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Kolmogorov-Kraichnan Scaling in the Inverse Energy Cascade of Two-dimensional Plasma Turbulence

Ghassan Y. Antar[1],

1. *Fusion Energy Research Program, University of California San Diego, 9500 Gilman Dr., La Jolla, CA 92093-0417, USA*

Magnetically confined plasmas are promising candidates for future energy production by fusion reaction. The study of turbulence in these plasmas is emphasized by the fact that turbulent fluctuations enhances the radial transport perpendicular to the magnetic field lines and thus reduces considerably the confinement times of particles and energy. Turbulence in magnetically confined plasmas like tokamaks or linear devices is two-dimensional due to the strong magnetic field $\vec{B}$. Experimental investigations in the laboratory was conducted using mainly soap films and conducting liquids subject to electro-magnetic forces.

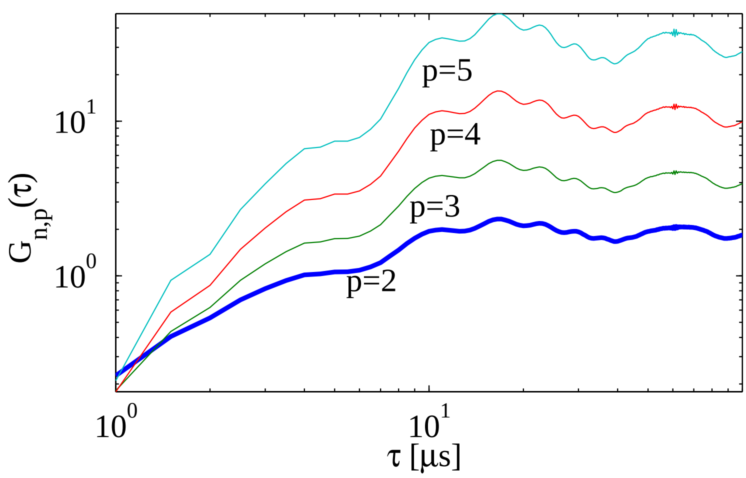

Figure 1: $G_{n,p}(\tau)$ *vs* τ for $p = 2$, 3, 4 and 5. Note that no scaling region can be identified.

Both of these methods use incompressible fluid flows. In two-dimensional turbulence, there exist two scaling regions on either side of the production scale where turbulence is forced, the scaling being reflective of self-similarity. The enstrophy transfer rate (β) feeds turbulence to scales r smaller than the production scale leading to an energy with the form $E = C\beta^{2/3}r^2$ using dimensional analysis *à la* Kolmogorov. The second domain extends over scales larger than the production scale and is called the energy or the inverse cascade sub-range. It is maintained by an energy transfer rate (ε) leading to $E = C'\varepsilon^{2/3}r^{2/3}$. The above two relations can be generalized to the pth order leading to $E_p \sim r^p$ and $E_p \sim r^{p/3}$ in the enstrophy and energy cascade respectively ($E_p(\vec{r}) = \langle(\vec{v}(\vec{x}) - \vec{v}(\vec{x}+\vec{r}))^p\rangle$).

The linear increase of the scaling exponent with p is called the Kolmogorov-Kraichnan scaling.

This letter presents the first comprehensive results on the scaling of density and velocity fields up to 10th order in the PISCES linear device and up to 6th order in the MAST spherical tokamak. We found that the Kolmogorov-Kraichnan theory rather accurately describes turbulence inside the plasma. Furthermore, we believe that the experimental results presented here are also important for the neutral fluid community as they provide evidence that the Kolmogorov-Kraichnan scaling describes accurately the energy cascade of turbulent fluctuations in compressible flows. More details of the results presented here can be found in the reference [G. Y. Antar, Phys. Rev. Lett. 91, 055002 (2003)].

