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Coupled dust-lattice modes in dust-plasma crystals

V. V. Yaroshenko, A. V. Ivlev, and G. E. Morfill

Max-Planck-Institut für Extraterrestrishe Physik, D-85741, Garching, Germany

INTRODUCTION

Dust-lattice (DL) modes and their stability are extensively studied both theoretically and in many experiments using discharge plasmas in the laboratory and under microgravity conditions (see Ref.[1], and references therein). In the laboratory experiments, the highly negatively charged particles levitate in the sheath region of the horizontal negatively biased electrode where there is a balance between the gravitational and electrostatic forces acting in the vertical direction. If the particles are magnetized by an external field, the magnetic force can also contribute to the force balance and supports the particle levitation. There are a few characteristic features of the particle trapping in discharge plasmas, which results in anisotropic character of dust-dust interactions: (i) the anisotropy due to vertical ion flows and formation of an ion "wake"; (ii) a dependence of equilibrium grain charge on the vertical particle position in the sheath region; (iii) if the particles are magnetized by an external inhomogeneous magnetic field, there exists a vertical nonuniformity of induced magnetic moments. We show that anisotropy in the interaction potential influences particle equilibrium state and causes linear DL mode coupling in a model of a one-dimensional particle string.

The horizontal (along the x-axis) string is formed by identical spherical dust particles of a radius a, charge $-Q<0$, mass M, and magnetic moment m induced by a vertical magnetic field, B ($m=\alpha B,\ \alpha=(\mu-1)a^3/(\mu+1)$, with magnetic permeability μ). The ion wake is modelled by a point-like effective positive charge located at a distance $l<\Delta$ beneath the particle, where Δ is an interparticle distance. The dust-dust interactions are described by a combination of the screened Debye potentials of the particle charge itself, its effective wake charge and magnetic dipole term

$$\varphi_n(\mathrm{r})=\frac{Q_n\exp\left(-|\mathrm{r}-\mathrm{r}_n|/\lambda\right)}{|\mathrm{r}-\mathrm{r}_n|}-\frac{q_n\exp\left(-|\mathrm{r}-\mathrm{r}_{nq}|/\lambda\right)}{|\mathrm{r}-\mathrm{r}_{nq}|}+\frac{((\mathrm{r}-\mathrm{r}_n)\mathrm{m}_n)}{|\mathrm{r}-\mathrm{r}_n|^3}, \qquad (1)$$

Here Q_n (q_n) denotes a temporary particle (wake) charge, vector $\mathrm{r}_n(\mathrm{r}_{nq})$ relates to the n-th particle (wake) position and λ is a screening length. When a grain n acquires a small vertical displacement around its equilibrium z_n, the particle magnetic moment and charge can be

approximated by $m_n \approx m_0(1+\beta z_n)$, $Q_n \approx Q_0(1+\varepsilon z_n)$, and $q_n \approx q_0(1+\varepsilon z_n)$ with $\beta = (dm/dz)_0$ and $\varepsilon = (dQ/dz)_0$. The subscript "0" implies the equilibrium grain position.

The equilibrium balance equation providing a trapping of magnetized grains in vertical direction involves the electric force, the force related to the vertical gradient of the external magnetic field and a term arising due to the particle-wake interactions with neighboring grains

$$Mg = Q_0 E_0 + m(\partial B/\partial z)_0 - 2\tilde{q} l M \Omega_0^2, \qquad (2)$$

where $\tilde{q} = q_0/Q_0$, $\kappa = \Delta/\lambda$ and $\Omega_0^2 = Q_0^2(1+\kappa)\exp(-\kappa)/(M\Delta^3)$.

DL MODES IN A HORIZONTAL STRING OF PARAMAGNETIC PARTICLES

Dispersion relation. Following the lines of recent treatments of DL waves (e.g. [2-4]), one can obtain the dispersion relation describing the perturbations in particle positions $x_n, z_n \propto \exp(-i\omega t + ikn\Delta)$ as a condition of solvability for the system of linearized momentum equations in the closest neighbor approximation. This can be written in the form typical for the linear coupling of two modes,

$$(\omega^2 + 2i\gamma\omega - \Omega_{\parallel}^2)(\omega^2 + 2i\gamma\omega - \Omega_{\perp}^2) + \Omega_{coupl}^4 = 0 \qquad (3)$$

with modified longitudinal

$$\Omega_{\parallel}^2 = 4\left[(1-\tilde{q})\psi(\kappa)\Omega_0^2 + \Omega_1^2\right]\sin^2(k\Delta/2) \qquad (4)$$

and transverse DL wave frequency

$$\Omega_{\perp}^2 = \Omega_v^2 - 4\left[(1-\tilde{q})\Omega_0^2 + 3\Omega_1^2\right]\sin^2(k\Delta/2) + 4\varepsilon\Delta\Omega_{Qq}^2\cos^2(k\Delta/2). \qquad (5)$$

Here $\psi(\kappa) = 2 + \kappa^2/(1+\kappa)$ and a frequency Ω_1 relates to the magnetic dust-dust interactions through $\Omega_1^2 = 3m_0^2/(M\Delta^5)$. The frequency describing the vertical oscillations of a single particle Ω_v is also modified by the magnetic field [3], yielding $\Omega_v^2 = -\left[Q_0(\varepsilon E_0 + E_0') - \beta B_0^2(\beta^2+\gamma)\right]M^{-1}$ with $\gamma = B_o''/B_0$ (here we assume that $\Omega_v^2 > 0$). There are two terms resulting from the combined effect of the particle-wake and equilibrium charge gradient: a new hybrid frequency $\Omega_{Qq}^2 = \tilde{q} l \Omega_0^2/\Delta$, and a mode coupling coefficient

$$\Omega_{coupl}^4 = 4(\tilde{q} l \Phi(\kappa)\Omega_0^4/\Delta)\left[\tilde{q} l \Phi(\kappa)\Omega_0^2/\Delta - \varepsilon\Delta(1-\tilde{q})\Omega_0^2 - \beta\Delta\Omega_1^2\right]\sin^2(k\Delta) \qquad (6)$$

where $\Phi(\kappa) = 1 + \psi(\kappa)$.

