

## Comparison of the Properties of Quasi-isodynamic Configurations for Different Numbers of Periods

M.I. Mikhailov<sup>1</sup>, W.A. Cooper<sup>2</sup>, M.F. Heyn<sup>3</sup>, M.Yu. Isaev<sup>1</sup>, V.N. Kalyuzhnyj<sup>4</sup>, S.V. Kasilov<sup>4</sup>, W. Kernbichler<sup>3</sup>, V.V. Nemo<sup>4</sup>, C. Nührenberg<sup>5</sup>, J. Nührenberg<sup>5</sup>, M.A. Samitov<sup>1</sup>, V.D. Shafranov<sup>1</sup>, A.A. Skovoroda<sup>1</sup>, A.A. Subbotin<sup>1</sup>, K. Yamazaki<sup>6</sup>, R. Zille<sup>5</sup>

<sup>1</sup> *Russian Research Centre "Kurchatov Institute", Moscow, Russia*

<sup>2</sup> *CRPP, Association Euratom-Confédération Suisse, EPFL, Lausanne, Switzerland*

<sup>3</sup> *Institut für Theoretische Physik, Technische Universität Graz, Graz, Austria*

<sup>4</sup> *IPP, NSC "Kharkov Institute of Physics and Technology", Kharkov, Ukraine*

<sup>5</sup> *Max-Planck-Institut für Plasmaphysik, IPP-EURATOM Association, Germany*

<sup>6</sup> *National Institute for Fusion Science, Oroshi-cho 322-6, Toki 509-5292, Japan*

### Abstract

Properties of quasi-isodynamicity (qi)-optimized [1] configurations with poloidally closed contours of constant  $B$  on magnetic surfaces for different numbers of periods are discussed. It is shown that with increasing number of periods, the stability- $\beta$  limit increases and such favorable properties as long-time collisionless fast particles confinement, small neoclassical transport and small bootstrap current, can be maintained.

### Introduction

In Ref. [1] it was shown that in stellarators with poloidally closed contours of constant  $B$  on magnetic surfaces, good long-time collisionless confinement of fast particles can be achieved by approaching the qi condition for deeply to moderately trapped particles. This condition requires the contours of the second adiabatic invariant,  $\mathcal{J} = \oint V_{\parallel} dl$ , to be constant on magnetic surfaces. In Ref. [2], by computational optimization of a six-period configuration with  $\langle \beta \rangle = 5\%$ , the possibility was shown to satisfy the qi condition for all reflected particles. The configuration found was stable against Mercier and resistive-interchange modes and has a small effective ripple and bootstrap current [3,4]. Because of these attractive features of qi configurations, it was interesting to study the possibility to increase the  $\beta$  value for the  $N = 6$  system and to consider configurations of such type for larger and smaller numbers of periods. To clarify (at least partially) the dependencies of the properties of these systems on the number of periods, the result of the optimization for  $N = 3$ ,  $N = 6$  and  $N = 9$  configurations are presented here. While the optimization is not fully completed yet, some characteristic features and dependencies of the configuration properties upon the number of periods are clearly seen.

### Initial configuration and main goals of optimization

The optimizations were carried out on the NEC SX-5 supercomputers himiko (Germany) and prometeo (Switzerland) with the VMEC code [5] for equilibrium calculations, the JMC code [6] for the transition to magnetic (Boozer) coordinates and the MCT code [7] for direct calculation of particle drift orbits. As the starting point, the boundary magnetic surface of the  $N = 6$  configuration optimized earlier [2] was used. The shapes of the cross-sections at the beginning and half of the period are similar to that of the stellarator W7-X [8]: the cross-section in the region of minimal  $B$  has triangular shape and that in the region of maximal  $B$  is bean-shaped. The initial boundary magnetic surfaces for  $N = 3$  and  $N = 9$  configurations were constructed by changing the aspect ratio of the  $N = 6$  configuration. For all three configurations the attempt was

made to find the stability-  $\beta$  limit while maintaining good collisionless confinement as well as small effective ripple and bootstrap current.

