

# Fourier Analysis and Interpretation of Computational Results on Combined Toroidicity, Ellipticity and Triangularity Effects on the Alfvén Modes Excited in Pre-Heated, Low Aspect Ratio Tokamaks

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**Introduction.** Recently, the combined effects of plasma non-uniformity on the Alfvén modes excited in pre-heated low aspect ratio tokamaks have been rigorously investigated by Cuperman *et al.*, (see Poster at this Conference, Cuperman *et al.*, 2004). Specifically, the authors studied the case of START (Sykes, 1994; Wilson, 1994) which is characterized by finite toroidicity, ellipticity and triangularity, with consideration of consistent temperature, pressure, magnetic field and neoclassical and bootstrap currents profiles.

In such a case, the following plasma non-uniformity-related physical processes may occur (e.g., Cheng and Chance, 1986, Betti and Freidberg 1991,1992):

1) Toroidicity-induced gaps and corresponding Alfvén eigenmodes (TAEs), due to the coupling of the  $m$ -th and  $m + 1$ -th poloidal harmonics at  $r = r_0$  where  $q(r_0) = (m + 1/2)/n$  and one has  $k_{||m} = -k_{||m+1}$ ; the eigenfrequency is  $\omega_{TAE} = c_A/2qR$  ( $q$  – the safety factor,  $c_A$  – the Alfvén speed,  $R$  – the major radius,  $k_{||}$  – the parallel wave number).

3) Triangularity-induced gaps and corresponding eigenmodes (non-circular Alfvén eigenmodes, NAEs) due to the coupling of the  $m$ -th and  $m + 3$ -th poloidal harmonics, near  $\omega_{NAE} = 3\omega_{TAE}$ .

Thus, the results of the rigorous computational investigation mentioned above (see Poster at this Conference) clearly indicate that, due to their global structure, the Alfvén eigenmodes induced by the combined plasma non-uniformity features considered in this work (toroidicity, ellipticity and triangularity) may be quite efficient for, e.g., plasma heating, non-inductive current drive and turbulent transport suppression barriers in the pre-heated stage of low aspect ratio tokamaks.

In order to deeper understand and interpret these global results – including, besides plasma non-uniformity, effects such as mode-conversion, reflection, tunneling and deposition – we have carried out their thorough Fourier analysis. A full account of the results obtained as well as their discussion are here presented.

## 1 Results and discussion

Figure 2 illustrates the dependence of the total power deposition,  $P_{tot}$  on the applied wave frequency for two sets of toroidal and poloidal pump wave numbers,  $N$  and  $M$ : the solid (dotted) curve represents the case  $M = 2$ ,  $N = 2$  ( $M = 2$ ,  $N = 0$ ) (see Cuperman *et al.*, this Conference). For the Fourier analysis, we considered the following relevant eigenmodes values:

(i) Peak values of non-uniformity induced eigenmodes, denoted by  $B_1, B_2$  and  $B_3$ ; and for comparison, the corresponding eigenmodes for FWs with  $N = 0$ ,  $M = 2$ , viz.  $B_1^0, B_2^0, B_3^0$ ;

(ii) Illustrative spectral positions differing by those of the peak eigenmodes, for both  $N = 2$ ,  $M = 2$  and  $N = 0$ ,  $M = 2$  FWs wave frequency number, viz.  $b_1$ ,  $b_2$ ,  $b_3$  and  $b_1^0$ ,  $b_2^0$ ,  $b_3^0$ , respectively.

The results of the study are presented in Figures 3 and 4. They indicate the following:

1. In a non-uniform plasma of the START-like, fast waves characterized by a given set of  $(\omega, N, M)$ -values, do excite – via wave-plasma interactions also involving mode conversion, reflection and tunneling – secondary waves with poloidal wave numbers  $m$ , in a range  $|m| \gg |M|$  with different, field amplitudes (see Figures 3); specifically, in this case  $-6 \leq m \leq 6$ .
2. Over the entire frequency range  $\omega_{A,L} \leq \omega \leq \omega_{A,U}$ , (A,L and A,U represent respectively, the lower and upper thresholds of the Alfvén continuum) most of the excited modes (poloidal number  $m$ ) are present even if  $N = 0$ , i.e., are associated with the poloidal pump wave number,  $M$ ; their actual range and amplitudes depend on, and usually increase with the  $M$ -value.
3. The role of the toroidal pump wave feature (wave number  $N$ ) is indeed to couple part of these excited modes, thus, resulting in  $TAE, EAE$  and  $NAE$  eigenmodes; this is clearly seen in Figure 4.
4. Finally, it appears that conditions exists for simultaneous generation of more than one (two or three) types of non-uniformity induced gaps and eigenmodes. Thus, in the interpretation of the complex computational results, the possible *superposition* of several gaps and eigenmodes of one type (i.e.,  $TAE, EAE$  or  $NAE$ ) or of different types resulting in wider non-uniformity induced gaps and larger eigenmodes should be taken into account.

