

## Ion beam driven electromagnetic fluctuations at Earth's magnetopause

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### Abstract

The generation of both long and short wavelength (in comparison with the ion gyroradius) electromagnetic waves that are triggered by ion beams at the Earth's magnetopause has been investigated by using a fluid model for long wavelength limit, and a hybrid approach for short wavelength limit. It is shown that the ion beam with a speed greater than the ion thermal speed and the plasma density inhomogeneity with scalelength less than the electron skin-depth destabilize the electromagnetic waves of lower-hybrid frequencies.

The lower-hybrid drift (LHD) wave instability [1,2] has often been discussed as a potential candidate for the generation of the anomalous resistivity required to facilitate the magnetic field reconnection. Therefore, it is highly desirable to develop a clear understanding for the generation of LHD wave turbulence [3] and associated fluctuation spectrum [4] that causes nonthermal transport [2] of charged particles across the geomagnetic field lines.

Drake [5] made an analysis of turbulence and transport involving LHD waves in the presence of an equilibrium sheared electron flow with an application to the magnetopause current layer. They found that the free energy stored in the sheared electron flow can be coupled to the low-frequency (in comparison with the electron gyrofrequency  $\omega_{ce}$ ), short-wavelength (in comparison with the ion gyroradius  $\rho_i$ ) electrostatic waves. Lakhina and Tsurutani [6] investigated LHD waves by treating both electrons and ions as fluids, and have taken into account the parallel electron beam, ion drift relative to electrons, and density/magnetic field nonuniformity. Recently, we [7] have studied short wavelength electrostatic LHD waves that are generated by sheared electron flows and ion beams at the Earth's magnetopause by a hybrid approach, in which the ions are treated kinetically and the cold electrons are described by means of a fluid model. In the present paper, we examine both long and short wavelength (in comparison with the ion gyroradius) electromagnetic waves that are triggered by ion beams at the Earth's magnetopause.

We assume that the equilibrium magnetic field  $\hat{\mathbf{z}}B_0$  is along the  $z$ -direction and the gradient of the equilibrium plasma number density  $n_0(x)$  is along the  $x$ -direction. In the presence of low-frequency electromagnetic waves, the perturbed electron number density  $n_e$  and electron fluid velocity components perpendicular and parallel to  $\hat{\mathbf{z}}$ ,  $\mathbf{u}_{e\perp}$  and  $u_{ez}$ , are determined from

$$\partial_t n_e + \nabla_{\perp} \cdot (n_0 \mathbf{u}_{e\perp}) + n_0 \partial_z u_{ez} \approx 0, \quad (1)$$

$$\mathbf{u}_{e\perp} \approx \frac{c}{B_0} \hat{\mathbf{z}} \times \nabla_{\perp} \phi - \frac{cT_e}{eB_0 n_0} \hat{\mathbf{z}} \times \nabla n_e + \frac{cD_t}{B_0 \omega_{ce}} \nabla_{\perp} \phi, \quad (2)$$

$$\partial_t u_{ez} \approx \frac{e}{m_e} \left( \partial_z \phi + \frac{D_t}{c} A_z \right) - \frac{3V_{Te}^2}{n_0} \partial_z n_e, \quad (3)$$

where  $D_t = \partial_t + \mathbf{u}_{e0} \cdot \nabla$ ,  $\partial_z = \partial/\partial z$ ,  $\partial_t = \partial/\partial t$ ,  $A_z$  is the  $z$ -component of the vector potential,  $\phi$  is the scalar (electrostatic) potential,  $V_{Te} = (T_e/m_e)^{1/2}$  is the electron thermal speed,  $\omega_{ce} = eB_0/m_e c$  is the electron gyrofrequency,  $c$  is the speed of light in vacuum,  $e$  is the magnitude of the electron charge, and  $m_e$  is the electron mass. It is obvious from Eq. (2) that  $\mathbf{u}_{e0} = -(cT_e/eB_0 n_0) \hat{\mathbf{z}} \times \nabla n_0$ . Equations (1)–(3) are indicating that the current along the magnetic field direction is carried by electrons only. They are closed by an equation for  $A_z$ , which comes from the parallel component of Ampere's law

