

## Effects of a current hole on the fast ion distribution in tokamaks

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### 1. Introduction

In this paper we propose a qualitative theoretical approach for the study of fast ion behavior in tokamak plasmas with a current hole (CH) [1-5]. For the equilibrium magnetic configuration in a CH tokamak we employ a simplified model that is based on an analytical approximation of the poloidal flux function shape [6]. For that the radial extent  $r_*$  of the plasma core area with zero rotational transform is used as a basic specific parameter. The modification of the fast ion orbit topology, as caused by the current hole, is studied. The model allows for manageable analytical expressions that describe completely the CH effect on the ion confinement domains as well as on the boundaries separating the regions of different type of orbits in the constants-of-motion space. Subject of investigation is the CH effect, both on the convective transport of fast ions induced by their slowing down as well as on the fast ion diffusive transport caused by pitch angle scattering. The influence of slowing down on the distribution function of NBI ions in CH tokamak plasmas will be evaluated.

### 2. Model magnetic field

The effect of the toroidal current profile on the particle orbit is completely described by the poloidal magnetic flux  $\Psi(r)$  inside a given magnetic flux surface (FS) with an equatorial distance  $r$  from the magnetic axis. For CH tokamak equilibria, this function takes on the simple form [6]

$$\Psi(r) \square \bar{\Psi}(x, x_*) \equiv \Psi'_a \psi(x - x_*), \quad \psi(y) = y\Theta(y), \quad (1)$$

where  $x = r/a$  is the normalised radius of the magnetic flux surface,  $\Psi'_a$  is a parameter determined mainly by the total plasma current and the plasma shape,  $\Theta$  represents the Heaviside step function and  $x_* = r_*/a$  measures the effective radial size of the current hole. Taking  $\Psi(r)$  from Eq. (1) and introducing  $\xi$  as well as the normalised poloidal gyroradius  $d = mcVa/(e\Psi'_a)$  as new variables for the particle velocity, we derive the orbit equation

$$(\psi - \psi_0 + dh_0\xi_0)^2 - d^2h[h - (1 - \xi_0^2)h_0] = 0, \quad (2)$$

where  $h = A + x\cos\chi$  is the normalised major radius with the plasma aspect ratio  $A = R_d/a$ ,  $\chi$  is the poloidal angle coordinate and the subscript "0" denotes the value that corresponds to the initial point ( $x=x_0$ ,  $\chi=\chi_0$ ) in the toroidal tokamak cross section, which for all orbits can be chosen as the crossing point with the equatorial plane ( $\cos\chi_0 = \pm 1$ ) at their respective minimum FS radius  $x_0$ . Note that orbit equation (2) is quadratic in the spatial coordinates  $x, \chi$  as well as in the velocity coordinates  $V, \xi$ , hence this allows an analytical treatment of particle behavior for orbit widths of the order of the plasma radius.

The current hole effect on confinement becomes exceptionally strong in the case of relatively low plasma currents, i.e. lower than the critical value  $I_{cr}$  [6] required to confine the fattest banana orbits crossing the central plasma region,

$$I_{cr} \cong \frac{F}{Z_i} \sqrt{\frac{\mu_i E}{A}} \frac{1}{1 - \sqrt{x_*}} ; \quad (3)$$

here the particle energy  $E$  is in  $MeV$ , the plasma current  $I$  in  $MA$ ,  $Z_i$  and  $\mu_i$  represent the fast ion charge and mass numbers, and  $1.5 > F > 1$  is a geometrical factor determined by plasma non-circularity [7]. Figure 1 displays the confinement domains for  $3.5MeV$  alphas in the

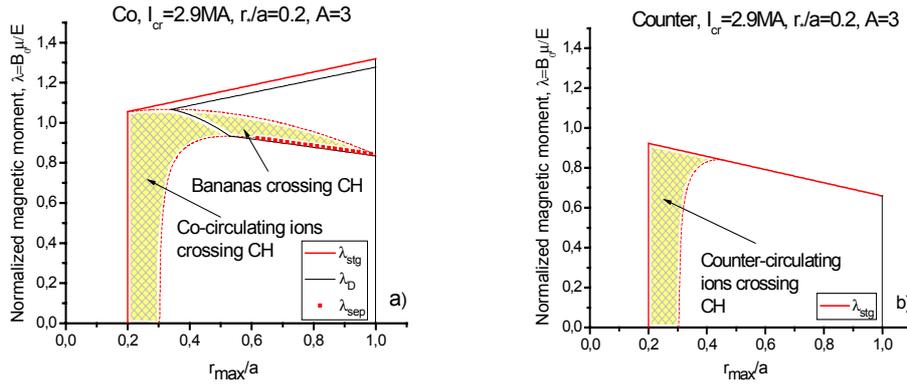


Fig.1: Confinement domains of  $3.5MeV$  alphas in the case of JET-like flux surface shapes [6] and  $I=I_{cr}$  for a) co-going and b) counter-going orbits in the case of a small current hole ( $r^*/a=0.2$ ).

