

Nonlocal electron transport in degenerate plasmas

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Abstract

The nonlocal electron transport is analyzed to construct hydrodynamic equations to study the non equilibrium behavior of a degenerate electron gas in plasmas. Electron transport is modeled by the semiclassical Boltzmann equation where collisions processes are described by the relaxation operator. The Boltzmann equation is analytically solved in the diffusive approximation and the nonlocal transport coefficients are deduced.

I Introduction

Understanding nonlocal transport in a degenerate electron gas is of a great importance in many physical situations such as in inertial confinement fusion, in white dwarf stars and in metal surface. The hydrodynamic equations provide a powerful tool in analyzing the properties of an inhomogeneous degenerate electron system. These equations are obtained by taking different velocity moments of the Boltzmann kinetic equation which describe at time t the evolution of the one-particle distribution function in phase space (\vec{r}, \vec{p}) . The untruncated hierarchy of such equations is completely equivalent to the Boltzmann equation and all kinetic informations are not lost in such a hierarchy. Usually, the infinite system is truncated to the first three equations, for the density $n(\vec{r}, t)$, the flow velocity $\vec{V}(\vec{r}, t)$ and the temperature $T(\vec{r}, t)$. The higher order moments computed from the kinetic theory in terms of the lower moments constitute the closure relations of these fluid equations. The aim of this work is to compute the nonlocal transport coefficients of partially degenerate electron gases.

II. The kinetic model

The basic equation of our model is the Boltzmann equation that include the electron-electron and the electron-ion collision contributions. In planar geometry and steady state approximation it reads

$$v_x \frac{\partial f}{\partial x} - \frac{e}{m_e} E \frac{\partial f}{\partial v_x} = C_{ei}(f) + C_{ee}(f) \quad (1)$$

where $f(x, v_x, v)$ is the electron distribution function (DF) and the other variables have their usual meaning. In Eq. (1) the collisions relaxes the DF towards the Fermi-Dirac distribution function

$$f_{FD} = \left(1 + \exp\left(\frac{\varepsilon - \mu}{T_e}\right) \right)^{-1} \quad (2)$$

where ε , is the electron kinetic energy and μ is the chemical potential. The anisotropic electron-ion collision operator is modeled by the Lee and More model [1], *i.e.*, $C_{ei}(f) = -v_{ei}(v)f$, where $v_{ei}(v) = n_e Z e^4 \ln \Lambda / (4\pi \varepsilon_0^2 m_e^2 v^3)$ is the electron-ion collision frequency, Z is the ion charge, n_e is the electron density and $\ln \Lambda$ is the Coulomb logarithm. The quantum effects in the cross section are included in $\ln \Lambda$ [1]. For simplicity we assume that $C_{ee}(f) = -v_{ee}(f - f_{FD})$ and $v_{ee} = \alpha(Z)v_{ei}(v_t)$ where $v_t = (T_e/m_e)^{1/2}$ is the thermal velocity. The parameter $\alpha(Z)$ is defined to recover the transport properties in the nondegenerate electron gas limit [2]. We rewrite Eq. (1) on the fluid velocity frame $\vec{V}(x) = V(x)\hat{x}$ and expand the DF on the Legendre polynomial basis

$$f(\vec{v}, x) = \sum_{n=0}^{\infty} f_n(v) P_n(v_x/v)$$

obtaining, in the diffusive and the Lorentz approximations, the following set of equations,

$$\sqrt{\frac{2}{3}} v_t y^{1/2} \frac{\partial f_1}{\partial x} - \sqrt{\frac{2}{3}} v_t \frac{\partial M_1^2}{\partial x} \frac{\partial f_{FD}}{\partial y} \frac{(6F_{1/2} - 2F_{-1/2}y)}{5F_{-1/2}F_{3/2} - 9(F_{1/2})^2} - \sqrt{\frac{2}{3}} v_t \frac{\partial M_1^1}{\partial x} \frac{\partial f_{FD}}{\partial y} \frac{(-10F_{3/2} + 6F_{1/2}y)}{5F_{-1/2}F_{3/2} - 9(F_{1/2})^2} = -v_{ee}(f_0 - f_{FD}) \quad (3)$$

$$\sqrt{\frac{2}{3}} v_t y^{1/2} \frac{\partial f_0}{\partial x} + \sqrt{\frac{8}{15}} v_t y^{1/2} \frac{\partial f_2}{\partial x} - \sqrt{\frac{2}{3}} \frac{e v_t y^{1/2}}{T_e} \frac{\partial f_{FD}}{\partial y} E = -v_{ei}(v) f_1 \quad (4)$$

$$\sqrt{\frac{8}{15}} v_t y^{1/2} \frac{\partial f_1}{\partial x} - \frac{4}{3\sqrt{5}} y \frac{\partial f_{FD}}{\partial y} \frac{dV}{dx} = -v_{ei}(v) f_2 \quad (5)$$

where $y = \varepsilon/T_e$, F_j are the Fermi-Dirac integrals and we used the notation $M_n^m = \int_0^{\infty} y^m f_n dy$.