Modifications of wave frequencies. In a 1D particle string without magnetic field and wake effects, the dispersion relation (3) describes two independent longitudinal and transverse DL modes. The characteristic wave frequencies $\Omega_{\parallel}$ and $\Omega_{\perp}$ reduce to the basic form:

$\Omega_{\|}^2 = 4\Omega_0^2\psi(\kappa)\sin^2(k\Delta/2)$ and $\Omega_{\perp}^2 = \Omega_v^2 - 4\Omega_0^2\sin^2(k\Delta/2)$. Inclusion of the particle magnetization modifies both wave frequencies increasing the values $\Omega_{\|}$ and $\Omega_{\perp}$ due to the magnetic dust-dust interactions ($\sim\Omega_1$), but the DL modes remain decoupled [3]. Note that for typical discharge conditions, a contribution of the magnetic terms in $\Omega_{\|}$ and $\Omega_{\perp}$ can be of the same order of magnitude or even larger than Ω_0, even in moderate magnetic fields ($B_0 \geq 0.1\,\mathrm{T}$). The ion -focusing effect leads to the coupling between the DL waves but decreases squared frequencies $\Omega_{\|}^2$ and $\Omega_{\perp}^2$ by a factor $(1-\tilde{q})$. In the case of a strong wake, $\tilde{q} \to 1$ this can significantly reduce (almost nullify) the contribution due to the Coulomb dust-dust interactions, so that the magnetic interactions will dominate. Finally, the combination of particle-wake effect, charge and magnetic field nonuniformity modifies the main frequency of the DL waves and a mode coupling coefficient Ω_{coupl}^4 according to Eqs. (4)-(6).

Stability analysis. The sign of Ω_{coupl}^4 determines the stability of the DL waves and a scenario of mode interactions. Typically $\left|\Omega_{coupl}^4\right| << 1$, and the mode interact mostly in the vicinity of the intersection point ($k = k_0, \omega = \omega_0$) of the two decoupled (initially stable) longitudinal and transverse waves [3]. When $\Omega_{coupl}^4 > 0$, a confluence of the transverse and longitudinal dispersion curves occurs in the long-wavelength ($k << k_0$) and short wavelength ranges ($k >> k_0$) separately, thus leading to the gap in the wavenumber domain and resultant instability of the coupled mode (Fig. 3 in Ref. [4]). The growth rate of this instability, $\mathrm{Im}\,\omega \approx -\gamma + \Omega_{coupl}^2\sin(k_0\Delta)/\omega_0$, indicates that this hybrid mode is unstable, when the neutral gas friction (γ) is sufficiently weak. In the opposite case Ω_{coupl}^4<0, the DL waves remain stable, but demonstrate a specific linear mode conversion: the long-wavelength transverse branch ($k << k_0$) joins with the short wavelength longitudinal DL wave ($k >> k_0$) and *vice versa* leading to a frequency gap in a vicinity of the intersection point (see Fig. 2 in Ref. [4]).

While the wake term between brackets in Eq.(6) is always positive, the combination of other two can have different signs dependently on the relative contributions of $\beta = (dm/dz)_0$ and $\varepsilon = (dQ/dz)_0$. It turns out that a strong wake ($\tilde{q} \to 1$) effect diminishes the contribution of the equilibrium charge gradient in Ω_{coupl}^4. The latter is then determined by the competition between the particle-wake and magnetic interactions. In a case of a weak wake effect, $\tilde{q} < 1$,

Ω_{coupl}^4 is mostly dependent on the gradients of the magnetic field and particle charge. Increase a magnetic field or making it more nonuniform apparently leads to the domination of magnetic interactions in the DL mode coupling process. This implies that the characteristics of DL mode coupling in the regime of sufficient magnetic field can be effectively controlled externally, thus opening new possibilities for studies of collective effects in strongly coupled systems.

CONCLUSIONS

The propagation of dust lattice modes in the 1D string (monolayer) formed by paramagnetic particles was studied with due care for the effects of spatial gradients of external magnetic field, equilibrium particle charge and formation of the ion wake. It is shown that the combination of these factors modifies the levitation condition and can also essentially affect the frequencies of DL waves. The main conclusion is that the DL modes exhibit a more complicated behavior than the classical DL in an nonmagnetized particle string without grain-wake interactions. In particular, while the particle magnetization can essentially increase the main frequencies of DL modes even in moderate magnetic fields $B_0 \geq 0.1$, the wake effect decreases these and can significantly reduce the contribution of Coulomb dust-dust interactions. Furthermore, the combination of three anisotropic factors not only modifies the wave frequencies of transverse and longitudinal modes, but also leads to mutual DL wave interactions. A scenario of the mode coupling is strongly dependent on relations between the gradient and wake terms: these can induce either instability if the coupling coefficient is positive or stable mode conversion in a case of a negative coupling term. In the regime of sufficient magnetic fields, the mode coupling can be effectively controlled externally, thus opening new possibilities for studies of collective effects in strongly coupled systems. The experimental investigations of the mode coupling in the medium of paramagnetic particles can also provide a tool for determining some plasma parameters. For example, observations of a certain threshold of the resonance instability for known magnetic field profile would permit determining an equilibrium charge gradient or estimating ion-wake parameters.

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