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# Neoclassical Momentum Transport and Radial Electric Field in Tokamak Transport Barriers

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## Introduction

During tokamak operation in enhanced confinement regimes, ion thermal transport in the core and barrier regions is observed to be at a level comparable to that predicted by neoclassical theory.[1,2] The radial angular momentum transport is nevertheless anomalous, typically an order of magnitude larger than the neoclassical prediction.[2] This predicted viscosity is substantially smaller than the heat diffusivity, scaling like a diffusion coefficient for the Pfirsch-Schlüter rather than the banana regime, even though the collisionality is low.[3]

In a toroidally rotating plasma, heavy impurity ions are subject to a large centrifugal force, pushing them to the outside of the torus, so their distribution over a flux surface is non-uniform.[4] This led to a large enhancement in the predicted neoclassical particle transport, of the order of $\varepsilon^{-3/2}$ with $\varepsilon$ the inverse aspect ratio.[5,6] This is the ratio of the diffusion coefficient in the banana regime to that in the Pfirsch-Schlüter regime. The effect of such poloidal impurity redistribution on radial angular momentum transport has not yet been considered.

Radial transport can be characterised by the transport matrix, $L$, which links radial gradients of plasma parameters – ion pressure ($p_i$), ion temperature ($T_i$), and toroidal angular velocity ($\omega$) – to the radial ion flux of particles ($\Gamma$), heat (q), and angular momentum ($\Pi$). We have calculated the three transport coefficients $L_{31}$, $L_{32}$ and $L_{33}$ which describe radial angular momentum transport.

The bulk plasma is assumed to consist of deuterium ions, density $n_i$, mass $m_i$, in the banana regime and a single heavy, highly charged and therefore collisional, impurity species. The impurity density, $n_z$, is low, so angular momentum carried by impurities may be neglected and the impurity charge z is large, thus $\alpha = n_z \mathrm{z}^2 / n_i \sim O(1)$. The bulk plasma rotates subsonically, with Mach number $M_i$ satisfying: $M_i^2 = m_i \omega^2 R^2 / 2T_i \sim 1/\mathrm{z} << 1$. This ordering allows for sonic impurity rotation and therefore their strong poloidal redistribution, as shown, for example, in Ref. 4: $n_z = n_{z0} \exp\left[\left(1 - \frac{T_e}{T_i + T_e} \mathrm{z} \frac{m_i}{m_z}\right) \frac{m_z \omega^2 \left(R^2 - R_0^2\right)}{2T_z}\right]$.

**Formulation**

Taking the plasma to be magnetised, the parameter $\delta$ is small: $\delta = \rho_i / L_r << 1$, where $\rho_i$ is the ion gyroradius and $L_r$ the macroscopic radial scale length. Cross-field transport occurs on a timescale which is second order in $\delta$.[3] Expanding the bulk ion distribution function, $f$, in $\delta$, gives: $f = f_0 + f_1 + f_2 + ...$ with $f_1 \sim \delta f_0$.

$f_2$ is required to calculate the angular momentum flux from its definition as a velocity moment, but it must be determined using the gyroaveraged Fokker-Planck equation for the bulk ions, so is not readily tractable. Thus a flux-friction relation for Π, requiring $f$ only to first order in $\delta$, is used. Assuming no angular momentum sources are present:[3]

$$\Pi = -\frac{m_i}{e} \langle \int \mathrm{d}^3\mathrm{v} \, \frac{1}{2} m_i R^2 \mathrm{v}_\phi^2 \, C(f_1) \rangle$$

