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A theoretical analysis of the effects of radio frequency waves on resistive wall modes

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The application of a high power radio frequency (rf) waves to a magnetically confined plasma gives birth to nonlinear interactions between the rf wave and collective plasma modes. One of the well-established examples of the nonlinear rf-plasma wave interaction will be the stabilization of interchange modes in mirror plasmas by the generation of a ponderomotive force (PMF) due to the applied rf. In the present study, we examine whether the similar enhancement of MHD stability by rf PMF will work on a tokamak plasma, for which the driving force for the instability is generally much stronger than that of the a mirror plasma. The required rf power should be within some practical level. Specifically, we present a preliminary analysis of the rf effects on resistive wall modes (RWM).

The rf-driven perturbed PMF, neglecting the magnetization current and parallel component of rf wave field, can be written[1],

$$f_P = -\frac{\varepsilon_0}{4}\sum_{\mu=L,R}\xi\frac{\partial\chi_\mu}{\partial r}\left(\frac{\partial|E_\mu|^2}{\partial r}+\frac{E_{\mu+}E_\mu^*}{\xi}+\frac{E_{\mu-}^*E_\mu}{\xi}\right)=-\frac{\varepsilon_0}{4}\sum_{\mu=L,R}\xi\frac{\partial\chi_\mu}{\partial r}\left(\frac{\partial|E_\mu|^2}{\partial r}+\frac{\partial\chi_\mu}{\partial r}A_\mu|E_\mu|^2\right) \quad (1)$$

where

$$\chi_\mu = -\sum_s\frac{\omega_{ps}^2}{\varpi_0(\varpi_0+\sigma_\mu\Omega_s)}\cong\frac{\omega_{pi}^2}{\sigma_\mu\Omega_i(\varpi_0+\sigma_\mu\Omega_i)},\quad \frac{\partial\chi_\mu}{\partial r}=\frac{\chi_\mu}{n(r)}\frac{\partial n(r)}{\partial r}+O(\beta)\cong\frac{\chi_\mu}{n(r)}\frac{\partial n(r)}{\partial r}$$

is the L and R component of susceptibility tensor,

$E_{\mu\pm}=\xi\dfrac{\partial\chi_{\mu\pm}/\partial r}{1+\chi_{\mu\pm}-c^2k_\pm^2/\varpi_\pm^2}E_\mu\cong\xi\dfrac{\partial\chi_\mu/\partial r}{1+\chi_\mu-c^2k_\pm^2/\varpi_0^2}E_\mu$ is the upper and lower sideband fields generated by the beating of the rf wave and the plasma wave, and

$$A_\mu \cong \frac{1}{1+\chi_\mu - c^2k_+^2/\varpi_0^2} + \frac{1}{1+\chi_\mu - c^2k_-^2/\varpi_0^2} = \frac{1}{1+\chi_\mu - c^2(k_0^2+2\vec{k}\bullet\vec{k}_0+k^2)/\varpi_0^2} + \frac{1}{1+\chi_\mu - c^2(k_0^2-2\vec{k}\bullet\vec{k}_0+k^2)/\varpi_0^2}$$

$$= -\frac{\varpi_0^2}{c^2(k^2+2\vec{k}\bullet\vec{k}_0)} - \frac{\varpi_0^2}{c^2(k^2-2\vec{k}\bullet\vec{k}_0)} = -2\frac{\varpi_0^2}{c^2}\frac{k^2}{k^4-4(\vec{k}\bullet\vec{k}_0)^2},$$

$$k^2 = m^2/r^2 + k_z^2 = m^2/r^2 + n^2/R^2, \quad k_0^2 = l^2/r^2 + k_\parallel^2, \quad \vec{k}\bullet\vec{k}_0 = ml/r^2 + k_z k_\parallel.$$

Here, $k(k_0)$ represents the plasma (rf) wave vector, and the other notations follow standard definitions being used in plasma physics. The first term in Eq. (1) represent the direct PMF, and the second term is the sideband coupling (SBC) term.

