*31st EPS Conference on Plasma Phys. London, 28 June - 2 July 2004 ECA Vol.**28G**, P-4.103 (2004)*

Low Dimensional Models for Transport Barrier Oscillations in Tokamak Edge Plasmas

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Abstract. Low-dimensional models for relaxation oscillations of transport barriers in tokamak edge plasmas are presented. These models are based on three-dimensional resistive ballooning turbulence simulations where a transport barrier is generated by an imposed ExB shear flow. This barrier exhibits quasi-periodic relaxation oscillations which are governed by the intermittent growth of a mode at the barrier center. A one-dimensional model for the dynamics of this mode coupled to the evolution of the pressure profile reproduces barrier oscillations in the case of finite (frozen) ExB shear flow. The effect of the ExB shear is found to be different from a simple modification of the instability threshold. A proper orthogonal decomposition of the dynamics reveals the relevant radial structures of the mode which is different from a linear mode. A dynamical system for the corresponding amplitudes exhibits oscillations whose frequency dependency on ExB shear is identical to the 3D simulation observations.

Relaxations oscillations in fusion plasmas have strong impact on high confinement modes. These modes are characterized by the appearance of a quasi-periodically relaxing transport barrier. The relaxation is characterized by an increase of turbulent transport through the barrier and a decrease of the pressure inside the barrier. These relaxations are linked to so called edge localized modes (ELMs) which are believed to be magneto-hydrodynamical (MHD) modes driven by the edge pressure gradient (ballooning) and/or the edge current (peeling).

Here, we propose low dimensional models (1D and 0D), derived from 3D resistive ballooning mode (RBM) turbulence simulations where a transport barrier is generated via an imposed $E \times B$ shear flow.

The 3D turbulence simulations are based on a system of normalized reduced resistive MHD equations for the electrostatic potential ϕ and pressure p [1],

$$\partial_t \nabla_\perp^2 \phi + \left\{\phi, \nabla_\perp^2 \phi\right\} = -\nabla_\parallel^2 \phi - \mathbf{G} p + \nu \nabla_\perp^4 \phi \quad (1)$$

$$\partial_t p + \{\phi, p\} = \delta_c \mathbf{G} \phi + \chi_\parallel \nabla_\parallel^2 p + \chi_\perp \nabla_\perp^2 p + S(r) \quad (2)$$

Eq. (1) corresponds to the charge balance in the drift approximation involving the divergence of the polarization current, the parallel current, and the diamagnetic current, and viscosity (μ), respectively. Eq. (2) is derived from energy conservation, where $\chi_\parallel$ and $\chi_\perp$ represent, respectively, parallel and perpendicular collisional heat diffusivities, $S(r)$ is an energy source, and δ_c is essentially the ratio between the pressure gradient length and the major radius of the torus. The curvature operator $\mathbf{G}$ arises from the compressibility of diamagnetic current and $E \times B$ drift. The parallel current is evaluated using a simplified electrostatic Ohm's law, $j_{||} \sim -\nabla_{||}\phi$. Magnetic flux surfaces are represented by a set of concentric circular tori, where the coordinates (r, θ, ϕ) correspond to the minor radius, and the poloidal and toroidal angles. The Poisson bracket is $\{\phi, .\} = r^{-1}(\partial_r\phi\partial_\theta - \partial_\theta\phi\partial_r)$, the curvature operator is $\mathbf{G} = \sin\theta\partial_r + \cos\theta r^{-1}\partial_\theta$.

In RBM turbulence simulations in the presence of a transport barrier, this barrier relaxes quasi-periodically, even in the case of a frozen shear flow [2]. The relaxations are found to be governed by the transitory growth of a mode localized at the barrier center. This allows for the construction of a 1D model for the dynamics of this mode coupled to the evolution of the pressure profile.

