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# Three dimensional relativistic, fluid Weibel instability

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**Abstract**

The Weibel instability is an electromagnetic instability that can generate a quasi-static magnetic field in the wake of an ultra-intense, ultra-short laser pulse propagating in an underdense plasma. Recently, attention has been paid to this instability also in the overdense plasma regime where current filaments are observed in large scale PIC numerical simulations. Here we study the evolution of the Weibel instability in the 3D fluid, relativistic, collisionless limit in the case of two initially counterstreaming electron beams. The aim is to understand the typical magnetic structures that can be expected to form as a consequence of the development of the Weibel instability.

## Introduction

The penetration into an overdense plasma of electron beams driven by the interaction of a superintense laser pulse with a critical plasma surface has attracted much attention in the context of laser driven inertial fusion. Indeed, the energy transport in the overdense plasma by particle beams is one of the key issues of the fast ignition scheme.

The possibility of generating electron beams by an ultra-intense laser pulse impinging on a critical surface plasma layer has been demonstrated, in the two dimensional [1, 2, 3] as well as three dimensional case [4, 5, 6], by kinetic PIC numerical simulations where current filamented structures elongated in the laser pulse direction, starting from the critical surface layer, have been observed together with a corresponding magnetic field. From these results, assuming that such fast hot electron beams have been immediately neutralised by a colder, denser return current produced by the overdense plasma, other authors have studied the dynamics of the counterpropagating beams [7] in the 2D plane perpendicular to the beam direction and, for very long times, for a cylindrical beam [8]. Indeed such currents are unstable with respect to the Weibel instability [9, 10, 11] (in this case driven by electron momentum anisotropy) which is considered to be responsible for the magnetic structures observed in the overdense plasma simulations. However, a comparison between the typical spatial structures observed in the simulations and those produced by the development of the Weibel instability has not been performed in detail.

Motivated by the role attributed to the Weibel instability in the electron transport process, we discuss the characteristic time and length scales which characterize the Weibel-driven three-dimensional structures in the typical conditions of the 2D PIC simulations in order to understand whether the current and magnetic field structures observed in the PIC simulations of such an overdense plasma can be attributed to the development of the Weibel instability.

## Governing equations and initial conditions

We normalize all quantities by using a characteristic density $\bar{n}$, the speed of light $c$ and the electron plasma frequency $\bar{\omega}_p = (4\pi\bar{n}e^2/m_e)^{1/2}$. Therefore, the electron skin depth, $d_e = \bar{\omega}_p/c$, is equal to one. We also define the characteristic fields as $\bar{E}(\bar{B}) = m_e c\omega_{pe}/e$. Then, the relativistic two-fluids (cold) electron dimensionless equations read:

$$\frac{\partial n_a}{\partial t} = \nabla \cdot \mathbf{j}_a \, ; \quad \frac{\partial \mathbf{p}_a}{\partial t} + \mathbf{v}_a \cdot \nabla \mathbf{p}_a = -(\mathbf{E} + \mathbf{v}_a \times \mathbf{B}) \tag{1}$$

$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + \sum_a \mathbf{j}_a \, ; \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} ; \quad \nabla \cdot \mathbf{E} = 1 - \sum_a n_a \, ; \quad \nabla \cdot \mathbf{B} = 0 \tag{2}$$

$$\mathbf{B} = \nabla \times \mathbf{A} \, ; \quad \mathbf{v}_a = \frac{\mathbf{p}_a}{(1+p_a^2)^{1/2}}, \quad \mathbf{j}_a = -n_a \mathbf{v}_a, \quad a = 1, 2. \tag{3}$$

In this model, the ions are assumed as a fixed neutralizing background since the instability develops on the electron time scale. Eqs. (**??**)-(**??**) are integrated in the 3D numerical domain $-L_x/2 < x < L_x/2$, $-L_y/2 < y < L_y/2$, $-L_z/2 < z < L_z/2$. At the initial time two inhomogeneous electron beams are directed in opposite directions along the $z$-axis with densities and velocities given by:

$$n_{0,1} = \text{ const.}; \;\; n_{0,2} = \text{ const.}; \qquad \mathbf{v}_{0,1} = v_{0,1} f(x,y) \mathbf{e}_z \, ; \quad \mathbf{v}_{0,2} = -\mathbf{v}_{0,1} n_{0,1}/n_{0,2},$$