For the evaluation of rf ponderomotive force using Eq. (1), eigenmode rf electric field profiles should be calculated. In the present work, we consider only the fast wave excitation with lowest radial and azimuthal wave numbers. Then, following Ref. [2], the wave equation can be treated in a perturbation method using the relation, $|E_L(r)| << |E_R(r)|$ and the L and R components are decoupled to yield[2],

$$\nabla^2_{\text{perp}} E_R(r,\theta) + 2\kappa_R(r) E_R(r,\theta) = 0. \qquad (2)$$

In the present study, we consider only E_R component for the analysis. For a parabolic density profile, $n(r) = n_0\left(1-(r/a)^2\right)$, the analytic solution of Eq. (2) is found to be,

$$E_R(r) = C(\frac{r}{a})^{l-1}\exp(-q^2r^2/2)\,M(\frac{l}{2} - \frac{\sigma}{4}|l|q^2r^2),$$

where *M(a/b/x)* is the confluent hypergeometric function, and

$$f = \varpi_0/\Omega_{ci}, \quad \tilde{k}^2 = 2(\varpi_{pi}^2(0)/c^2)f^2/(1+f), \quad q^2 = \tilde{k}/a, \; k_r^2 = \tilde{k}^2 - 2k_\parallel^2, \; \sigma = k_r^2/q^2,$$

$$k_\parallel^2 = \left(\varpi_{pi}^2(0)/c^2\right)\left[f^2/(1+f)\right]\left[(1-(a/b)^2)/(l+1-(a/b)^2)\right].$$

The other notations, again, follow standard definitions. Figure 1 shows the radial profile of the l=1 E_R component for various values of central densities. As the central density increases, the radial mode number increases resulting in a large radial gradient of rf electric field.

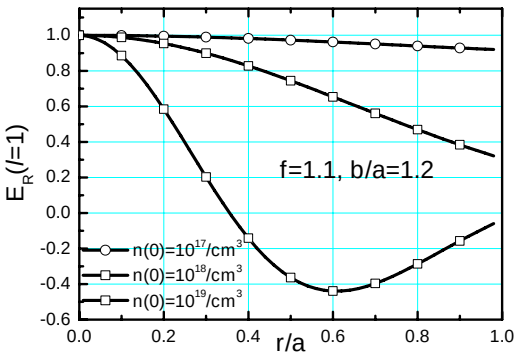

Fig. 1. Radial profile of E_R for various central densiies.

With the perturbed PMF given in Eqs. (1) and (2), the potential energy of a plasma is modified into the form,

$$\delta W = \delta W_{MHD} + \delta W_{RF} = \delta W_{MHD} - \frac{1}{2}\int_V dV \vec{\xi}^* \bullet \vec{f}_p$$

$$= \delta W_{MHD} + \frac{\varepsilon_0}{8}\sum_{\mu=L,R}\int_V dV|\xi|^2 \frac{\partial \chi_\mu}{\partial r}\left(\frac{\partial |E_\mu|^2}{\partial r} + \frac{\partial \chi_\mu}{\partial r} A_\mu |E_\mu|^2\right).$$

Then, the Euler-Lagrange equation inside the plasma satisfying the variational principle can be written

$$\frac{d}{dr}\left(f_{MHD}\frac{d\xi}{dr}\right) - g\xi = 0,\ g = g_{MHD} + g_{RF},\ g_{RF} = \frac{\varepsilon_0}{8}\sum_{\mu=L,R}\frac{\partial \chi_\mu}{\partial r}\left(\frac{\partial |E_\mu|^2}{\partial r} + \frac{\partial \chi_\mu}{\partial r} A_\mu |E_\mu|^2\right), \quad (3)$$

where f_{MHD} and g_{MHD} are the standard MHD operators given in the text[3]. The dispersion relation for external kink modes in the presence of a resistive wall at $r=b$ is well-known and given by the expression

$$\gamma\tau_w = -\left[2|m|/(1-(a/b)^{2|m|})\right]\left[(1+R_m)/(\Lambda_m + R_m)\right],$$

where

$$\tau_w = \mu_0\sigma_w bd, \quad \Lambda_m = 1+\left[(1+(a/b)^{2|m|})/(1-(a/b)^{2|m|})\right],$$

$$R_m = |m|/(a^2[k^2]_a)\left[a(d\xi/dr)_a/\xi_a - (1-q_a(n/m))/(1+q_a(n/m))\right].$$

The problem of rf influence on RWM, then, is reduced to finding $(d\xi/dr)_a/\xi_a$ by solving Eq. (3) for given mode numbers and edge q-value. The Wesson form of q-profile, $q(r) = q_a\hat{r}^2/(1-(1-\hat{r}^2)^{q_a/q_0})$ with q_0=1.3, q_a=2.9 and copper resistive wall at $r=b$ is used in the present study.