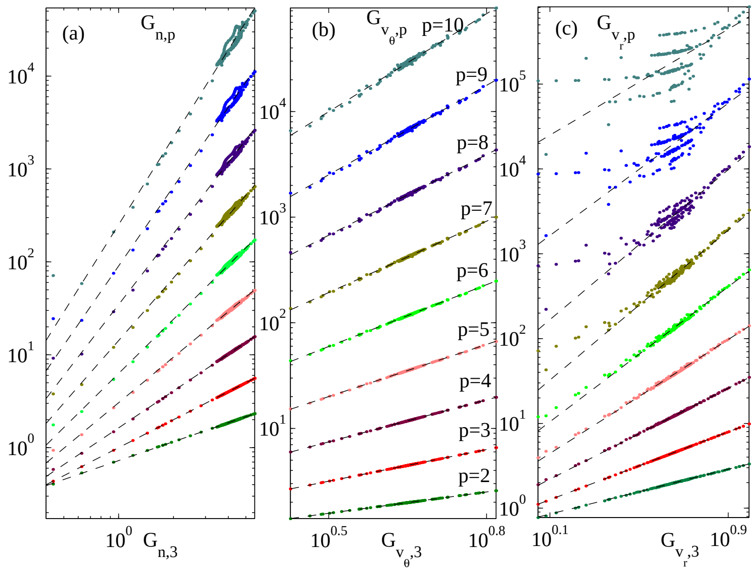

Figure 2: The structure functions of pth order as function of the third-order moment for the density (a), v_θ (b) and v_r (c). Note the good scaling that is reached except for v_r and for $p \geq 8$.

The generalized structure function $G_{n,p}(\tau) = \langle |n(t) - n(t+\tau)|^p \rangle$ is used. Hereafter, the subscripts, n, v_r, v_θ, denote density, radial and poloidal velocity fluctuations. In Fig. 1 is plotted $G_{n,p}(\tau)$ as function of τ for $p = 2$, 3, 4 and 5. Clearly one cannot use the self-similarity (SS) assumption (τ^{ξ_p}) to determine the scaling exponents ξ_p because no scaling region is detected. This is probably due to the production-dissipation energy spectrum which extends over a large region affecting the shape of G_p. In neutral fluids, the lack of self-similarity was also reported and the difficulty is overcome by using the extended self-similarity (ESS) method. This technique scales moments among each other rather than scaling moments against spatial or temporal separation. Therefore, the constraint of having an inertial range far from the production and dissipation scales of the system is relaxed. In the case where an inertial range exists, it was shown that either the ESS or SS methods yield the same scaling law. The ESS method does not exclude deviations from the Kolmogorov law nor other possible scalings. Because the scaling of the third-order moment can be deduced from the Navier-Stokes equation ($\langle|\Delta v(r)|^3\rangle \sim r$), the scaling $\langle|\Delta v|^p\rangle = A\langle|\Delta v|^3\rangle^{\xi_p}$ is used here. In principle, however, one can use any moment to conduct the scaling.

In Fig. 2 is shown the scaling of $G_{n,p}$, $G_{v_r,p}$ and $G_{v_\theta,p}$ with respect to the third-order moments. It is clear that the scaling is considerably enhanced when compared to Fig. 1 and extends over almost the entire domain. A linear fit in the logarithmic plot yields the scaling exponents for the density ($\xi_{n,p}$) as well as for the velocity fluctuations ($\xi_{v_r,p}$ and $\xi_{v_\theta,p}$). To our knowledge, this is the first time such scaling is obtained in magnetically confined plasmas. The corresponding scaling exponents are plotted in Fig. 3(a) with respect to p. Note that the values of $\xi_{v_r,p}$ should be disregarded for $p \geq 8$ because no scaling is detected. Moreover, in Fig. 3(b) $\xi_{n,p}$ is shown for different operational parameters in PISCES. Within the experimental precision, the dependence of $\xi_{n,p}$ on p is linear with a slope equal to 1/3 in agreement with the Kolmogorov-Kraichnan theory for two-dimensional turbulence in the energy cascade sub-range.