$$\nabla_{\perp}^2 A_z \approx \frac{4\pi e n_0}{c} u_{ez}. \quad (4)$$

We are interested in the dispersion properties of the electromagnetic waves whose frequency is much higher than the ion gyrofrequency (viz.  $\omega \gg \omega_{ci}$ ). We, therefore, can neglect the influence of the magnetic field on the ion dynamics. For long (in comparison with the ion gyroradius  $\rho_i$ ) wavelength ( $k_{\perp} \rho_i \ll 1$ ) modes, the perturbed ion number density  $n_i$  and ion fluid velocity  $\mathbf{u}_i$  are determined from

$$d_t n_i + \nabla \cdot (n_0 \mathbf{u}_i) \approx 0, \quad (5)$$

$$d_t \mathbf{u}_i \approx -\frac{e}{m_i} \nabla \phi - \frac{3V_{Ti}^2}{n_0} \nabla n_i, \quad (6)$$

where  $d_t = \partial_t + \mathbf{u}_0 \cdot \nabla$ ,  $V_{Ti} = (T_i/m_i)^{1/2}$  is the ion thermal speed,  $T_i$  is the ion temperature, and  $m_i$  is the ion mass. On the other hand, for short (in comparison with ion gyroradius) wavelength ( $k_{\perp} \rho_i \gg 1$ ) the perturbed ion number density is given by  $n_i = \int f_i d\mathbf{v}$ , where  $f_i$  the perturbed part of the ion distribution function satisfying the linearized ion Vlasov equation

$$\partial_t f_i + \mathbf{v} \cdot \nabla f_i - \frac{e}{m_i} \nabla \phi \cdot \frac{\partial f_0}{\partial \mathbf{v}} = 0, \quad (7)$$

where  $f_0(\mathbf{r}, \mathbf{v})$  is the unperturbed part of the ion distribution function, and is assumed to be a local Maxwellian shifted by a constant ion-drift velocity  $\mathbf{u}_0$ .

For long wavelength waves ( $k_{\perp}\rho_i \ll 1$ ) we express the perturbed electron and ion number densities,  $n_e$  and  $n_i$ , in terms  $\phi$  by Fourier-decomposing (1)-(6) [i.e. assuming that  $n_e$ ,  $\mathbf{u}_{e\perp}$ ,  $u_{ez}$ ,  $n_i$ ,  $\mathbf{u}_i$ , and  $\phi$  are proportional to  $\exp(-i\omega t + ik_y y + ik_z z)$ ], and solving the resultant equations for  $n_e$  and  $n_i$ , the electron and ion susceptibilities ( $\chi_e$  and  $\chi_i$ ) for  $\omega \gg k_y u_{e0}$  are given by

$$\chi_e = \frac{1 + \alpha_1(k_y V_D \omega / k_z^2 c_e^2 - k_y^2 \omega^2 / k_z^2 \omega_{ce}^2)}{k^2 \lambda_{De}^2 (1 - \alpha_1 \omega^2 / 3k_z^2 V_{Te}^2)}, \quad (8)$$

$$\chi_i = -\frac{\omega_{pi}^2}{(\omega - k u_0 \cos \theta)^2 - 3k^2 V_{Ti}^2}, \quad (9)$$

where  $\alpha_1 = 1 + 1/k_y^2 \lambda_e^2$ ,  $\lambda_e = c/\omega_{pe}$  is the electron skin-depth,  $\omega_{pe} = (4\pi n_0 e^2 / m_e)^{1/2}$  is the electron plasma frequency,  $\lambda_{De} = (T_e / 4\pi n_0 e^2)^{1/2}$  is the electron Debye radius,  $\omega_{pi} = (4\pi n_0 e^2 / m_i)^{1/2}$  is the ion plasma frequency,  $V_D = c_e^2 k_n / \omega_{ce}$ ,  $k_n = (1/n_0)(\partial n_0 / \partial x)$ ,  $c_e = (T_e / m_e)^{1/2}$  is the electron acoustic speed, and  $\theta$  is the angle between  $\mathbf{k}$  and  $\mathbf{u}_0$ . The dispersion relation  $1 + \chi_e + \chi_i = 0$  for  $k^2 \lambda_{De}^2 \ll 1$  is given by

$$\frac{1 + \alpha_1(k_y V_n \omega / k_z^2 c_a^2 - k_y^2 \omega^2 / k_z^2 \omega_{ce}^2)}{1 - \alpha_1 \omega^2 / 3k_z^2 V_{Te}^2} = \frac{k^2 c_i^2}{(\omega - k u_0 \cos \theta)^2 - 3k^2 V_{Ti}^2}, \quad (10)$$