Figure 2(a) shows the comparison of growth rate as a function of C with (triangle) and without (circle) SBC effects. The PMF stabilize the RWM while SBC

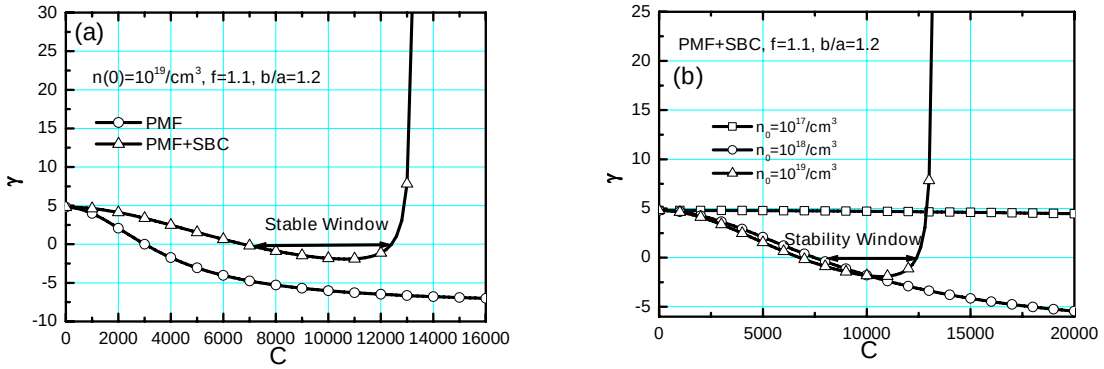

Fig. 2. (a) Effects of PMF and SBC, and (b) appearance of stability window in rf power.

destabilize it, which is consistent with the theory. It can be also seen in Fig. 1(a) that a complete stabilization is possible when C exceeds some value, and the required power is estimated to be $P(W/m^3) \approx 4\varepsilon_0 \varpi_0 <|E_-|^2>_{vol} \cong 2\varepsilon_0 \varpi_0 C_0^2 \cong 1.2\times 10^5 (W/m^3)$ which yields ~ 4 MW power in a medium size tokamak. When the central density is too low, the RWM can not be stabilized due to the smallness of field gradient (Fig. 1). In a higher density, the stability is achieved but a stability window in rf power appears. Further increase of the rf power beyond some critical value of C converts the RWM into the ideal mode. For a given rf power and b/a, the increase of $f=\omega_0/\Omega_{ci}$ value diminishes the stabilizing influence of rf PMF (Fig. 3(a)). For a fixed value of f, the rf enhancement of the RWM stability can be interpreted as the increase of the critical wall position beyond which the RWM is converted into the ideal mode, as can be seen in Fig. 3(b).

Summarizing the results, we have carried out a preliminary analysis of the rf influence on RWMs in a tokamak plasmas. Toroidal extension of the present study is remained as a future work for the elucidation of rf effects on RWMs.

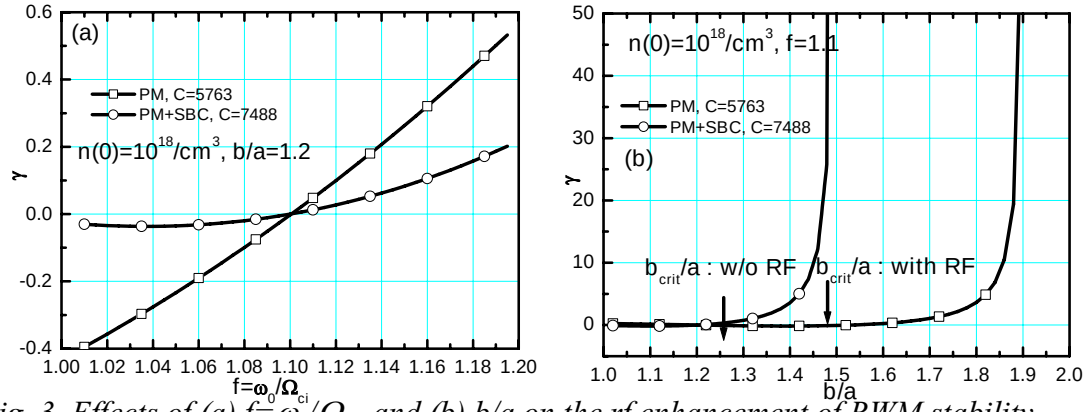

Fig. 3. Effects of (a) $f=\omega_0/\Omega_{ci}$, and (b) b/a on the rf enhancement of RWM stability.

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