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Nonlinear simulation of magnetic reconnection with a drift kinetic electron model

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The process of reconnection allows for a change of magnetic topology inside a plasma. It is an important process for eruptive phenomena in astrophysical plasma, and the sawtooth relaxation in laboratory plasma close to thermonuclear conditions. The sawtooth relaxation is associated with the collisionless electron tearing instability, caused by the electron inertia. A thorough treatment therefore requires a kinetic model for the electron dynamics [1]. In this contribution, we report on the numerical simulation of the electron tearing instability by solving the Vlasov equation directly.

Drift kinetic model

We use a Cartesian coordinate system, with small variation ($\partial/\partial z = 0$) in the z-direction. The equilibrium magnetic field is dominated by a constant component B_0, such that the total field is written as $\boldsymbol{B} = B_0\boldsymbol{e}_z + \boldsymbol{B}_{pol} = B_0\boldsymbol{e}_z + \nabla\Psi\times\boldsymbol{e}_z$ We use the drift kinetic approximation for the electron dynamics, where the perpendicular velocity $\boldsymbol{v}_\perp$ is approximated by the $\boldsymbol{E}\times\boldsymbol{B}$ drift velocity. The perpendicular motion of the electrons is treated with a kinetic model. It is convenient to introduce the generalised impulse $p_z = m_e v_z - e\Psi$. Since we assume that fields do not depend on the variable z, the generalised impulse p_z is a constant of motion [2]. The dynamical evolution is obtained from solving the drift-kinetic equation for a distribution function $f^e(x, y, p_z, t)$. In a suitably normalised form, it is given by

$$\frac{\partial f^e}{\partial t} + [\Phi - \frac{\rho_*}{2d_e\rho_{ep}}(p_z + \Psi)^2, f^e] = 0 \qquad (1)$$

where we use the definition $[h, f] := \boldsymbol{e}_z \cdot \nabla h \times \nabla f$. The coordinates are normalised on a characteristic length L, and the time scale is the poloidal Alfven time $\tau_{Ap} = L(\mu_0\rho)^{1/2}/B_{pol}$. The poloidal flux Ψ and the electric potential Φ are normalised on LB_{pol} and B_0L^2/τ_{Ap}. The parameters used in (1) are the ion sound Larmor radius $\rho_* = (kT_e m_i)^{1/2}/(eB_0L)$, the collisionless skin depth $d_e = c/(L\omega_{pe})$, and the poloidal electron Larmor radius $\rho_{ep} = v_{te}/(L\Omega_{cep})$, with the thermal electron speed v_{te} and $\Omega_{cep} = eB_{pol}/m_e$.

The parallel velocity of the ions v_{zi} is negligible compared to the electron velocity v_{ze}. The current is mainly carried by the electrons $J_z \approx J^e$, such that Ψ is obtained from $f^e(x, y, p_z, t)$

with Ampére's law

$$\Delta\Psi - \frac{1}{d_e^2}\Psi = \frac{1}{d_e^2}\int p_z f^e dp_z \tag{2}$$

The perpendicular motion of the ions is described by a fluid equation for the electric potential Φ [3]

$$\frac{\partial \Delta\Phi}{\partial t} + [\Phi, \Delta\Phi] = [\Psi, \Delta\Psi] \tag{3}$$

Equations (1),(2) and (3) describe the dynamic evolution of the system. It is a fluid-like equation system with two space dimensions and the generalised impulse p_z as a parameter.