Restricting to large $\alpha$, only bulk ion–impurity collisions need be described by the collision operator, $C$. Due to the disparate masses, the interaction may be described analogously to that of electrons and ions - the first term represents pitch-angle scattering and the second momentum conservation:[7]

$$C = \nu_{iz}(\Psi,\theta)\left( L(f_1) + \frac{m_i \mathrm{v}_\parallel}{T_i} \mathrm{V}_{z\parallel}(\Psi,\theta) f_0 \right)$$

where $\nu_{iz} = \left(3\pi^{1/2}/4\tau_{iz}\right)\left(\mathrm{v}_{\mathrm{Ti}}/\mathrm{v}\right)^3$, $\mathrm{v}_{\mathrm{Ti}} = \left(2T_i/m_i\right)^{1/2}$, $\tau_{iz} = 3(2\pi)^{3/2} \varepsilon_0^2 m_i^{1/2} T_i^{3/2} / n_z \mathrm{z}^2 e^4 \ln\Lambda$, the Lorentz operator $L = (1/2)(\partial/\partial\xi)\left(1-\xi^2\right)(\partial/\partial\xi)$ with the pitch angle $\xi = \mathrm{v}_\parallel / \mathrm{v}$ and the angle $\theta$ measures poloidal position on the flux surface, with $\theta = 0$ at the outboard side. The $\theta$ dependence of $C$ results from the non-uniform poloidal impurity distribution.

For the parallel impurity velocity, $\mathrm{V}_{z\parallel}$, we extract $\mathrm{V}_{z\perp}$ from the impurity fluid momentum equation and take the impurity flux to be divergence free, to lowest order in $\delta$. Thus:

$$\mathrm{V}_{z\parallel} = -\frac{\mathrm{I}}{\mathrm{B}}\left( \frac{\partial\Phi}{\partial\Psi} + \frac{1}{n_z z e}\frac{\partial p_z}{d\Psi} - \frac{m_z \omega^2}{2ze}\frac{\partial R^2}{\partial\Psi} \right) + \frac{\mathrm{K}(\Psi)\mathrm{B}}{n_z}$$, where $\mathrm{I} = R\mathrm{B}_\phi$, $\Phi$ = electrostatic potential

The constant of integration, K, is determined by multiplying the momentum equation by $\mathrm{B}/n_z$ and flux surface averaging. Using $\langle \mathrm{B}\nabla_\parallel \mathrm{A} \rangle = 0$ for any A periodic in $\theta$, $\langle \mathrm{B}\mathrm{R}_{\mathrm{zi}\parallel} / n_z \rangle = 0$, with the bulk ion-impurity friction $\mathrm{R}_{\mathrm{zi}\parallel} = -\int m_i \mathrm{v}_\parallel C(f_1) \mathrm{d}^3\mathrm{v}$, and K is found upon substitution of $C$.

To determine $f_1$ we follow Ref. 3 and transform the drift kinetic equation to a rotating frame, then expand in powers of $\delta$. Plasma on a flux surface is shown to rotate toroidally, as a rigid body, with angular velocity $\omega$, such that $\omega = -\partial\Phi/\partial\Psi$. This gives the connection between the evolution of toroidal rotation via angular momentum transport and the development of the radial electric field, in neoclassical regions.

With H the bulk ion Hamiltonian and $\mu$ the adiabatic magnetic moment, the equation in order $\delta$ gives $f_1$ (details of $A_j$ and the coefficients $\alpha_j$ are given in Ref. 3):

$$\mathrm{v}_{\|}\nabla_{\|} f_1 - C(f_1) = -\frac{e_i}{T_i}\mathrm{v}_{\|}\nabla_{\|}\Phi_1 f_0 - \mathrm{v}_{\|} f_0 \sum_j^3 \mathrm{A}_j(\Psi)\nabla_{\|}\alpha_j$$

$A_j$ are the three radial gradients, where $\mathrm{A}_3 = \omega'/\omega$ with $' \Rightarrow \partial/\partial\Psi$ is the rotation shear.