case of JET-like flux surface shapes with  $F=1.3$  [6] and  $I=I_{cr}$ . The abscissa measures the maximum radial coordinate  $r_{max}$  of the guiding center orbit in terms of the plasma radius  $a$ , and the ordinate axis scales the magnetic moment  $\mu$  in terms of  $E/B(x=0)$ , where  $B$  represents the magnetic field. A particular confinement domain is bounded by  $\lambda = \lambda_{stg}$  featured by stagnation orbits. Further seen is the region of trapped ions bordered by  $\lambda_D$  of D-shaped orbits. The orbits of fattest bananas are associated with  $\lambda_{sep}$  characterizing the possibility of transition from co- to counter-motion. Comparison of confinement domains in

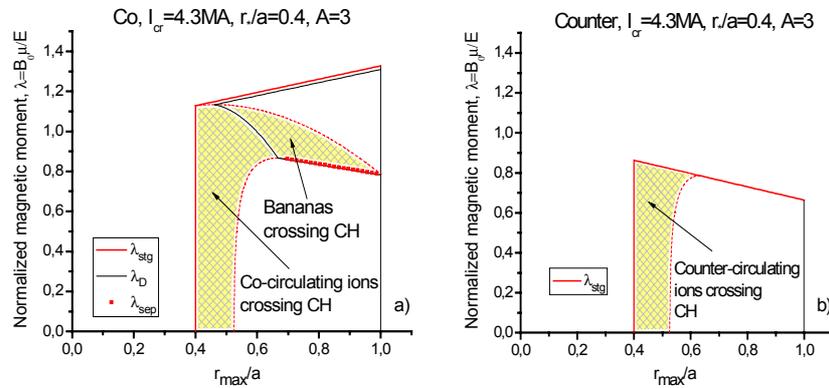


Fig.2: Confinement domains of  $3.5MeV$  alphas in the case of JET-like flux surface shapes [6] and  $I=I_{cr}$  for a) co-going and b) counter-going orbits in the case of large current hole ( $r^*/a=0.4$ ).

Figs. 1 and 2 suggests that an increase of the current hole size from  $r_{\#}/a = 0.2$  to  $r_{\#}/a = 0.4$  is equivalent to 30% reduction of the plasma current.

### 3. Bounce averaged Fokker-Planck equation

The fast ion distribution function in tokamak plasmas is adequately described by the bounce averaged Fokker-Planck equation in constants-of-motion (COM) space  $\mathbf{c} \equiv \{c^1, c^2, c^3\}$ ,

$$\frac{\partial f}{\partial t} = \frac{1}{\sqrt{g_{\mathbf{c}}}} \frac{\partial}{\partial c^i} \sqrt{g_{\mathbf{c}}} \left( \langle U^i \rangle - \langle D^{ij} \rangle \frac{\partial}{\partial c^j} \right) f + \langle S \rangle, \quad \sqrt{g_{\mathbf{c}}} = \int_{r_{\min}}^{r_{\max}} dr \sqrt{g}, \quad i, j = 1, 2, 3. \quad (4)$$

Here  $U^i$  and  $D^{ij}$  are transport coefficients describing convective and diffusive transport in  $\{r, \mathbf{c}\}$  space,  $S$  is the fast ion source term,  $\sqrt{g}$  is the Jacobian for the transformation from Eulerian coordinates  $\{\mathbf{r}, \mathbf{V}\}$  to COM space,  $r_{\max}$  and  $r_{\min}$  are the maximum and minimum values of  $r$  along the orbit and  $\langle \dots \rangle$  means bounce averaging in accordance with the rule

$$\langle \dots \rangle = \int_{r_{\min}}^{r_{\max}} dr \sqrt{g} (\dots) / \int_{r_{\min}}^{r_{\max}} dr \sqrt{g} \equiv \int_{r_{\min}}^{r_{\max}} dr (\dots) / \dot{r}(r, \mathbf{c}) / \int_{r_{\min}}^{r_{\max}} dr / \dot{r}(r, \mathbf{c}), \quad (5)$$

