

Nonlinear interaction of an ultraintense electromagnetic wave and the self-created electron-positron plasma

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Abstract

The nonlinear interaction between the electron-positron pairs produced by an electromagnetic wave and the wave itself in a plasma is investigated for a circularly polarized wave using the relativistic Vlasov equation with a source term based on the Schwinger formula for the pair creation rate.

The production of electron positron pairs in a constant, spatially homogeneous electric field was first investigated in Refs. [1]. This quantum field theory effect lies outside perturbation theory and its experimental verification would test the validity of the theory in the region of strong fields. The most probable way of detecting the e^+e^- pair production is to use a time-varying ultra intense laser field [2], as indicated by recent developments of laser technology [3] and by the proposed methods for reaching $I_{Sch} = 4.65 \times 10^{29}$ W/cm², corresponding for a $\approx 1\mu m$ laser, to an electric field equal to the critical Schwinger field $E_{Sch} = 1.32 \times 10^{16}$ V/cm [4].

The problem of the backreaction of the produced particles on the background field was discussed extensively in a number of papers on the particle formation process in high energy hadronic interactions as well as under the action of electric fields [5]. While the effect of the produced particles on the electric field was taken into consideration and a kinetic equation coupled

to Maxwell equations was used, the spatially homogeneous time dependent electric field adopted was not a solution of Maxwell equations in vacuum.

We consider the process of e^+e^- pair production in a cold collisionless plasma by an e.m. field which is an actual solution of the Maxwell equations, as well as the backreaction of the produced pairs on the e.m. field [6]. In order to elucidate the role of the magnetic field component of the e.m. wave on the e^+e^- pair production, we consider a planar, circularly polarized wave propagating in an underdense collisionless e^+e^- plasma, with amplitude A_0 , frequency ω , and wave vector k in the laboratory frame. For a wave in a plasma the Lorentz invariant $\mathcal{F} = (\mathbf{E}^2 - \mathbf{B}^2)/2 = \Omega^2 A_0^2/2$ where Ω is the Langmuir frequency. Therefore, in a plasma, e^+e^- pairs can be produced by a planar e.m. wave, as was shown in Ref.[6]. A Lorentz transform to the reference frame moving with the group velocity v_g of the wave transforms the e.m. field into a purely electric field, that rotates with constant frequency, and with no associated magnetic field, $\mathbf{E} = \Omega A_0 (\mathbf{e}_x \sin \Omega t - \mathbf{e}_y \cos \Omega t)$. This transformation reduces the problem to the situation where the pairs are produced by a time-varying electric field.

The propagation of the circularly polarized wave in the boosted frame is analyzed using the relativistic kinetic equation

$$\partial_t f_\alpha + e_\alpha \mathbf{E}(t) \cdot \partial_p f_\alpha = q_\alpha(|\mathbf{E}|, p), \quad (1)$$

for the positron (electron) d.f. $f_\alpha(\mathbf{p}, t)$ with $\int f_\alpha(\mathbf{p}, t) d^3p / (2\pi)^3 = n_\alpha$ the number of positrons or electrons per unit volume in the boosted frame, and e_α is their charge. The source term in Eq.(1)

$$q_\alpha(|\mathbf{E}|, p) = 2e^2 |\mathbf{E}(t)|^2 \exp[-\pi m^2 / |e\mathbf{E}(t)|] \delta(\mathbf{p}) \quad (2)$$

is proportional to the quasiclassical probability of e^+e^- production under the action of the constant electric field. Since the characteristic pair production time c/l_c , $l_c = \hbar/mc$ is negligible with respect to the wave period, we take the time dependence as parametric. We assume that the pairs are produced at rest.

The e^+e^- pair production leads to the appearance of a time-dependent electric dipole which generates a polarization current $d\mathbf{E}/dt = -4\pi \mathbf{j}_{tot} = -4\pi (\mathbf{j}_{cond} + \mathbf{j}_{pol})$, with

$$\mathbf{j}_{\alpha, cond}(t) = e \int f_\alpha(\mathbf{p}, t) \frac{\mathbf{p}}{\mathcal{E}} \frac{d^3p}{(2\pi)^3}, \quad \mathbf{j}_{\alpha, pol}(t) = \frac{\mathbf{E}(t)}{|\mathbf{E}(t)|^2} \int q_\alpha(\mathbf{p}, t) \mathcal{E} \frac{d^3p}{(2\pi)^3}, \quad (3)$$