For this purpose, the pressure is decomposed into a mean profile $\bar{p}(r,t)$ and a perturbation $\delta p = \tilde{p}(r,t)e^{i(m\theta - n\varphi)}$ localized at the barrier center ($r = r_0$), i.e. $\nabla_{||}^2 \delta p \sim (r - r_0)^2 \delta p$. For simplification, a cylindrical curvature operator $\mathbf{G}$ is assumed, $\mathbf{G} \to r^{-1}\partial_\theta$, and the number of fields is reduced by assuming a linear relation between potential and pressure fluctuations, $\tilde{\phi} = ik_\theta/(\gamma_0 k_\perp^2)\tilde{p}$ with $k_\theta = m/r_0$ and $k_\perp$ representing the poloidal and perpendicular wave numbers. Here γ_0 is the linear growth rate in the presence of a mean pressure gradient κ and in absence of dissipation and $E \times B$ shear flow. The poloidal shear flow is assumed to have the form $\bar{u}_\theta = \partial_r \bar{\phi} = \omega_E(r - r_0)$. The evolutions equations for the pressure become,

$$
\begin{aligned}
\partial_t \bar{p} &= -2\gamma_0 \partial_x |\tilde{p}|^2 + \chi_\perp \partial_x^2 \tilde{p} + S &\quad (3)\\
\partial_t \tilde{p} &= \gamma_0 (-\partial_x \bar{p} - \kappa_0)\tilde{p} - i\omega_E' x\tilde{p} - \chi_{||} x^2 \tilde{p} + \chi_\perp' \partial_x^2 \tilde{p} &\quad (4)
\end{aligned}
$$

where $x = r - r_0$, $\omega_E' = k_\theta \omega_E$, $\chi_\perp' = k_\theta \omega_E$, and $\kappa_0 = k_\theta^2 \chi_\perp/\gamma_0$. In absence of shear flow ($\omega_E = 0$), the system evolves to a stationary state. With increasing $\omega_E > 0$, the system first shows regular oscillations (Fig. 1a) and then reproduces relaxation oscillations (Fig. 1b). The mechanism for relaxations oscillations is as follows. During a quiescent phase, the pressure gradient increases on a slow timescale. When crossing the linear instability threshold fluctuations start growing rapidly and are stabilized by the velocity shear only after a time delay of the order $\tau = \left(\frac{1}{4}\chi_\perp\omega_E'\right)^{-1/3}$ [2]. This essentially non linear mechanism reveals that the role of the velocity shear is different from a modification of the linear instability threshold. Indeed, if the coupling term with the shear flow is replaced by a shift of the instability threshold, no

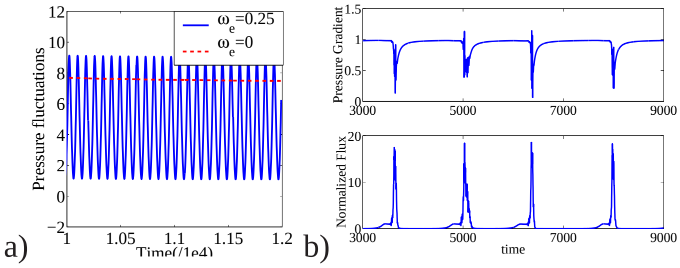

Figure 1: a)Time evolution of the pressure fluctuations $\sqrt{\int \tilde{p}^2}$ with and without frozen shear flow b)Time evolution of pressure gradient $[|\partial_r \bar{p}|/(\Gamma_{tot}/\chi_\perp)]$ and turbulent flux $\left[2\gamma_0 |\tilde{p}|^2/\Gamma_{tot}\right]$ at the barrier center observed from the 1D model (3-4)

relaxations oscillations are observed except if the instability term is further modified (e.g. by a Heavyside function [3]).

In the last part of this paper, a system of amplitude equations (0D model) is constructed, describing the evolution of the amplitudes of the relevant radial structures and reproducing main features of the oscillations observed in the previous 3D and 1D models.

