

## Influence of thermal electron motion on surface wave discharge characteristics

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The electron temperature and the radial inhomogeneity of the plasma density are considered in studying the wave characteristics and the axial structure of surface-wave-sustained plasmas. It is shown that the relative degree of the thermal effects depends on the radial density profile and the plasma column radius.

### I. INTRODUCTION

Plasmas sustained by traveling electromagnetic surface waves have been widely studied in view of various technical applications as well as fundamental research [1]. The electron thermal motion has been usually neglected in studying their discharge characteristics, but its effects may be important in inhomogeneous plasmas. The inhomogeneity of the plasma density gives rise to the resonant enhancement of the radial electric field and possible generation of fast electrons near the wall where the wave frequency is close to the local plasma frequency [2]. The electric field is singular at this local plasma resonance in collisionless plasmas but this singularity can be removed by including collisions. As the thermal motion also resolves the singularity, its effect as well as that of collisions should be considered together to examine the enhancement of the electric field near the plasma resonance and the influence on dispersion characteristics. In this work, the dispersion characteristics of the surface electromagnetic waves and the axial density profile are investigated including the electron temperature and the radial inhomogeneity of the plasma density.

### II. GOVERNING EQUATIONS

An azimuthally symmetric wave, that is, a transverse magnetic mode is considered for a plasma contained in a cylindrical glass tube of the inner radius  $a$  and the outer radius  $b$  with vacuum and a metal enclosure of the inner radius  $d$  outside. Using the momentum equation for the electrons, the perturbed current density can be obtained in terms of the perturbed electric field and charge density [3]:

$$\mathbf{J}_1 = \frac{i\omega_{pe}^2 \epsilon_0 f}{\omega + i\nu_e} \mathbf{E}_1 + \frac{iv_{te}^2}{\omega + i\nu_e} (\rho_1 \nabla \ln f - \gamma_e \nabla \rho_1), \quad (1)$$

where  $\epsilon_0$ ,  $\omega_{pe}$ ,  $\nu_e$ ,  $v_{te}$ ,  $\gamma_e$ ,  $f$  are the permittivity of free space, the electron plasma frequency, the electron-neutral collision frequency for momentum transfer, the electron thermal speed, the ratio of specific heats, the electron density profile normalized to the central density at the axis, respectively. Here, the ion equations are ignored as the ions are too slow to respond to the high frequency electron perturbation.

The perturbed charge and current are related to the electric and magnetic fields through the Maxwell equations. Assuming the spatial dependence  $\exp[i f k(z) dz]$  and normalizing the variables by the relations

$$\begin{aligned}
r &= \lambda\xi, & \lambda &= \frac{c}{\omega}, & \kappa &= k\lambda, & \delta &= 1 + i\frac{\nu_e}{\omega}, & \mathcal{E}_r &= E_r \\
\mathcal{E}_z &= -iE_z, & \mathcal{B}_\theta &= cB_\theta, & \Omega &= \frac{\omega}{\omega_{pe}}, & \varrho &= \frac{1}{\tau} \frac{\lambda}{\epsilon_0} \rho, & \tau &= \frac{\delta}{\gamma_e} \frac{c^2}{v_{te}^2},
\end{aligned} \tag{2}$$

the Maxwell equations yield

$$\frac{d\varrho}{d\xi} = \frac{\varrho}{\gamma_e} \frac{d \ln f}{d\xi} - \varepsilon_p \mathcal{E}_r + \kappa \mathcal{B}_\theta, \tag{3}$$

$$\frac{1}{\xi} \frac{d}{d\xi} (\xi \mathcal{E}_r) = \tau \varrho + \kappa \mathcal{E}_z, \tag{4}$$

$$\frac{d\mathcal{E}_z}{d\xi} = \kappa \mathcal{E}_r - \mathcal{B}_\theta, \tag{5}$$

$$\frac{1}{\xi} \frac{d}{d\xi} (\xi \mathcal{B}_\theta) = \kappa \varrho + \varepsilon_p \mathcal{E}_z, \tag{6}$$

where  $\varepsilon_p = 1 - f/(\Omega^2\delta)$  is the relative permittivity of the cold plasma.