Numerical simulation

In this paper, we consider a dynamic evolution inside a box with periodic boundary conditions. We consider a simple periodic equilibrium with $L = \pi$ and $\Psi_0 = \cos x$. With a box size of $x = \pm L$ and $y = \pm 2L$, fluid theory suggests this equilibrium to become unstable [3] against the collisionless tearing instability. The initial equilibrium distribution function f_0 is given by a shifted Maxwellian

$$f_0^e(x, p_z) = \frac{1}{\sqrt{\pi}\rho_{ep}} \exp\left(-\frac{(p_z - d_e^2(d^2\Psi_0/dx^2) + \Psi_0)^2}{\rho_{ep}^2}\right) \tag{4}$$

The parameters are chosen to be $\rho_* = \rho_{ep} = 1$ and $d_e = 0.25$. Analysis of the corresponding ideal MHD equation shows that we are in the large Δ' regime ($\Delta' d_e \geq 1$), where fluid calculations showed a rapid growth of magnetic islands [4].

The structure of the governing equations allows for the use of a pseudo-spectral method [5] that advances in time the Fourier representation of the dynamic equations. The code has been optimised for vector-parallel machines as the NEC-SX6. The numerical results presented in this paper are obtained with a resolution of 512×128 (x, y) modes, and 51 values for the impulse p_z. The numerical runs are initialised with the equilibrium (4) and a superimposed mode derived from the linear fluid theory. It is essential to start with a quiet initial function, because in a Vlasov simulation, initial noise is not damped away in the course of the run.

The exponential growth of the instability is shown in figure 1. The instability develops linearly up to $t \approx 90\tau_{Ap}$, where the island width becomes larger than the skin depth d_e. In the early non-linear phase, the growth rate seems to accelerate, as observed in fluid simulations [3]. Figure 2 shows contour plots of Ψ, Φ, current and vorticity in the nonlinear phase at $t = 110\tau_{Ap}$. In contrast to fluid simulations, the fields do not show symmetry around the centre, especially apparent on the electric potential. This confirms earlier results of a Vlasov simulation [6], and is due to the absence of parity symmetry of the governing equations (1,2,3). The current density and the vorticity develop strong gradients along the reconnecting field line. These fine structures become more concentrated in time, such that the scale can no longer be

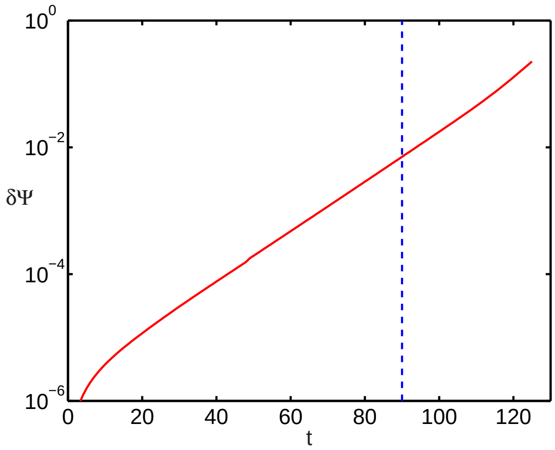

Figure 1: Logarithmic plot of the function $\delta\Psi_X := (\Psi - \Psi_0)|_{\text{x-point}}$ versus time, measuring the deviation from the equilibrium flux at the X-point. The dotted vertical line marks the begin of the nonlinear phase

resolved with the truncated Fourier expansion at later times.

Conclusion

In this paper we have analysed the nonlinear development of the collisionless tearing instability in the framework of a drift-kinetic model for the electron dynamics. The results confirm results of an earlier paper on the same subject [6], and extends them to smaller values of the collisionless skin depth $d_e = 0.25$. Our results suggest a faster than exponential growth in the early nonlinear phase of the instability. We observe as well an asymmetry of the resulting fields. It seems, however, that the field structure becomes closer to the fluid case for small values of d_e.

In the present simulation, electron acceleration is difficult to detect. Further studies with higher resolution are therefore required to explore the more realistic cold electron regime $\rho_* < d_e < 1$. For an application to tokamaks, it is necessary to adopt cylindrical geometry for the simulation.