We perform a subsidiary expansion of $f_1$, in the small ratio of the bulk ion collision frequency to the typical banana orbit bounce frequency. The two lowest order equations give:[7] $f_1 = \mathrm{F} + \mathrm{g}$ where $\nabla_{\|}\mathrm{g} = 0$ , $\mathrm{F} = \left(\frac{e\Phi_1}{T_i} - \sum_j \mathrm{A}_j\alpha_j\right) f_0$

and: $\oint v_{iz}\left(L(\mathrm{g}+\mathrm{F}) + \frac{m_i \mathrm{v}_{\|}}{T_i}\mathrm{V}_{z\|} f_0\right)\mathrm{dt} = 0$, with the integral taken along a complete orbit.

**Results**

Applying the above for arbitrary flux surface geometry and impurity distribution we find:

$$\Pi = -\frac{p_i \mathrm{I}^2 \omega \mathrm{R}_0^2}{\hat{\Omega}_\mathrm{i}^2}\frac{1}{\langle\tau_{iz}\rangle}\left(l_{31}\mathrm{A}_1 + l_{32}\mathrm{A}_2 + l_{33}\mathrm{A}_3\right) \qquad \text{where} \quad l_{33} = l_{33}^{(1)} + l_{33}^{(2)}$$

$$l_{31} = \langle\frac{\mathrm{nr}^2}{\mathrm{b}^2}\rangle - f_c\langle\mathrm{nr}^2\rangle - \frac{1-f_c\langle\mathrm{nr}^2\rangle}{\langle\frac{\mathrm{b}^2}{\mathrm{n}}\rangle - f_c}(1-f_c) \qquad l_{32} = -\frac{3}{2}l_{31} - \langle M_\mathrm{i}^2\rangle\left[\langle\frac{\mathrm{nr}^4}{\mathrm{b}^2}\rangle - f_c\langle\mathrm{nr}^2\rangle^2 - \frac{\left(1-f_c\langle\mathrm{nr}^2\rangle\right)^2}{\langle\frac{\mathrm{b}^2}{\mathrm{n}}\rangle - f_c}\right]$$

$$l_{33}^{(1)} = -(2l_{32} + 3l_{31}) \qquad l_{33}^{(2)} = -\frac{5}{2}\frac{\mathrm{R}_0^2\mathrm{B}_0^2}{\mathrm{I}^2}\left[\frac{15\mathrm{B}_0^2}{4}\int_0^{\mathrm{B}_{\min}^{-1}}\frac{\langle\frac{\mathrm{nr}^2}{\mathrm{b}}\frac{\mathrm{B}_\phi^2 - \mathrm{B}_\mathrm{p}^2/2}{\mathrm{B}_0^2}\sqrt{1-\lambda\mathrm{B}}\rangle^2}{\langle\mathrm{n}\sqrt{1-\lambda\mathrm{B}}\rangle}\lambda d\lambda - \langle\mathrm{nr}^4\left(\frac{\mathrm{B}_\phi^2 - \mathrm{B}_\mathrm{p}^2/2}{\mathrm{B}^2}\right)^2\rangle\right]$$

with $\mathrm{n} = n_z/\langle n_z\rangle$, $\mathrm{r}^2 = R^2/\langle R^2\rangle$, $\mathrm{b}^2 = \mathrm{B}^2/\langle\mathrm{B}^2\rangle$, $\mathrm{R}_0^2 = \langle R^2\rangle$, $\mathrm{B}_0^2 = \langle\mathrm{B}^2\rangle$, $\hat{\Omega}_\mathrm{i} = e\mathrm{B}_0/m_i$ , $\mathrm{B}_{\min(\max)}$ = magnetic field strength on out(in)board side, $\lambda = \mathrm{v}_\perp^2/\mathrm{v}^2\mathrm{B}$ , $\lambda_c = \mathrm{B}_{\max}^{-1}$ defines the passing-trapped boundary and $f_c = \frac{3}{4}\langle\mathrm{B}^2\rangle\langle n_z\rangle\int_0^{\lambda_c}\frac{\lambda\, d\lambda}{\langle n_z\sqrt{1-\lambda\mathrm{B}}\rangle}$, where $n_z$ may vary strongly over a flux surface.[7] The expressions may be evaluated numerically using experimental data.

