Gyrokinetic Simulation of Neoclassical and Turbulent Transport

J.A. Heikkinen[1], S. Janhunen[2], and T.P. Kiviniemi[2]

[1] *VTT Processes, Euratom-TEKES Association, P.O. Box 1608, FIN-02044 VTT, Finland*
[2] *Helsinki University of Technology, Euratom-TEKES Association, FIN-02015 HUT, Finland*

Introduction

Electromagnetic gyrokinetic (GK) 3D plasma simulation for toroidal magnetic fusion devices has become a standard tool for transport analysis. However, based on delta f technique and analytic GK field equations [1], transport phenomena involving wide orbit effects, steep gradients, and rapid dynamic changes in profiles become difficult if not impossible to model with the present GK methods. In the present work, we describe an implicit solution method for the full f plasma quasineutrality, where the ion density change by polarization drift is evaluated directly from the ion orbit motion in terms of the unknown electrostatic potential at each time step. Being also full f and treating electrons kinetically within the drift-kinetic approximation, the present nonlinear method can provide a rigorous treatment of such global and dynamic transport phenomena like transport barrier generation and wide orbit effects. Within this method, one can also model the neoclassical and turbulent transport at saturation.

A full f nonlinear 3D gyrokinetic code ELMFIRE implementing an analytical GK method has been developed [2], and has been upgraded to incorporate the described direct solution method. The validation of the code with respect to linear ITG and TEM instability growth is described in Ref. [3]. The developed code is applied for global transport analysis in an FT-2 tokamak plasma involving lower hybrid (LH) heating. In the latter, both an internal transport barrier and edge H mode have been observed in strongly heated discharges. The transport coefficients and electrostatic field together with the density and temperature profiles are solved for the heated plasma and compared with the published experimental results.

Direct polarization method

In the original paper by W.W. Lee [1], the ion polarization density $(q/mB) \int (\Phi - \langle\Phi\rangle)\partial\langle f\rangle/\partial\mu d\vec{v}$ was derived within the gyrokinetic formalism. Here, Φ is the electrostatic potential, f is the ion distribution function, μ is the magnetic moment, $d\vec{v}$ denotes the velocity phase space differential, q, m are the ion charge and the mass, respectively, and B is the magnetic field. The gyroaverage around the ion Larmor rotation is denoted by $\langle..\rangle$. This perturbation of the ion density by the ion polarization drift $\vec{v}_p = (1/\Omega B)d\langle\vec{E}\rangle/dt$ has been derived assuming f is unperturbed by $\vec{v}_p$. Here, $\Omega = qB/m$ is the ion cyclotron frequency, and $\vec{E}$ is the electric field. In order to apply this perturbation, one must either know (or guess) the (background) ion distribution, or construct it from the particle simulation together with the solution of the gyrokinetic equation for the potential Φ. The latter was used in [2], while in other works, f has been guessed. As both of these techniques are nonrigorous, and for long simulations with steep gradients or wide orbits in configuration space are useless, the perturbation

in ion density by $\vec{v}_p$ is here directly calculated from the particle orbits during simulation. This direct calculation is made implicit in $\vec{E}$ by evaluating the ion density change δn_j at a cell j by the k'th ion polarization shift during the time step dt as

$$
\begin{aligned}
\delta n_j &= \frac{1}{\Omega B dV_j} \sum_i \sum_p \sum_l w_i [f_{xij} a_l \Phi_{pl} + f_{yij} b_l \Phi_{pl}] \\
&\quad - \frac{1}{\Omega B dV_j} \sum_i [\langle E_x^m \rangle f_{xij}^m + \langle E_y^m \rangle f_{yij}^m],
\end{aligned} \tag{1}
$$