For an uniform plasma the wave equations can be solved analytically yielding the dispersion relation in a closed form [4]. The finite difference method is used to numerically obtain the dispersion relation and the electromagnetic fields for an inhomogeneous plasma. For a complex wave number it is convenient to write the governing equations with  $\kappa$  eigenvalue; the wave equations are rearranged in the form

$$\left( \mathbf{I} \frac{d}{d\xi} + \mathbf{a} \right) \cdot \Psi = \kappa \mathbf{b} \cdot \Psi \tag{7}$$

where

$$\mathbf{a} = \begin{bmatrix} -\frac{1}{\gamma} \frac{d \ln f}{d\xi} & \varepsilon_p & \cdot & \cdot \\ -\tau & 1/\xi & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & -\varepsilon_p & 1/\xi \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & 1 & \cdot \\ \cdot & 1 & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot \end{bmatrix}, \quad \Psi = \begin{bmatrix} \varrho \\ \mathcal{E}_r \\ \mathcal{E}_z \\ \mathcal{B}_\theta \end{bmatrix}. \tag{8}$$

Although analytic solutions are available in the dielectric and vacuum regions, they can not be incorporated into the current form of the eigenvalue equation since the fields are given in terms of the modified Bessel functions whose argument have the factor  $\sqrt{\kappa^2 - \epsilon}$ , where  $\epsilon$  is the relative dielectric constant, so that the system of the equations is not linear in  $\kappa$ . Therefore, the fields should be found numerically in these regions as well as in the plasma and matched by applying appropriate boundary conditions at the interfaces. The wave equations with the relevant boundary conditions lead to the overall equation of the form  $\mathbf{A}\Psi = \kappa\mathbf{B}\Psi$  which is solved to obtain  $\kappa(\equiv \beta + i\alpha)$ , being generally complex, for a given value of  $\omega/\omega_{pe}$ .

The axial profiles of the density and temperature can be found from the particle and power balance relations combined with the axial damping rate  $\alpha$  [1,5,6]. Taking  $n(r) \simeq n_0 J_0(\mu r/a)$ ,  $T_e$  and  $\mu$  are determined from the Bohm condition at the sheath and  $T_e = -U_*/\ln[\Theta/\Theta_0]$ . Finally, the axial profile of the plasma density is obtained from

$$\frac{dn}{dz} = -\frac{2\alpha n}{1 - \frac{n}{\alpha} \frac{d\alpha}{dn} + \frac{n}{\Theta} \frac{d\Theta}{dn}}, \tag{9}$$

where  $\Theta = \nu_* U_*$ ,  $\Theta_0 = \nu_*^0 U_*^0$ ,  $\nu_* = \nu_*^0 e^{-U_*/T_e}$  ( $U_*$  and  $\nu_*$  are the excitation energy and frequency, respectively), and the particle balance equation is used to express  $\Theta$  in terms of the relevant plasma parameters.

### III. RESULTS

For illustration of the results, numerical calculations are conducted for the following parameters: the wave frequency  $\omega/2\pi = 2.45$  GHz, the relative dielectric constant of the tube  $\epsilon_d = 4.7$ , the outer radius of the tube  $b = a + g$  and the conductor boundary radius  $d = 3a$ . The inner radius  $a$  and the tube thickness  $g$  are taken to be 5 or 10 mm and 1 mm, respectively. In addition, the wave is supposed to be launched at  $z = 0$ , where the density is taken to be  $100n_{cr}$  or  $\Omega = 0.1$  ( $n_{cr} = \epsilon_0 m \omega^2 / e^2$ ).

Figures 1 and 2 show the phase diagram and the axial density profile for the case of  $p = 1$  Torr and  $a = 5$  mm. The solid and dotted lines represent the real and imaginary parts, respectively, and the heavy and light lines do the results of the cold and the warm plasma models, respectively. The warm plasma model reveals the resonance structure in the phase diagram, but this does not affect the axial density profile because the imaginary parts of the wave number are virtually same in the weak damping region ( $\alpha \ll \beta$ ).

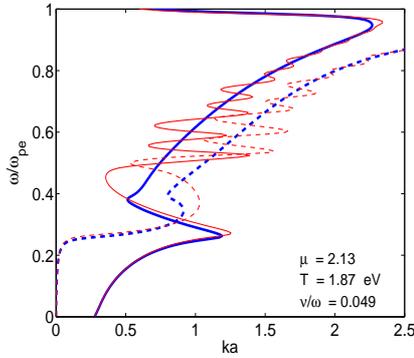


FIG. 1. Phase diagram for a plasma for  $p = 1$  Torr and  $a = 5$  mm.

