabilizing shell at about twice the minor radius, robustly stable equilibria up to β=6% are achievable. Recent experiments raise questions as to the applicability of linear MHD stability in stellarators since the predicted stability limits appear to be significantly exceeded. A context for interpreting this question, consistent with tokamak experience, is discussed; both the equilibria and nonlinear consequences need to be more carefully considered. Nonlinear stability is analyzed by computing solutions with highly resolved discontinuities to effectively simulate current sheets and islands. This yields β limits in better agreement with measured values.

I. Linear Ideal MHD Stability Limits

A first step in evaluating the prospects for any proposed fusion experiment is to determine reference equilibria and test the linear stability properties. For an equilibrium based on a scaled-up three-period NCSX configuration with β = 4.1% and A = 4.47, the linear MHD stability was determined using the TERPSICHORE [1] code. The stability results for this equilibrium are restricted to a range of external conformal conducting walls between a_w = 1.7 and 2.7 times the minor radius of the plasma. For the wall in this range, the plasma is stable.

A sequence of higher β equilibria was then constructed with the volume, average major radius, average minor radius, and vacuum toroidal field held fixed as β was increased through uniformly scaling the pressure profile. Also, the calculation was a fixed boundary calculation. The major change in the equilibria is that the magnetic axis shifts outward by about 5% as β increases. This sequence was then tested for ideal stability. With a conformal stabilizing shell at twice the plasma minor radius, robustly stable equilibria up to 6% are achievable. The computed growth rates were sensitive to mesh size but the stability boundaries were unchanged.

A two-period variant, the MHH2 stellarator, with good quasi-axisymmetry comparable to that in a tokamak with toroidal ripple, which has some engineering advantages, was also considered. This can be generated by a set of only eight moderately twisted modular coils; a fixed-boundary MHH2 equilibrium was reproduced with β = 4.52%. Figure 1 shows a comparison with the three-field period equilibrium. The main shape difference is the lower order shaping near $N_{fp}\phi \sim 180$ deg. The rotational transform profiles, however, are very different; for MHH2, the ι profile monotonically decreases over most of the minor radius, increasing again only near the edge. For the three field period equilibrium, the converse is true.

A series of increasing β MHH2 equilibria was then constructed with $\langle\beta\rangle = 4.52\%$, 5.71%, and 6.92% as for the previous case. The ι profile was adjusted to force the average current density to vanish as β increases; the bootstrap current was not self consistently calculated. These equilibria were tested for ideal MHD stability. The Mercier criterion indicates stability at all surfaces up to β=5%. The TERPSICHORE studies find the growth rates are small and insensitive to wall position. They remain small up to β=6.9% indicating marginal stability to free boundary modes.

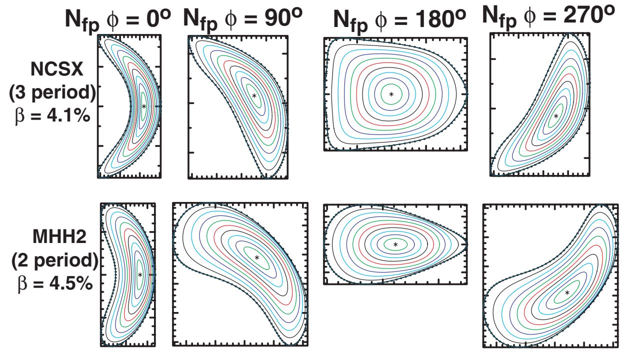

Fig. 1. Comparison of the three- and two-period candidate equilibria at four toroidal cross sections.

II. Stellarator β Limits

W7-AS and LHD have exceeded predicted β limits by significant margins; the calculated local limits of less than 2% were exceeded, with β reaching above 3% in both devices. Stellarator designs that are based on optimizing against linear ideal MHD stability are therefore questionable. There are two possible resolutions based on experience with tokamak stability limits. First, the discharge equilibria are not necessarily those used in the stability predictions. Second, the nonlinear consequences of linearly unstable ideal modes need to be understood; heuristically, internal modes surrounded by a fairly robustly stable outer shell can be expected to be benign and not limit β directly.

A. Equilibrium Questions. Experience with tokamaks has shown in recent years that the ι (or q) profile is not necessarily what theoretical arguments suggest it should be. It is important to use actual reconstructed discharge equilibria and test the sensitivity of stability predictions to profile and boundary variations within the experimental errors [2]. Reducing these errors is crucial to obtaining unambiguous predictions. Critical parameters are the last closed flux surface, the q and pressure gradient profiles, and the bootstrap current, especially at the edge. Considerable success has been achieved in demonstrating the quantitative validity of ideal limits, mode structures and growth rates, when these are properly accounted for [2].

For stellarators, experimental measurements of the pressure and ι profiles at finite β, like those in large tokamaks, are needed to pin down the apparent violation of predicted stability limits. For example, the bootstrap current is often assumed negligible but may not necessarily be so. Even the nested surfaces assumption may not be the case in stellarator discharges.

B. Nonlinear Stability Consequences. Experience with tokamaks has also shown that some predicted ideal limits can be violated but that the nonlinear consequences do not directly limit β or lead to disruptions. Examples are sawteeth and ELMs. Also, local Mercier, and even ballooning stability, are generally considered ignorable when unstable only over a small region. This is fairly well understood intuitively but the same understanding is not yet available for stellarators.

For stellarators, there are good experimental and theoretical justifications for ignoring local stability. For global modes, the nonlinear consequences need to be examined on a case-by-case basis. Nonlinear stability can be analyzed by computing weak solutions for 3-D equilibria, with discontinuities to simulate current sheets and island. The key is to accurately resolve the discontinuities by ensuring that the ideal equations are in conservative form.