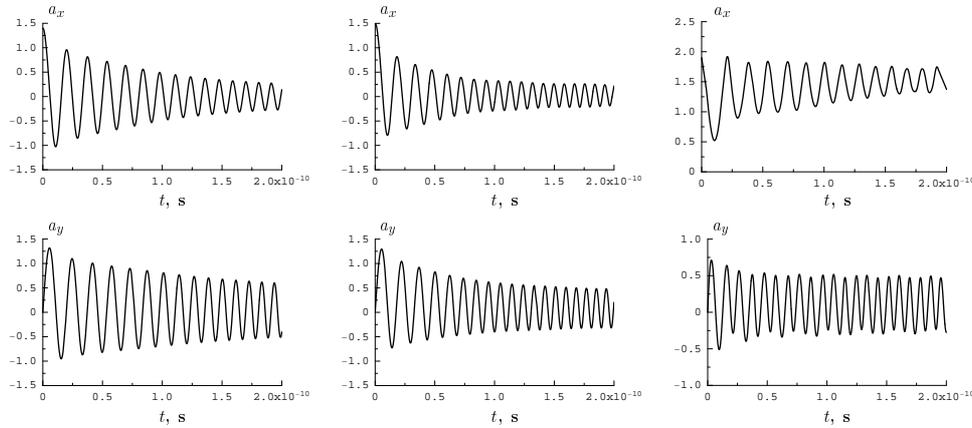


Figure 1: Time evolution in the moving frame of the x and the y -components of $\mathbf{a}(t)$ for different initial amplitudes: 1.4×10^5 (a), 1.5×10^5 (b), 1.9×10^5 (c) with initial plasma density $n_0 = 10^{19} \text{ cm}^{-3}$ in the moving frame.

where $\mathcal{E} = (m^2 + p^2)^{1/2}$. From the solution of Eq.(1), with $\mathbf{a} = e\mathbf{A}/m$, $\mathbf{e} = e\mathbf{E}/m^2$, we have

$$\begin{aligned} \frac{d\mathbf{a}(t)}{dt} = -m\mathbf{e}(t), \quad \frac{d\mathbf{e}(t)}{dt} = \frac{\Omega^2}{m}\mathbf{a}(t) - \frac{em}{2\pi^2}\mathbf{e}(t) \exp\left[-\frac{\pi}{|\mathbf{e}(t)|}\right] \\ + \frac{\kappa}{8\pi^3 m^2} \int_0^t \frac{[\mathbf{a}(t) - \mathbf{a}(s)]|\mathbf{e}(s)|^2}{[1 + |\mathbf{a}(t) - \mathbf{a}(s)|^2]^{1/2}} \exp\left[-\frac{\pi}{|\mathbf{e}(s)|}\right] ds \end{aligned} \quad (4)$$

Here $\kappa = 8\pi e^2 m^4$ and the factor m^4 stands for the inverse of the invariant Compton 4-volume $m^4 = c/l_c^4 \approx 0.14 \times 10^{53} \text{ cm}^{-3} \text{ s}^{-1}$. Numerical solutions of this system are presented in Fig.1 for initial amplitudes $a = 1.4 \times 10^5$ (a), $a = 1.5 \times 10^5$ (b), $a = 1.9 \times 10^5$ (c). The pair production leads to the damping of the wave and to the nonlinear up-shift of its frequency due to the increase of the plasma pair density. Since the pair production rate depends on the field amplitude exponentially, an unbalanced damping of the field components can occur and lead to a change of the field polarization as shown in Fig.2, where the projections of the polarization vector are presented for the same set of initial parameters as in Fig.1.

In Fig.2a we see the damping of the x -component of the electric field and the transition from circular to elliptic polarization with the major axis of the ellipse directed along the y -axis. In Fig.2b we see a rotation of the principal axes of the ellipse. In Fig.2c the initial pair production rate is so large that

the first wave oscillation cycle cannot be completed, leading to oscillations of the x -component of the wave vector potential around a non-zero mean value. This shift of the center of the oscillations of the x -component of the vector potential leads to a reduction of the oscillation frequency of this wave component so that the x and the y -components of the wave oscillate at different frequencies.

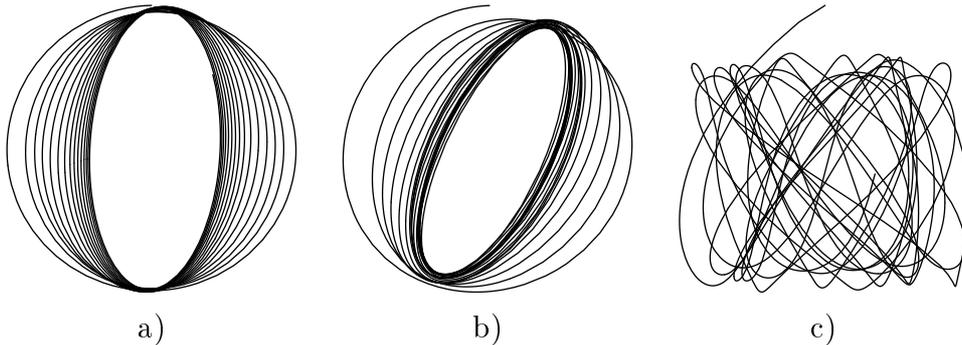


Figure 2: Trajectories of the projections of the electric field polarization vector for the same set of initial conditions as in Fig.1.

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