where the gyroaveraged electric field $\langle \vec{E} \rangle = \sum_p \sum_l (a_l \Phi_{pl} \hat{x} + b_l \Phi_{pl} \hat{y})$ has been interpolated from the potential grid values Φ_{pl} around each Larmor circle point x_p, y_p, and summed over these points to obtain the Larmor average. w_i is the weight of the i'th subparticle on the Larmor circle of the particle k. We have $d\vec{s} = dx\hat{x} + dy\hat{y}$ for the polarization shift of the k'th ion perpendicular to the magnetic field with $\hat{x}$ and $\hat{y}$ denoting the unit vectors on the polarization plane. The particle cloud fraction derivatives $f_{xij} = df_{ij}/dx$ and $f_{yij} = df_{ij}/dy$ for the shift of the i'th subparticle cloud in each direction are obtained from the fraction f_{ij} of the cloud of the i'th particle within the cell j. dV_j is the volume of the j'th cell. The quantities with the upper index m arise from the equality $d\vec{s} = (1/\Omega B)(\langle \vec{E} \rangle - \langle \vec{E}^m \rangle)$, where $\vec{E}^m$ is the electric field at the start of the time step dt, and $\vec{E}$ is that (unknown) at the end of the time step. From the quasineutrality, $n^{ion} = n^{el}$, one obtains after summing Eq.(1) over all particles k the gyrokinetic equation $\sum_k \delta n_{kj} + \sum_k \sum_i w_{ki}^{ion} f_{kij}^{ion}/dV_j = \sum_k w_k^{el} f_{kj}^{el}/dV_j$, where $\sum_k \sum_i w_{ki}^{ion} f_{kij}^{ion}/dV_j$ denotes the ion density obtained in the cell j after all the drifts except the polarization drift. From this equation, one may evaluate the potential Φ at each grid point after the time step dt. $\langle \vec{E}_x \rangle$ and $\langle \vec{E}_y \rangle$ are stored for each particle after each time step from the resolved Φ, and are thus available as $\langle \vec{E}_x^m \rangle$ and $\langle \vec{E}_y^m \rangle$ at the next time step. In contrast to other methods, here the ions are advanced according to polarization drift keeping the sampled f consistent with particle motion.

The present technique increases somewhat the CPU and memory requirements in comparison to the method with a known f, but with a nine point interpolation for the field and using four subparticle points on each ion Larmor radius, only 20% increase in CPU requirement has been recorded over that with the standard gyrokinetic equation (with the known f). With a similar implicit solver for the electron density perturbation by the parallel electric field acceleration [2], superior stability properties of the algorithm have been observed making possible to use time steps much beyond the standard semi-implicit limit [4].

First gyrokinetic simulation evidence of ITB generation in a tokamak

In FT-2 tokamak[5], internal transport barrier has been observed with some 100 kW LH heating of ions. To model this experiment, simulations in a hydrogen or deuterium plasma with a minor radius of $a = 8$ cm, major radius of $R = 50$ cm, magnetic field $B = 2.2$ T, and plasma current of $I = 22$ kA have been conducted. At the outer edge, neutral particle ionization close to the limiter (with 1 cm decay length in the strength) provides the source of cold electron-ion pairs (at 15 eV) to replace the lost particles to the limiter. We have either started the simulation from the $T_e \sim 300$ eV, $T_i \sim 120$ eV plasma just after the onset of the LH heating (as in experiments) or
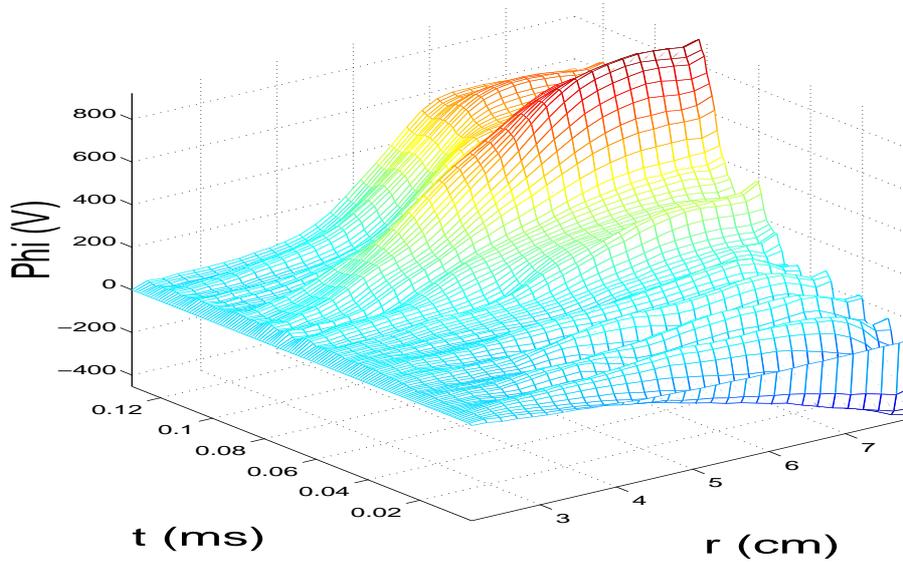
Figure 1: *Rapid growth in electrostatic potential is observed when ITB developes*

from well heated plasma with $T_i \sim 250$ eV. The LH power deposition profile is taken off-axis, as described in Ref. [5] by adjusting the LH wave intensity profile in our LH heating model. The density and temperature profiles are taken to be close to the experimentally measured ones at the simulation start. For the case, where the initial conditions are close to that of observed internal transport barrier generation (high ion temperature), the plasma potential (magnetic surface averaged) evolves as in Fig.1. Interestingly, after some 100 μs the radial electric field increases significantly in the middle of the plasma leading to a relatively strong shear in the $E_r \times B$ velocity at the radius $r \sim 6$ cm at $t \sim 110$ μs. As seen in Figs. 2–3, this leads to a simultaneously appearing knee-point in the ion temperature profile at $r \sim 6$ cm and a rapid collapse of the heat and particle diffusion coefficients inside this radius.