The MHD equations describing force balance can be put in conservation form: $\nabla\cdot \mathbf{T} = 0$, $\nabla\cdot \mathbf{B} = 0$ where the Maxwell stress tensor $\mathbf{T} = \mathbf{B}\ \mathbf{B} - (B^2/2 + p)\ \mathbf{I}$. Finite difference equations that respect the conservation form telescope into an approximate statement of force balance over the boundary when summed over a test volume: $\iint \mathbf{T} \cdot d\mathbf{S} = 0$. In a simplified example using Burgers equation as a model problem for an RFP in a slab, $x \in (1,+1)$: $2\Psi_x \Psi_{xx} = \left(\Psi_x^2\right)_x = \eta\Psi_{xxx}$, subject to boundary conditions $\Psi(-1) = \Psi(-1) = 0$, and $\Psi_x(-1) = 0$, a conservative difference scheme: $(\Psi_{n+1} - \Psi_n)^2 - (\Psi_n - \Psi_{n-1})^2 = \eta(\Psi_{n+2} - 3\Psi_{n+1} + 3\Psi_n - \Psi_{n-1})$ imposes force balance in the limit where $\eta = 0$, across the discontinuity at $x = 0$, so that the solution reduces to $\Psi = 1 - |x|$. However, a non- conservative finite difference scheme defined with an additional diffusive term, $\varepsilon(\Psi_{n+1} - 2\Psi_n + \Psi_{n-1})^2$, added to the left hand side, diverges when η is much smaller than the mesh size $\varepsilon \neq 0$. When $\eta \sim \varepsilon$, force balance is not maintained by a non-conservative scheme; this is exhibited numerically by inequality of the slopes on the two sides of the limiting solution $\Psi = 1 - |x|$. which characterize the magnetic field on opposite sides of a current sheet and simulates a chain of magnetic islands.

The NSTAB [4] equilibrium and stability code applies the MHD variational principle to compute weak solutions of the conservation form of the dynamical equations. Solutions are represented in terms of Clebsch Potentials. The code computes fixed boundary 3-D equilibria and nonlinear stability is tested by applying a mountain-pass theorem that requires a search for multiple or bifurcated equilibria. The code utilizes a finite difference scheme based on the above discussion that captures discontinuities with unusually fine resolution.

III. Discussion

It is significant that both the ARIES-CS equilibria have linear ideal MHD β limits above 5% since the evidence from experiments suggests that these may be lower limits. To resolve the question of the meaning of these limits requires high quality equilibrium reconstructions and stability calculations using the experimental equilibria to determine which limits are really being violated. Then, robust nonlinear calculations are needed to obtain an intuitive feel for the consequences of the instabilities predicted. This is not yet at hand and consequently, it is not clear at present how valuable linear MHD stability predictions using nested flux surface equilibria really are; they might, in fact, be quite misleading. Free boundary non-nested equilibria, and/or nonlinear stability studies may be needed to obtain meaningful predictions.

The existence of a nested flux surface equilibrium can be considered as either an equilibrium problem or a stability problem; unstable equilibria with nested surfaces will evolve to non-nested surfaces with lower energy if physically possible. Equilibrium codes can therefore be considered stability codes, since an equilibrium computed under certain constraints must be stable unless those numerically imposed constraints can be avoided by a physically valid motion. NSTAB makes use of this to determine nonlinear stability when bifurcated equilibria exist. The code does impose nested flux surfaces but permits well-resolved discontinuities to simulate islands. The major constraint imposed, is the fixed boundary; NSTAB can only guarantee guarantee stability to internal global modes.

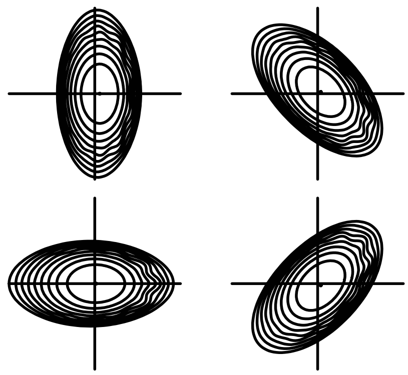

Fig. 2. Poincare map of the flux surfaces at four cross sections over one field period of a bifurcated LHD equilibrium at β = 0.04. For a broad pressure profile $p = p_0(1 - s^2)$, this solution developed a ballooning mode but remained marginally stable.

From a theoretical and numerical point of view, the first step then in resolving the questions formulated here, is to compare linear and nonlinear predictions and to compare both against experiments. The nonlinear stability test in NSTAB yields beta limits that are in substantial agreement with measured values; the calculations show that LHD is marginally stable at the largest values of β achieved and there is similar agreement for W7-AS.

IV. Conclusions

This work shows that one can obtain linear ideal MHD β limits for a compact stellarator reactor of order of 6%. Future cork will examine the sensitivity of the linear β limits for the two configurations to profile and shape variations. These are linear stability limits and there are questions as to their relevance. In the longer term, these questions need to be resolved by examining the stability of actual reconstructed discharge equilibria with measured ι and pressure, and the sensitivity to profile and boundary variations. Additional discrepancies should be resolved by nonlinear studies and routine comparisons with observations. This will entail repeating the nonlinear calculations for the ARIES-CS equilibria by searching for bifurcated equilibria as described in Section II.B. The correlation of computations with observations was, in fact, exploited to design the two field period MHH2 stellarator.

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