For this purpose, the relevant radial structures are determined by applying a proper orthogonal decomposition (POD) method [4] to a spatio-temporal signal obtained from the 1D model. The data are converted to an $M \times N$ matrix P_i^j in which columns correspond to time series. With the POD, the matrix is decomposed into a set of modes A_n, P_n which are orthogonal in space and time,

$$P_i^j = P(x_i, t_j) = \sum_n W_n A_n(t) P_n(r) \tag{5}$$

These modes are sorted in a series of decreasing weight W_n (Fig. 2a). The steepness of the weight distribution suggests that the dynamics of the system can be described using the first modes only (Galerkin approximation),

$$\bar{p}(r,t) = \alpha_0(r - r_0) + a_0(t) p_0(r), \tag{6}$$

$$\tilde{p}(r,t) = a_1^r(t) p_1^r(r) + a_1^i(t) p_1^i(r) \tag{7}$$

The subscript "$_0$", "$^r_1$" and "$^i_1$" corresponds to the mean pressure, the real and imaginary part of the perturbation, respectively. The projection of the 1D model equations onto these modes reveals the time evolution for each a_i

$$\partial_t a_0 = -\Gamma_0 a_0 + 2\delta_0 \left(a_1^R\right)^2 + 2\delta_0' \left(a_1^i\right)^2, \tag{8}$$

$$\partial_t a_1^R = \Omega(\omega_E) a_1^i + \Gamma_1 a_1^R - \delta_0 a_1^R a_0, \tag{9}$$

$$\partial_t a_1^i = -\Omega(\omega_E) a_1^R + \Gamma_2 a_1^i - \delta_0' a_1^i a_0 \tag{10}$$

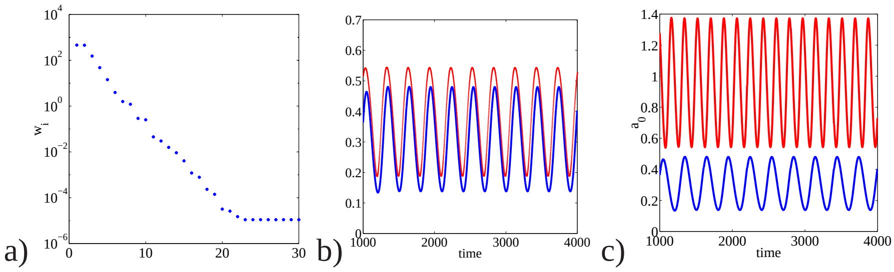

Figure 2: (a) Weight of the POD, (b) Time evolution of a_0 (in red) and $\sqrt{(a_1^r)^2+(a_1^i)^2}$ (in blue) observed from the 0D model (8-10) in the case of a frozen shear flow, (c) Influence of ω_E on oscillation frequency (red line : $\omega_E = 0.7$, blue line : $\omega_E = 0.8$)

where an analytic approximation of the modes p_0, p_1^r, p_1^i allows to calculate the coefficients $\Gamma_0, \Gamma_1, \Gamma_2, \delta_0, \delta_0^{'}, \Omega$ analytically. In particular, the influence of the velocity shear ω_E in this system is represented by the intermediate of Ω, where $\omega_E = 0$ is equivalent to $\Omega = 0$ and Ω decreases with ω_E (for large enough ω_E).

The system (8-10) reproduces oscillations (Fig. 2b) due to the coupling between p_1^r and p_1^i. Note that this coupling would be absent if p_1^r and p_1^i were linear eigenmodes of the 1D system. The variation of the oscillation frequency with ω_E is the same as in the 3D turbulence simulations, i.e. the oscillation frequency decreases with ω_E (Fig. 2c). Furthermore, in agreement with the higher order models, the system evolves to a stationary state for an $\omega_E = 0$.

In conclusion, we have derived a 1D model based on 3D turbulence simulations that reproduces relaxation oscillations of transport barriers. These relaxations are governed by the growth of a mode at the center of the barrier. This model reveals the essential role of the $E \times B$ shear flow, which is different from a modification of the linear instability threshold. A 0D model can be constructed systematically which reproduces main features of these oscillations and confirms the essentially non linear role of the velocity shear .

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