Initial runs showed that from the point of view of stability different values of  $\beta$  should be chosen for different numbers of periods. The optimization then was performed for the following  $\beta$  values:  $\langle \beta \rangle = 3.9\%$  for  $N = 3$ ,  $\langle \beta \rangle = 8.9\%$  for  $N = 6$  and  $\langle \beta \rangle = 10\%$  and  $\langle \beta \rangle = 15\%$  for  $N = 9$ . The stability-  $\beta$  limits found are close to, but do not coincide exactly with these choices. Because of such a strong dependence of the  $\beta$  limit on the number of periods, other properties of these configurations depend on the correlated triple aspect ratio, number of periods and  $\beta$ .

### Some characteristic features of qi configurations and their dependencies on the number of periods

#### *Characteristic geometry of the magnetic surfaces and value of mirror component*

The 3D view of the qi-optimized configurations for small ( $N=3$ ) moderate ( $N=6$ ) and large ( $N=9$ ) numbers of periods are shown in Fig. 1. The color here corresponds to the magnetic field strength on the boundary magnetic surface. It is seen that a significant mirror component of  $B$  is present in all configurations. The value of this component diminishes with increasing number of periods. Also, it is seen that for all cases, the plasma columns are almost straight in the vicinity of the  $B$  extrema. It is precisely the smallness of the magnetic axis curvature in the cross-sections with extrema of  $B$  that permits to avoid the formation of islands in the contours of  $B$  on magnetic surfaces and to close the contours of  $B$  poloidally. The deviation of the plasma column from the plane increases with diminishing the number of periods. In the table (see below) the aspect ratios and the ratios of  $B_{max}/B_{min}$  on boundary magnetic surfaces are shown for all three configurations.

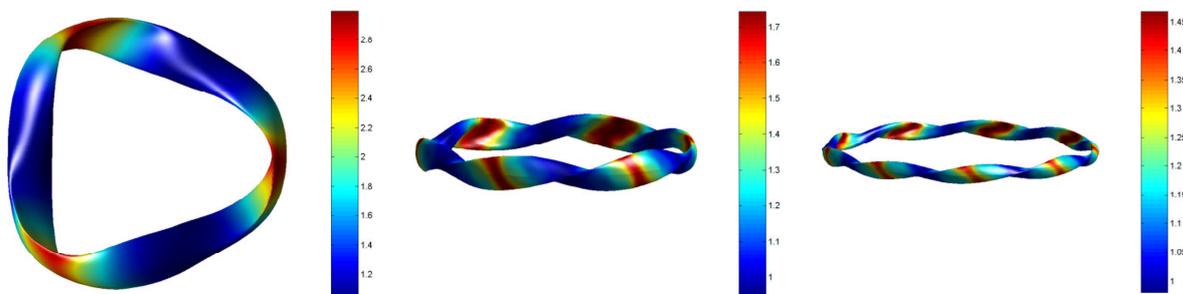


Fig.1. Boundary magnetic surfaces of the optimized  $N = 3$ ,  $N = 6$  and  $N = 9$  configurations. The color corresponds to the magnetic field strength on the boundary magnetic surfaces.

#### *Effect of $\beta$ value on particle confinement*

During the optimization, it was found that the collisionless confinement of fast particles is improved for configurations with larger number of periods (and higher  $\beta$  value), at least for particles with starting orbits at half of the plasma minor radius. This can be explained as follows. Because of the poloidally closed contours of constant  $B$  on magnetic surfaces in these configurations, the reflected particles are restricted to remain within their field period of birth and the trajectories of their banana centres can be described by the contours of the second adiabatic invariant. The value of  $\mathcal{J}$

is a function of the flux label and poloidal angle. In the ideal case of exact quasi-isodynamicity, the poloidal variation of  $\mathcal{J}$  should be zero. Computationally, it can be made small, but not exactly zero. Thus, the behavior of the contours of  $\mathcal{J}$  does not only depend on its poloidal variation, but on its radial dependence, too. With increasing  $\beta$ , the radial dependence of  $\langle \mathcal{J} \rangle_\theta$  becomes larger because of the diamagnetic effect, so that even for significant poloidal variation of  $\mathcal{J}$  it became possible to close its contours inside the plasma column. In spite of the small diamagnetic effect for the three-period configuration with  $\langle \beta \rangle = 3.9\%$ , approaching qi here permits to improve the fast particle confinement significantly, too.