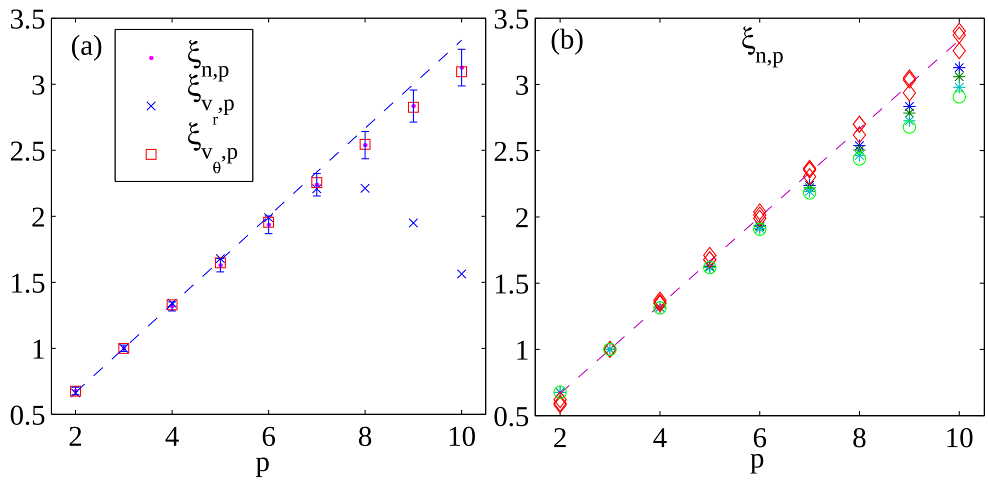

Figure 3: (a), $\xi_{n,p}$, $\xi_{v_\theta,p}$ and $\xi_{v_r,p}$ as functions of p resulting from the scaling shown in Fig. 2. (b), the different symbols represent the scaling exponents $\xi_{n,p}$ as function of p in the PISCES linear device under the following conditions: ($\diamond$), neutral hydrogen gas pressure of 2 mTorr and magnetic field strength equal to 800, 1200, 1600 and 2000 G, (*), magnetic field of 1200 G and hydrogen neutral pressure equal to 2, 3 and 4 mTorr, (o) helium gas at 2 mTorr and 1200 G. The dashed line in (a) and (b) represents the Kolmogorov-Kraichnan law in the energy cascade range $p/3$.

The same procedure, using the same type of diagnostic, is applied to data obtained on the Mega-Ampère Spherical Tokamak (MAST). In this toroidal device probes cannot stay long inside the last closed flux surface because of the high ion and electron fluxes which seriously perturb the measurement. For the present experiment, stationary plasma conditions were achieved for about 15 ms, the acquisition frequency being 1 MHz. Consequently, the maximum order moment that converges is about 6. In Fig. 4(a) is shown the structure functions $G_{n,p}$ plotted as function of $G_{n,3}$ indicating a satisfactory scaling. In Fig. 4(b) is shown the scaling exponents of the radial velocity and density. Here again, the scaling is in agreement with the linear $p/3$ prediction of the Kolmogorov-Kraichnan theory.

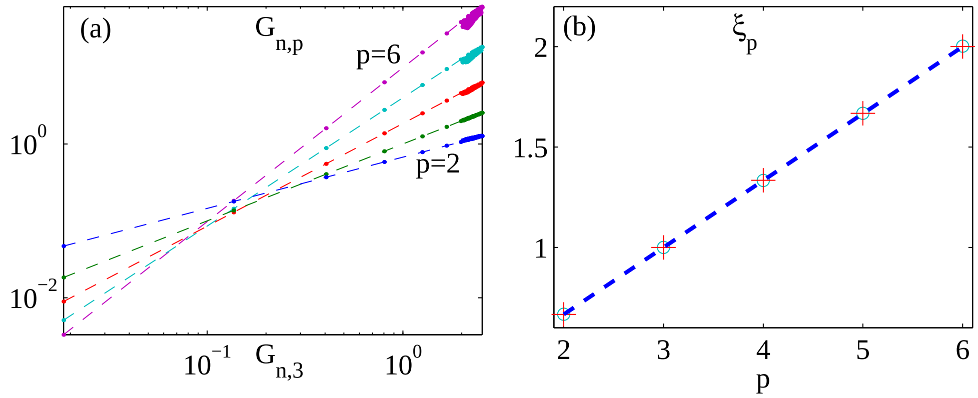

Figure 4: (a), $G_{n,p}$ *vs* $G_{n,3}$ in a logarithmic plot showing a large scaling region for p from 2 to 6. (b), the scaling exponents in MAST for the density (+) and the radial velocity fluctuations (o) as function of p. The discharge in MAST was in L-mode with line-average plasma density equal to 1.5×10^{20} m^{-3} and a plasma current of 670 kA. The probe is at a fixed position equal to 3 cm inside the last closed flux surface with stationary plasma parameters.

In conclusion, it is rather clear that the self-similarity assumption cannot be used to evaluate the scaling exponents. This is why the data could not be compared to the Kolmogorov-Kraichnan theory. However, this difficulty is overcome by using the so-called extended self-similarity where moments of different order are scaled with respect to each other. The scaling is considerably enhanced. The resultant scaling exponents increase linearly with the moments order. This is the case for the PISCES linear device and the MAST spherical tokamak. This is also the case for the density and the velocity fluctuations and does not depend on the pressure, the magnetic field and the type of gas. The linear increase has a slope of 1/3 indicating, without ambiguity and for the first time, that the studied fluctuations lie in the inverse cascade range. For more details please see [G. Antar *et al*, Phys. Rev. Lett. 91, 055002 (2003)].