References

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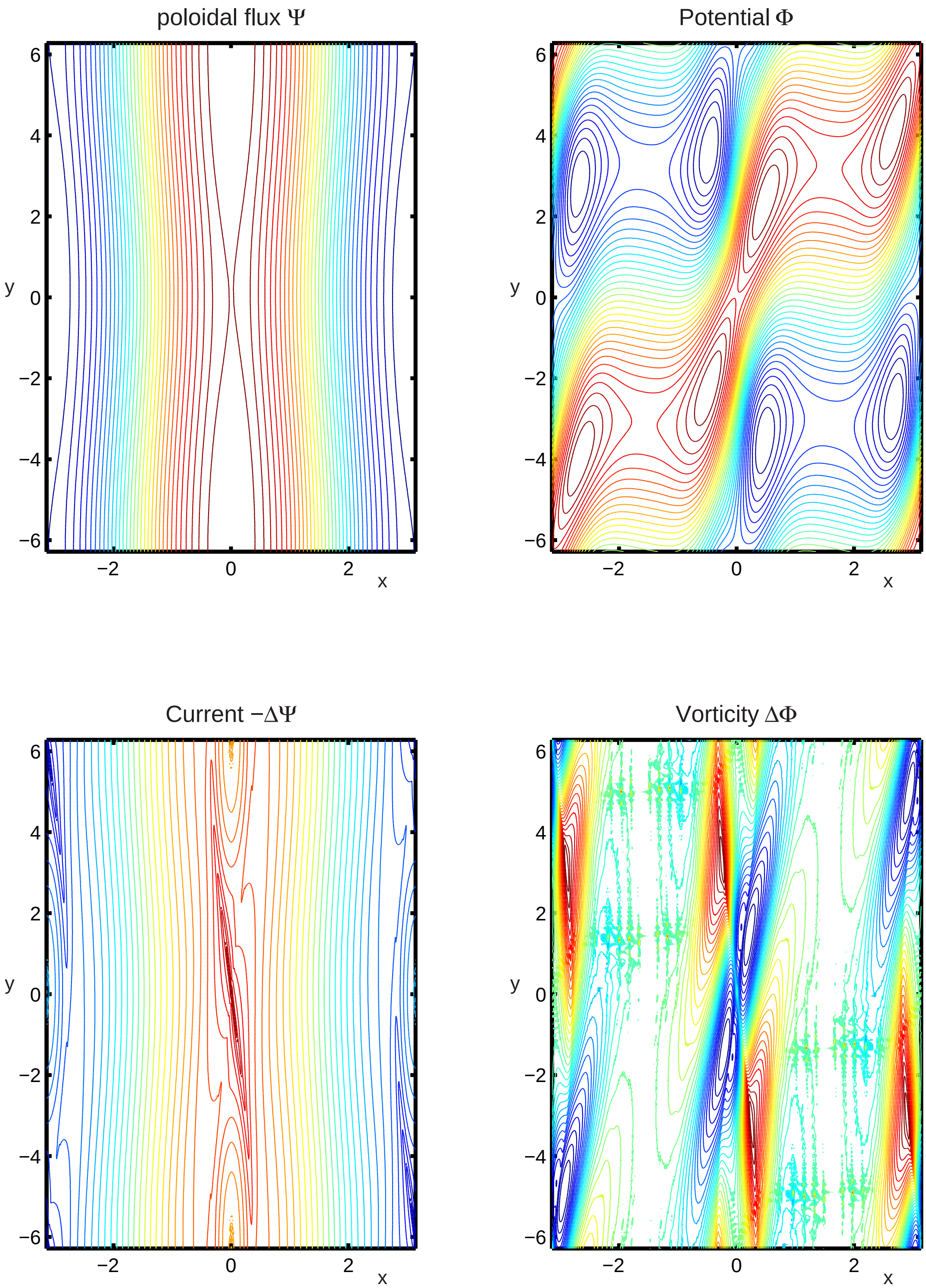

Figure 2: Contour plots of poloidal flux Ψ, electric potential Φ, current $-\Delta\Psi$ and vorticity $\Delta\Phi$ at $t = 110\tau_{Ap}$.