These conclusions are consistent with and extend those holding for large aspect ratio, inherently simpler, linear theory.

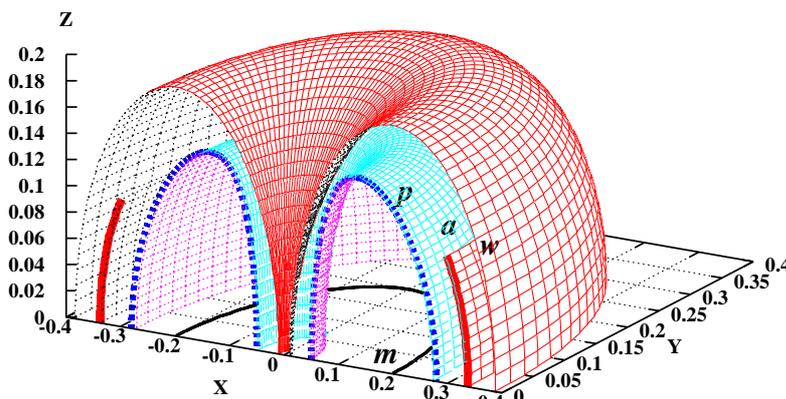


Figure.1 Simulated spherical tokamak configuration: **m** - magnetic axis, **p** - plasma vacuum interface, **a** - antenna, **w** - metallic wall. The equilibrium parameters are as follows: geometric axis 0.165m, magnetic axis 0.199m, inverse aspect ratio 0.697, toroidal current 65kA, on axis toroidal magnetic field 0.4895T,  $T_{0e} = 180\text{eV}$ ,  $n_{0e} = 2.5 \times 10^{20}\text{m}^{-3}$ , elongation 1.3, and triangularity 0.3.

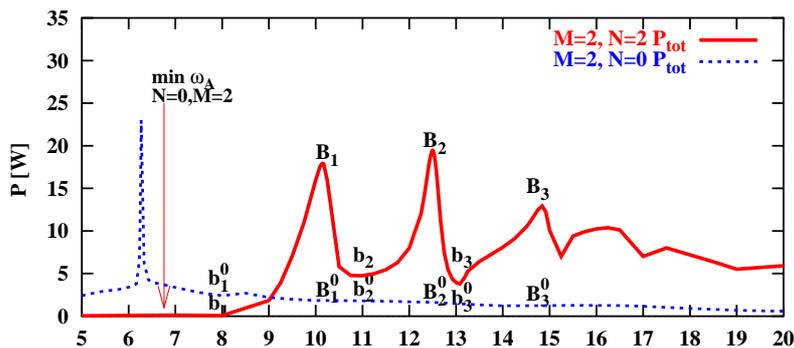


Figure.2 Dependence of the total power deposition,  $P_{tot}$  on the applied wave frequency for two sets of toroidal and poloidal pump wave numbers,  $N$  and  $M$ : the solid (dotted) curve represents the case  $M = 2, N = 2$  ( $M = 2, N = 0$ ) ;  $\min \omega_A$  indicates the cylindrical continuum Alfvén frequency.