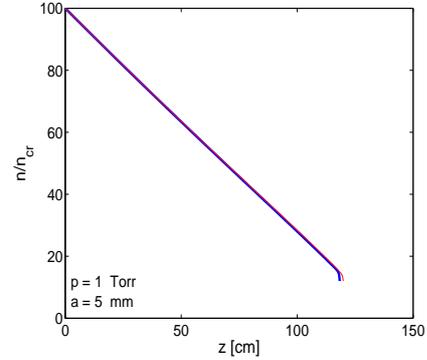


FIG. 2. Axial profiles of the plasma density for the case of Fig. 1.

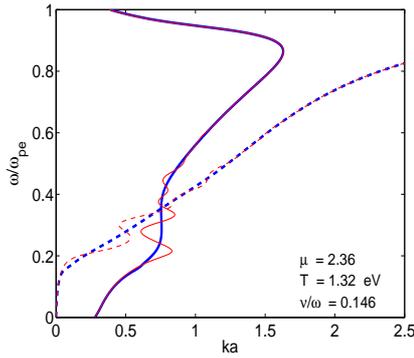


FIG. 3. Phase diagram for a plasma for  $p = 5$  Torr and  $a = 5$  mm.

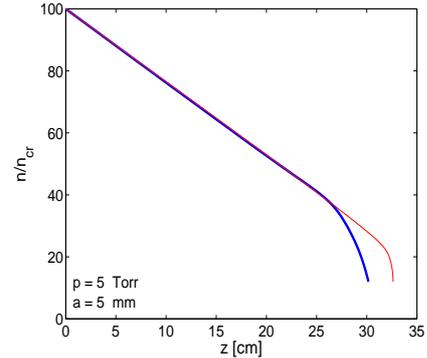


FIG. 4. Axial profiles of the plasma density for the case of Fig. 3.

As the pressure increases, the radial density profile becomes steeper, bringing the thermal modes to the higher density region as shown in Fig. 3. The damping rate  $\alpha$  is smaller from the warm plasma model than from the cold plasma approximation before it grows around  $\Omega \simeq 0.2$ , and this difference makes the wave propagate a little further before it damps out, as suggested by Fig. 4. The plasma column radius also affects the thermal resonance through its role in the mode spacing in the phase diagram as well as

the relation to the density inhomogeneity [4], affecting the axial damping rate and the axial density profile consequently as demonstrated in Fig. 5. Finally, the radial electric field profiles from the cold and warm models are compared in Fig. 6.

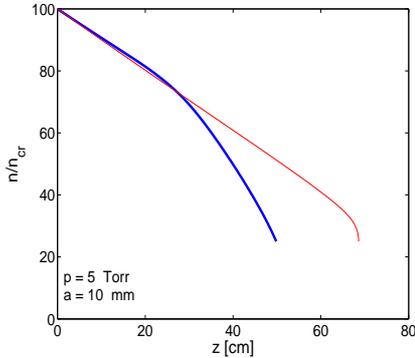


FIG. 6. Axial profiles of the plasma density for  $p = 5$  Torr and  $a = 10$  mm.

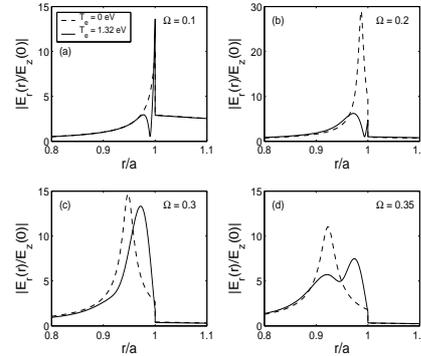


FIG. 5. Comparison of the radial electric field profiles for the cold and warm plasmas at the different values of  $\Omega$  for the case of Fig. 3.

#### IV. CONCLUSIONS

The electron temperature and the inhomogeneity of the plasma density are considered in studying the discharge characteristics of surface-wave-sustained plasmas. The electron temperature and collision frequency are apparently the important parameters indicating the effects of the electron thermal motion. As the temperature decreases or the collision frequency increases, the thermal effects become less significant. But these changes are usually associated by the pressure increase; the increase in the pressure as well as the plasma column radius results in a steeper radial density profile. The strong inhomogeneity shifts the thermal modes down to the higher density region in the phase diagram, affecting the axial damping rate and then the axial density profile consequently.

#### ACKNOWLEDGMENTS

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