$$f(x,y) = 0.5 \, [\tanh\left((r - r_0)/R\right) - \tanh\left((r + r_0))/R\right)] \, ; \qquad r = \sqrt{x^2 + y^2}$$

where $n_{0,1} + n_{0,2} = 1$ (total electron density), $n_{0,1} v_{0,1} + n_{0,2} v_{0,2} = 0$ (total current). The beams have a cylindrical shape of width $2r_0$ with velocity gradient length scale $R$. We perturb the beams by adding, at $t = 0$, a white random noise on the potential vector $\mathbf{A}$ with typical amplitude $\epsilon$. Since the instability generates a magnetic field in the $(x, y)$ plane perpendicular to the initial beams, i.e. $B_z \ll B_x, B_y$, we limit our initial noise to the $z$ component of the potential vector:

$$A_z(x,y) = \sum_{k_x} \sum_{k_y} a^+_{k_x,k_y} cos[k_x x + k_y y + \phi(k_x, k_y, z)] + a^-_{k_x,k_y} cos[k_x x - k_y y + \phi(k_x, k_y, z)] \tag{4}$$

$$a^{\pm}_{k_x,k_y} = \epsilon/\sqrt{k_x^2 + k_y^2} \, ; \qquad \phi(k_x, k_y, z) \, \equiv \, \text{random phase}$$

In the relativistic regime, the electrostatic two-stream instability contribution is strongly depressed [12], so that we may assume $E_z \simeq A_z$, in agreement with the simulation results.

## Results

We present two relativistic, inhomogeneous 3D simulations with $R = 1$, $r_0 = \pi$, $\epsilon = 10^{-6}$ $L_x = L_y = 4\pi$, $L_z = 8\pi, N_x = N_y = 256$, $N_z = 128$. In the first one (asymmetric) a fast, less dense electron beam is compensated by a slow, denser beam representing the return current. In the second one (symmetric) the two electron beams have the same density (and thus equal and opposite velocity). In both cases, the beams have a finite transverse width of the order of a few $d_e$. The asymmetric beam simulation can be considered as a typical model of the PIC observed current filaments where the initial fast particle beam, penetrated from the critical surface into the overdense plasma, is immediately compensated by a return current generated by the plasma in order to maintain quasi-neutrality. The parameters of the asymmetric simulation are: $n_{0,1} = 0.1$, $n_{0,2} = 0.9$, $v_{0,1} = 0.95$, $v_{0,2} = -0.10556$. In the symmetric simulation the parameters are: $n_{0,1} = n_{0,2} = 0.5, v_{0,1} = 0.95, v_{0,2} = -0.95$.

Since the initial beams are inhomogeneous, as pointed out in Ref. [13] in the one dimensional limit, the Weibel instability develops a *spatial resonant-type singularity* around which the

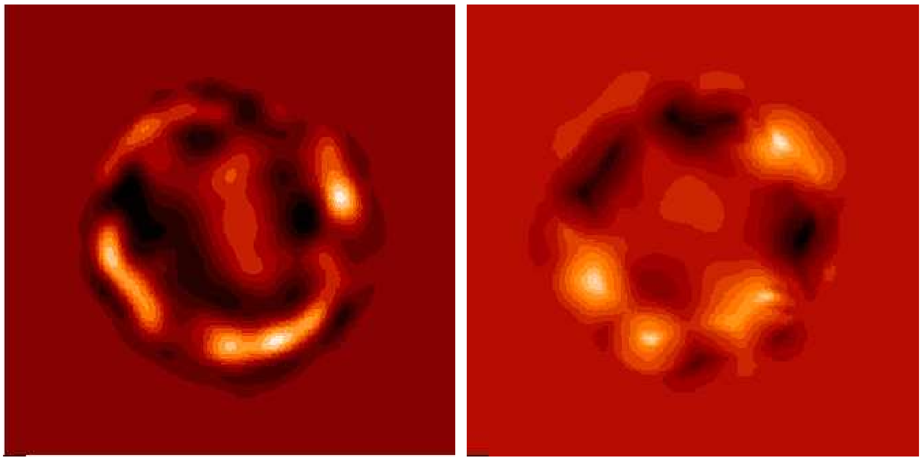

Figure 1: Shaded iso-contours of $E_z$ in the $(x, y)$ plane at $z = 0$, $t = 27$ for the asymmetric (left) and the symmetric (right) cases.

generated magnetic field becomes more and more concentrated. The singularity is located in the region of the beams velocity gradient and develops from any initial perturbation at a rate which depends on the value of the characteristic gradient of the electron velocities. The occurrence of a singularity which concentrates the magnetic field generated by the instability remains valid also in the cylindrical case both in the 2D (not shown here) and in the 3D case.