The results of direct calculations of collisionless  $\alpha$ -particle losses are shown in the table. The calculations were made for power-plant type parameters ( $B_0 = 5T, V = 1000m^3$ ). One thousand particles orbits are initiated at 1/2 of the minor plasma radius. The losses are shown for a time of flight  $\tau = 1sec$ .

#### *Effective ripple in qi configurations*

As a measure of the neoclassical transport in the  $1/\nu$  regime, the value of the effective ripple is used (see, e.g. Ref. [9]). The transport coefficients are proportional to  $\epsilon^{3/2}$  and inversely proportional to the square of the distance of the plasma column from the major axis. The value of effective ripples is directly connected with the poloidal variation of the second adiabatic invariant. Decreasing the poloidal variation of  $\mathcal{J}$  leads to diminishing the effective ripple. The radial dependence of  $\mathcal{J}$  is not important for the effective ripple. One can note that – while in the ideal case of exact qi, when the second adiabatic invariant is constant on magnetic surfaces, there should be no particle losses and the effective ripple should equal zero, in the cases realized here, a decorrelation is seen between collisionless losses and effective ripple: with increasing  $\beta$ , the closure of  $\mathcal{J}$  contours does not lead automatically to small effective ripple. Thus, for a large number of periods and large  $\beta$  values, an additional optimization to diminish the effective ripple was performed. As a result, for all configurations the values of effective ripple were made small enough. The characteristic values of  $\epsilon^{3/2}$  for 1/2 of the plasma minor radius are shown in the table.

#### *Bootstrap current in qi configurations*

Because of the poloidally closed contours of constant  $B$  on magnetic surfaces, the radial shifts of the fraction of trapped particle orbits with the same direction of parallel velocity change sign as a particle drifts poloidally. This leads to the possibility of small bootstrap current in the configurations considered here. Indeed, by optimization it has been demonstrated that the structural factor of the bootstrap current can be made much smaller here than in configurations with axially or helically closed contours of constant  $B$  on magnetic surfaces. The values of the structural factor of the bootstrap current at 1/2 of the plasma minor radius are shown in the table. The characteristic values of this factor for systems with axially or helically closed contours of  $B = \text{constant}$  are of the order of 5-10.

#### *Stability properties of qi configurations*

For all cases there were no problems to stabilize Mercier and resistive-interchange modes by optimization. The stabilization of these modes does not lead automatically to ballooning mode stability. Additional optimization toward ballooning-mode stability was carried out for several values of the parameters  $\theta_0, \zeta_0$  of the ballooning-mode equation. In addition, for the configurations with  $N = 6$  and  $N = 9$  periods, the

investigation of the global-mode stability with the CAS-3D code [10] was performed. The results are shown in the table. For the three-period configuration, the table shows the stability limit for ballooning modes. The corresponding calculations of global-mode stability will be made in the future. It is worth mentioning here that for the  $N = 9$  configuration optimized toward collisionless particle confinement and ballooning-mode stability only, the effective ripple became noticeably larger, but the global-mode stability limit was found to be significantly higher at  $\beta = 14.5\%$ . Further attempts are required to elucidate the possibility to increase the  $\beta$  limit for  $N = 9$  configurations.

**Table. Comparison of the configuration properties for different number of periods**

N \ Property	$A$	$B_{max}/B_{min}$	Losses	$\beta$ limit	$\epsilon^{3/2}$	$G_{BC}$	$\iota$ range
N=3	6.8	2.9	100	3.9%	0.002	0.7	0.67 - 0.71
N=6	12	1.8	0	8.4%	0.0012	0.05	0.86 - 1.0
N=9	24	1.5	0	10.5%	0.002	0.1	1.56 - 1.73

**Conclusions** It has been demonstrated that configurations of qi type can be realized with small, moderate and large number of periods through a qi optimization procedure. Approaching qi identifies configurations with good confinement of fast particles, small neoclassical transport and small bootstrap current. It has been shown that for these configurations, incrementing the number of periods leads to increasing the MHD stability limit while simultaneously maintaining small bootstrap current, in contrast to systems with quasi-helical or quasi-axial symmetry.

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