The coefficients are simplified in the limit $\varepsilon$<<1 (conventional aspect ratio) and strong impurity redistribution, such that $<\mathrm{n}\cos\theta> \sim \mathrm{O}(1)$. Taking $\varepsilon = \varepsilon(\theta)$, thus not restricting to circular equilibrium, $\mathrm{r} = 1 + \varepsilon\cos\theta + O(\varepsilon^2)$, $\mathrm{b} = 1 - \varepsilon\cos\theta + O(\varepsilon^2)$ and $M_{i0} = M_i(\mathrm{R}_0)$:

$$l_{31} = (1-f_c)\left(1 - \frac{1-f_c}{\langle \frac{1}{\mathrm{n}} \rangle - f_c}\right) \qquad l_{32} = -\left(\frac{3}{2} + M_{i0}^2\right) l_{31} \qquad l_{33}^{(1)} = 2M_{i0}^2 l_{31} \qquad l_{33}^{(2)} \sim O(\varepsilon^2)$$

Generalising to $\mathrm{r} = 1+\alpha$, $\mathrm{b} = 1+\beta$, with $\alpha, \beta \sim O(\varepsilon)$ and free to vary arbitrarily over the flux surface: $l_{33}^{(2)} = \frac{2}{5}\left(\langle \mathrm{n}(2\alpha+\beta)^2 \rangle - \langle \mathrm{n}(2\alpha+\beta) \rangle^2\right)$. Thus $l_{33}^{(2)}$ is zero to first order in $\varepsilon$ and sensitive in second order to only the first order shape of the flux surfaces. The resemblance to a statistical variance shows $l_{33} \geq 0$, as is required by entropy considerations.[7]

Schwartz's inequality gives $f_c \sim 1 - O(\sqrt{\varepsilon})$, thus with the classical diffusion coefficient $D_{cl} = \rho_i^2 / \tau_{iz}$, $q$ the safety factor and $L_{31} = -l_{31}\, p_i \mathrm{I}^2 \omega R_0^2 / \left(\hat{\Omega}_i^2 \langle \tau_{iz} \rangle\right)$:

$$L_{31} \sim L_{32} \sim D_{cl}\, q^2 / \varepsilon^{3/2} \qquad L_{33} \sim M_i^2 D_{cl}\, q^2 / \varepsilon^{3/2}$$

The angular momentum flux shows an enhancement of $\varepsilon^{-3/2}$ over previous predictions.[3]

The effect of rotation shear, $A_3$, as a driving force is increased by $M_i^2 \varepsilon^{-3/2}$ over previous results,[3] a small factor as $M_i^2 << 1$. However, pressure and temperature gradients become very effective driving forces in the presence of impurities and essentially dominate the transport. The large values of $L_{31}$, $L_{32}$ could allow spontaneous toroidal rotation to arise in a plasma, as with no external angular momentum source to balance $\Pi$: $\frac{\omega'}{\omega} = -\frac{1}{L_{33}}\left(L_{31}\frac{p_i'}{p_i} + L_{32}\frac{T_i'}{T_i}\right)$

**Conclusions**

As was seen in the case of particle transport,[6] poloidal redistribution of impurities significantly affects radial angular momentum transport in a mildly rotating tokamak plasma. At conventional aspect ratio, with strong redistribution and large $\alpha$, the flux is increased by a factor of $\varepsilon^{-3/2}$ compared to the usual prediction, making it typical of banana regime transport.

Radial pressure and temperature gradients are the primary driving forces of the flux, whereas previously only rotation shear had been effective.[3] Thus spontaneous toroidal rotation could arise in a plasma with no external angular momentum sources, as is frequently observed in experiments. The rotation direction depends on the edge boundary condition.

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