We should note that the invariance properties of the collision operators have been included in Eq. (3).

III The transport coefficients

We have solved analytically Eqs. (3)-(5) for weak nonlocal effects, *i.e.*, $\left(\lambda_{ei} \frac{\partial}{\partial x}\right)^2 < 1$, where $\lambda_{ei} = v_t / \nu_{ei}(v_t)$, and computed the explicit expression of the components of the DF, $f_0 - f_2$. Then we have deduced the non-vanishing components of the stress tensor Π_{xx} , the electron-ion momentum transfer R_x and of the heat flux q_x . For convenience we present the results in the Fourier space ($x \leftrightarrow k$) and at the lower order with respect to the small parameter $(\lambda_{ei} k)^2$, the local quantities are

$$q_x^{loc}(k) = -\frac{40}{3\sqrt{2}} \frac{F_4}{F_{1/2}} \left(1 - \frac{16(F_3)^2}{15F_2F_4}\right) \frac{n_e T_e}{m_e \nu_{ei}(v_t)} ik T_e \quad (6)$$

$$\Pi_{xx}^{loc}(k) = -\left(\frac{256}{45\sqrt{2}} \frac{F_3}{F_{1/2}}\right) \frac{n_e T_e}{\nu_{ei}(v_t)} ik V \quad (7)$$

$$R_x^{loc}(k) = -\left(-\frac{5F_{3/2}}{3F_{1/2}} + \frac{4F_3}{3F_2}\right) n_e ik T_e - \frac{1}{2\sqrt{2}} \frac{F_{-1}}{F_{1/2}} m_e n_e \nu_{ei}(v_t) V. \quad (8)$$

Taking into account the nonlocal contributions proportional to $(k\lambda_{ei})^2$, we obtain

$$q_x^{nloc}(k) = q_x^{loc}(\sim T_e) \left[1 - \frac{2\sqrt{2}}{15} \frac{\left(-13F_{11/2} + \frac{88F_3F_{9/2}}{3F_2} - \frac{16F_{7/2}(F_3)^2}{(F_2)^2}\right) (k^2 \lambda_{ei}^2 / \alpha(Z))}{F_4 \left(1 - \frac{16(F_3)^2}{15F_2F_4}\right)} \right] - \frac{64}{15\sqrt{6}} \left(\frac{13F_{11/2}}{\sqrt{3}F_{1/2}} - \frac{44F_{9/2}}{3\sqrt{5}F_{1/2}}\right) n_e T_e (k\lambda_{ei})^2 V \quad (9)$$

$$R_x^{nloc}(k) = R_x^{loc}(\sim V) \left[1 - \frac{16\sqrt{2}}{15\sqrt{3}} \frac{\left(\frac{16F_3}{\sqrt{3}F_{1/2}} - \frac{11F_{9/2}}{\sqrt{5}F_3}\right) (k\lambda_{ei})^2}{(F_{-1}/F_{1/2})} \right] + R_x^{loc}(\sim T) \left[1 - \frac{\left(\frac{11F_{9/2}}{6F_3} - \frac{2F_{7/2}}{F_2}\right) (k\lambda_{ei})^2}{\frac{5F_{3/2}}{3F_1} - \frac{4F_3}{3F_2}} \right] \quad (10)$$

$$\Pi_{xx}^{nloc}(k) = \Pi_{xx}^{loc}(\sim V) - \frac{64}{15\sqrt{6}} \left(\frac{13F_{11/2}}{\sqrt{3}F_{1/2}} - \frac{44F_{9/2}}{3\sqrt{5}F_{1/2}}\right) n_e T_e (k\lambda_{ei})^2. \quad (11)$$

The nonlocal part of the viscosity coefficient less important than the other contributions has been neglected in Eq. (11). We note in Eqs. (9)-(11) that the nonlocal effects reduce the transport coefficients and that the off-diagonal coefficients verify the Onsager symmetry. Converting back to the real space Eqs. (9)-(11) we obtain expressions corresponding to integro-differential operators. The resulting transport quantities constitute reliable closure relations for the hydrodynamic equations to describe inhomogeneous degenerate electron gas.

References

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- [2] L. Spitzer and R. Härm, Phys. Rev. **89**, 977 (1953).

Acknowledgments

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