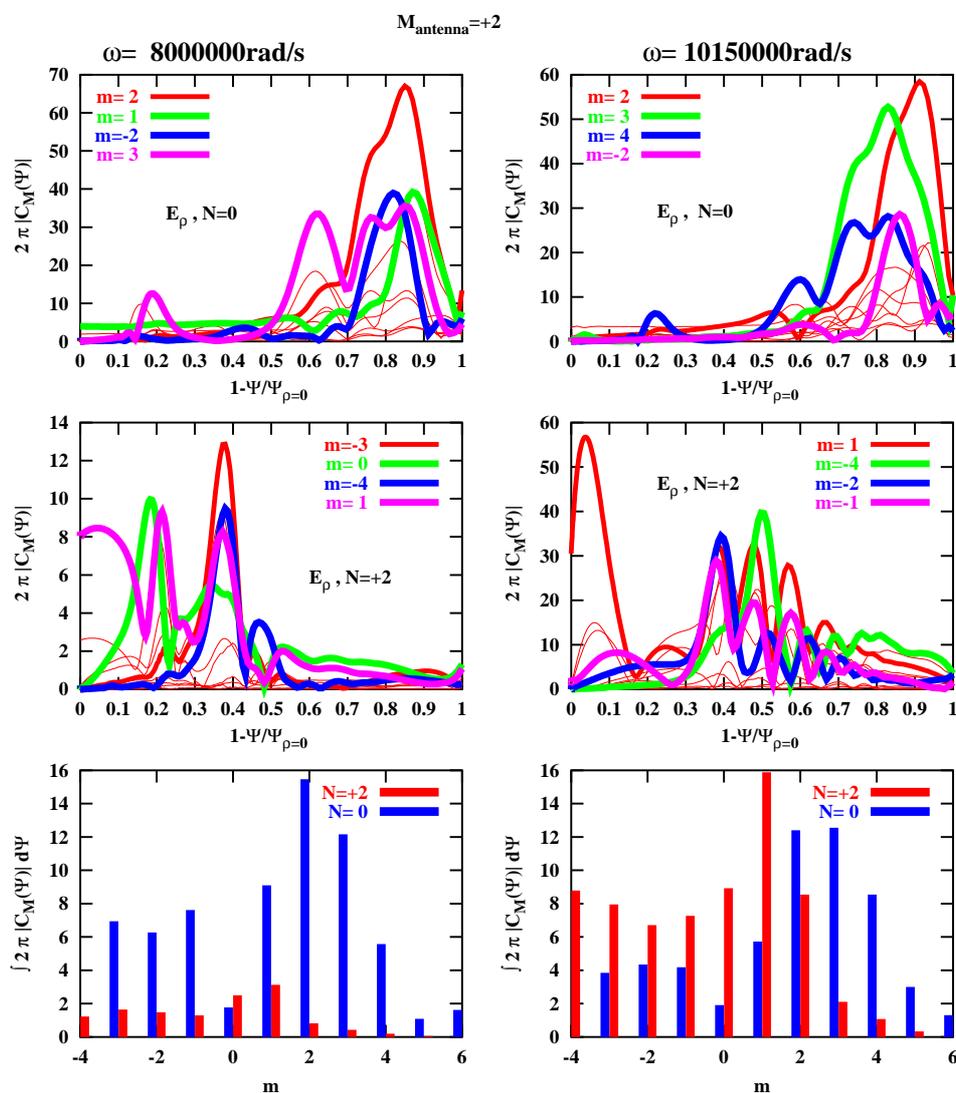


Figure.3 Fourier decomposition of the electric field components  $E_\rho$  for the cases  $\omega_{B_1} = 10.15 \times 10^6 \text{ rad/s}$ ,  $\omega_{B_1^0} = 10.15 \times 10^6 \text{ rad/s}$ ,  $\omega_{b_1} = 8.0 \times 10^6 \text{ rad/s}$ ,  $\omega_{b_1^0} = 8.0 \times 10^6 \text{ rad/s}$  (see Figure 2).

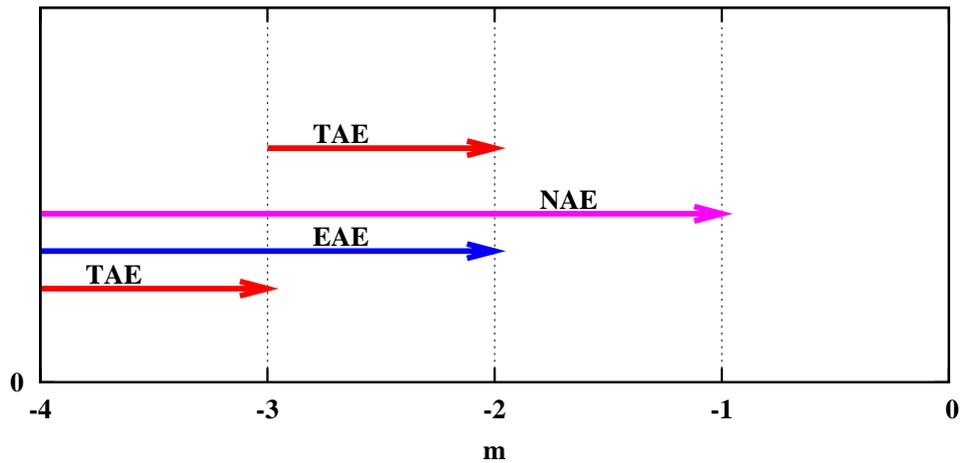


Figure.4 *The possible coupling modes in the cylindrical case.*

## References

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