where  $\dot{r}(r, \mathbf{c})$  is the radial component of the guiding center velocity. In the case of a mono-energetic source term  $S = S_0 \delta(E - E_0)$ , one can derive from Eq. (4) an approximate expression for the initial distribution function of fast ions as

$$f(E = E_0, \lambda, r_{\max}) \approx \langle S_0 \rangle / \langle U^1 \rangle, \quad (6)$$

if  $\mathbf{c} \equiv \{E, \lambda, r_{\max}\}$  and  $U^1$  denotes the energy slowing down term. We note that, in the case of a thin beam injected in the vicinity of the tokamak mid-plane (on-axis beam, vertical coordinate  $Z = Z_{\text{axis}}$ ) [8], the RHS of Eq. (6) allows for an explicit analytical form of the initial beam ion distribution. Fig. 3 displays the confinement domain and the beam trace in the  $\lambda, r_{\max}$ -plane for on-axis 105keV tritium beam ions in a JET CH plasma with  $I/B = 1.5 \text{ MA}/3.5 \text{ T}$ . Taking into account that, in this case,  $S_0 \sim S_{00} \delta(Z - Z_{\text{axis}})$  and

$$\dot{r}(r, \mathbf{c}) \approx Z - Z_{\text{axis}} = \pm \sqrt{r^2 - A^2 [h(r, \mathbf{c}) - 1]^2} = \sqrt{(r_{\max} - r)(r - r_{\min})} G(r, \mathbf{c}), \quad (7)$$

where  $G(r_{\min} < r < r_{\max}, \mathbf{c}) \neq 0$ , we obtain for  $\langle S_0 \rangle$  in the vicinity of the focusing point of the beam trace producing ions with near-stagnation orbits, where  $S_0$  is maximum,

$$\langle S_0 \rangle \approx \frac{S_{00}}{r_{\max} - r_{\text{stg}}(\lambda)}. \quad (8)$$

Numerical Fokker-Planck calculations [8] as well as an analytical derivation using Eq. (8) with  $S_{00} \sim \exp[-k^2(r_{\max} - r_f)^2/a_b^2]$  corresponding to a Gaussian distribution of beam neutrals perpendicular to the mid-plane with half-width  $a_b = 12 \text{ cm}$ ,  $r_f/a = 0.23$  and elongation  $k = 1.7$ , lead us to the initial distribution function around the focusing point (at  $\lambda = \lambda_f = 0.73$  and  $r = r_f = 0.23a$ ) as displayed in Fig. 4 for 105keV beam tritons in a CH discharge in JET. It is noted that the analytically obtained  $f(E_0, \lambda, r_{\max})$  agrees satisfactorily with the numerical distribution function [8] thus confirming the reliability of the approach used.

## 4. Summary

We conclude that the proposed analytical approach based on a simplified CH magnetic field can be used for investigating fast particle behavior in CH tokamak plasmas. Apparently, for the central plasma region, the analytical analysis developed provides results in excellent agreement with quantitative numerical Fokker-Planck predictions.

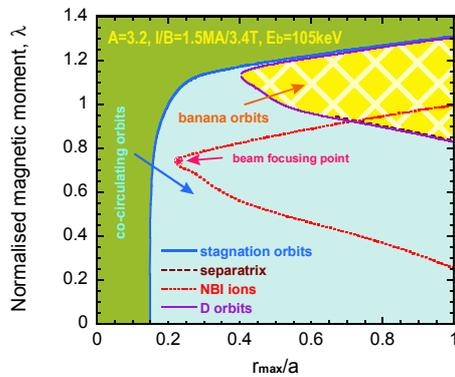


Fig.3: Confinement domain and beam trace of 105keV tritons in the  $\lambda$ ,  $r_{max}$ -plane for on-axis injection in a JET CH plasma with  $I/B = 1.5MA/3.5T$  [8]

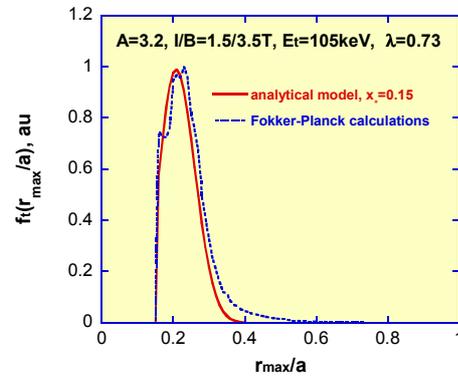


Fig.4: Distribution function of 105keV beam tritons vs  $r_{max}$  in the vicinity of the focusing point in the JET CH plasma of Fig.3

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