where  $c_i = (T_e / m_i)^{1/2}$  is the ion-acoustic speed. Equation (10) is a local dispersion relation for long wavelength coupled electromagnetic LHD waves and obliquely propagating fast ion acoustic waves. We have numerically solved this dispersion relation, and have found one positive real root and one imaginary root for  $\delta = 60^\circ$  (where  $\delta$  is the angle between  $\mathbf{k}$  and  $\hat{\mathbf{z}}$ ),  $\theta = 30^\circ$ , and for the plasma parameters corresponding to the Earth's magnetopause [8]:  $n_0 = 5 \text{ cm}^{-3}$ ,  $T_i = 1 \text{ keV}$ ,  $T_e = 0.2 T_i$ , and  $B_0 = 60 \text{ nT}$ . The real root  $\omega_r$  (corresponding to the frequency of the mode) is found to be  $\sim 0.1\omega_{ci}$  for  $u_0/V_{Ti} = 10$ ,  $k_n \lambda_e = 2 \times 10^{-3}$  and  $k_y \lambda_e = 2 \times 10^{-3}$ . However,  $\omega_r$  increases as we increase  $k_n \lambda_e$  or decrease  $k_y \lambda_e$ . We have also found that the imaginary root  $\omega_i$  (corresponding to the growth rate of the mode) is very small (insignificant) in comparison with the real frequency.

For short wavelength ( $k_{\perp}\rho_i \gg 1$ ) waves we can use  $\chi_e$  given in Eq. (8), but we have to re-derive the expression for  $\chi_i$  from  $n_i = \int f_i d\mathbf{v} = -k^2 \chi \phi / 4\pi e$ , we have [9]

$$\chi_i = \frac{1}{k^2 \lambda_{Di}^2} [1 + \zeta Z(\zeta)], \quad (11)$$

where  $\lambda_{Di} = (T_i / 4\pi n_0 e^2)^{1/2}$  is the ion Debye radius,  $Z(\zeta) = \pi^{-1/2} \int_{-\infty}^{+\infty} dt \exp(-t^2) / (t - \zeta)$  is the plasma dispersion function, and  $\zeta = (\omega - k u_0 \cos \theta) / \sqrt{2} k V_{Ti}$ . For  $\omega \gg k_y u_{e0}$  and

$k^2 \lambda_{De}^2 \ll 1$  the dispersion relation,  $1 + \chi_e + \chi_i = 0$ , yields

$$\frac{1 + \alpha_1 (k_y V_n \omega / k_z^2 c_a^2 - k_y^2 \omega^2 / k_z^2 \omega_{ce}^2)}{1 - \alpha_1 \omega^2 / 3 k_z^2 V_{Te}^2} = -\frac{T_e}{T_i} [1 + \zeta Z(\zeta)]. \quad (12)$$

Equation (12) is a local dispersion relation for short wavelength coupled electromagnetic LHD waves, and obliquely propagating electron-acoustic waves. We have numerically solved this dispersion relation, and have found two positive real roots (corresponding to low and high frequency modes) and one imaginary root (corresponding to the growth rate of the high frequency mode). For  $u_0/V_{Ti} = 1$ ,  $k_n \lambda_e = 0.1$ , and  $k_y \lambda_e = 1$  the real frequency  $\omega_r$  of high and low frequency modes are found to be  $\sim 0.1 \omega_{ci}$  and  $\sim 0.01 \omega_{ci}$ , respectively. However, in both cases  $\omega_r$  increases as we increase  $k_n \lambda_e$  or  $u_0/V_{Ti}$  or  $k_y \lambda_e$ . The high frequency mode is found to be unstable and its growth rate is found to be  $\sim 0.02 \omega_{ci}$  for  $u_0/V_{Ti} = 1$ ,  $k_n \lambda_e = 0.1$  and  $k_y \lambda_e = 1$ . We have also found that it increases as we increase  $k_n \lambda_e$  or  $u_0/V_{Ti}$  or  $k_y \lambda_e$ .

To conclude, the ion beam with a speed greater than the ion thermal speed and the plasma density inhomogeneity with scalelength less than the electron skin-depth is found to destabilize the electromagnetic waves of lower-hybrid frequencies. We hope that our present study will be useful in understanding the origin of short and long wavelength electromagnetic fluctuations that are observed at the Earth's magnetopause.

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