In Fig. 1 we draw the shaded iso-contours of $E_z$ in the plane $z = -L_z/2$ perpendicular to the beams direction. The asymmetric and symmetric case are shown in the left and in the right frames, respectively. We see that in both cases the Weibel generated magnetic field is concentrated in a ring shaped region corresponding to the region of the velocity gradient of the initial beams. A detailed study of the cylindrical resonance will be the object of a forthcoming paper.

In Fig. 2 we show the shaded isosurfaces of $E_z$. Positive (negative) values are represented by the red (blue) color, corresponding to a clockwise (anti clockwise) magnetic field. The asymmetric and symmetric cases are shown in the left and right frame, respectively. This figure shows very clearly that the resulting magnetic field is characterized, in the asymmetric case, by a bubble like shape with typical length scale of the order of a few $d_e$ both in the perpendicular and in the parallel directions (with respect to the beam direction), i.e. by $\ell_{||} \sim \ell_{\perp} \sim d_e$. On the other hand, in the symmetric case, the magnetic field is strongly anisotropic being elongated in the beams direction, i.e. $\ell_{||} \gg \ell_{\perp} \sim d_e$. Similar results (not presented here) have been obtained, aside for the localization of the magnetic field around the resonance for two initially homogeneous beams (i.e. $f(x, y) = 1$). The results discussed here are in agreement with the linear theory from which the maximum growth rate is obtained at $k_{||} \sim k_{\perp} \sim d_e$ in the asymmetric case and at $k_{||} \ll k_{\perp} \sim d_e$ in the symmetric case.

## Conclusions

In the PIC simulations, fast particles are observed to penetrate into the overdense plasma. It is generally believed that the corresponding currents generate filaments of magnetic field via the development of the Weibel instability. Such filamented magnetic structures are very elongated in the current direction with a typical length $L$ of the order of many electron skin depths ($L \sim 100 d_e$ or even longer). On the other hand, we have shown here that elongated magnetic structures are

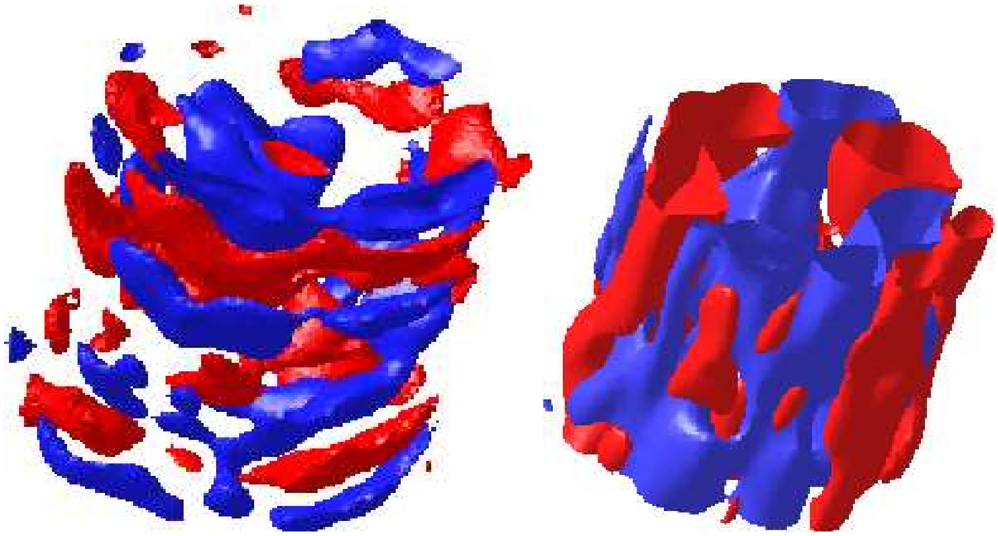

Figure 2: The shaded iso-surfaces of $E_z$ at $t = 27$ of the asymmetric and symmetric case, left and right frame, respectively. Red and blue colors refer to positive and negative values, respectively

generated by the Weibel instability only for nearly symmetric initial beams, while in the case of asymmetric beams (as expected in the laser plasma interaction context), the typical length scale in the beams direction is of the order of $d_e$. Finally, we have shown that in the case of spatially inhomogeneous beams the magnetic field is concentrated in the regions of large velocity gradients

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