The diagnostics of turbulence shows contemporary suppression of broadband modes and related $\vec{E} \times \vec{B}$ convective cells together with the reduction of the size of the cells. The appearance of the transport barrier is found for both deuterium and hydrogen ions (hydrogen was used in experiments) roughly at the same ion temperature threshold, 250 eV, on axis. Also, when the plasma is allowed to be heated gradually (in $100 - 200$ μs) from $T_i \sim 100$ eV level up, the transition turns on when the ion temperature reaches the above threshold. The ion heat diffusion coefficients $4-6$ m^2/s and < 1 m^2/s before and after the transition, respectively, inside the position of the ITB are close to those obtained in ASTRA interpretative modeling of the FT-2 discharge [4].

The strong growth of Φ is also obtained when surface-averaged samples of ion and electron densities are used in the GK equation for Φ, i.e., when the turbulence is suppressed. This implies that the potential difference seen in Fig.1 arises from neo-classical effects. This particularly strong poloidal $\vec{E} \times \vec{B}$ flow for low current tokamaks was studied in detail in Ref. [6] with the neoclassical Monte Carlo particle code AS-
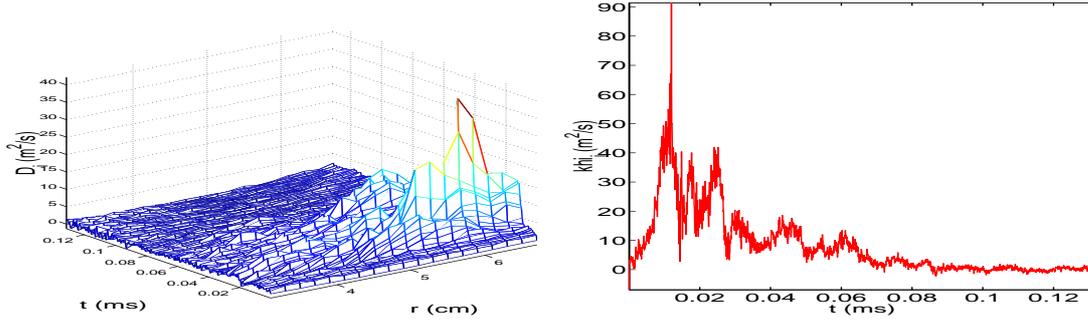
Figure 2: *a) Particle diffusion coefficient D_i and b) ion heat diffusion coefficient χ_i (at $r = 5$ cm).*
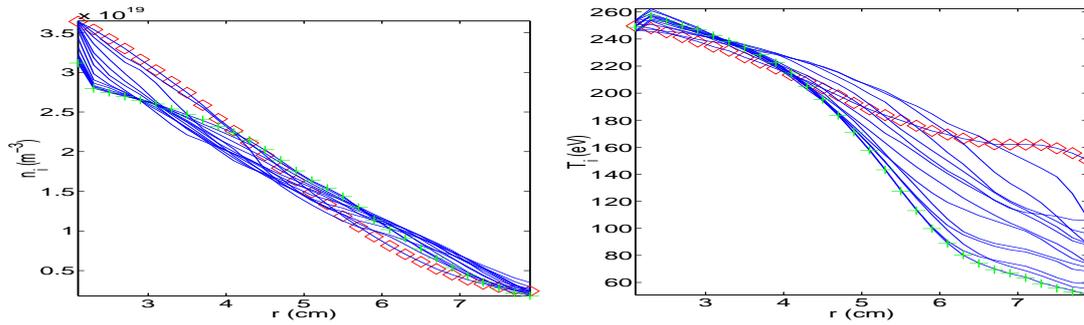

Figure 3: *Time traces of a) ion density n_i and b) temperature T_i. Here, diamonds denote the start and "+" the end of simulation.*

COT. In agreement with it, no strong potential difference is found at high plasma current $I > 35$ kA, and no transport barrier is observed within the line of experimental observations in FT-2. However, one should note that the code records a noteworthy Reynolds stress at the position of ITB when it is turned on.

In conclusion, a first direct ion polarization GK code has been written (to our knowledge) and is implemented in tokamak transport simulations. We find first evidence of ITB generation in close agreement with experimental observations for a heated tokamak. The low current FT-2 tokamak plasma simulation shows the importance of both the neoclassical field and Reynolds stress in creating the $\vec{E} \times \vec{B}$ poloidal flow shear needed to suppress the turbulent convection.

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