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Nonlinear Tearing Mode Reconnection

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The resistive tearing mode instability in slab geometry is a function of two parameters only: the plasma resistivity η, and the instability parameter Δ', defined in (4). According to the analysis of Furth *et al.* [1], the tearing mode is unstable if $\Delta' > 0$. Traditionally, the tearing mode has been studied mainly in the low-Δ' regime. This is associated with the reasoning that earlier stability of the system (i.e., $\Delta' < 0$) and continuity of Δ' as a function of the plasma parameters must imply that Δ' has to go through zero if the system is to become unstable. However, significant arguments can be presented in favour of the relevance of large Δ' tearing modes. For example, it is known that kinetic effects can rise the instability criterium to $\Delta' > \Delta'_{crit}$ [2]. Also, current sheets that are formed in experiments such as MRX [3] have very large aspect ratios and thus large Δ'. Finally, recent work by Turri *et al.* [4] on the MAST tokamak has found Δ' to lie in the range 1000–6000. We have performed a thorough scan of the parameter space defined by these two parameters. We have found that the regime $W\Delta' \sim 1$ is significantly different from the classic, low Δ' description, which comprises only three different evolutionary stages: FKR, Rutherford and saturation. For large values of Δ', the tearing mode evolution can be divided into five main stages: linear exponential growth, nonlinear slow-down, nonlinear speed-up, instability of the current sheet and saturation – see Fig. 1.

We use the Reduced MHD equations [5]:

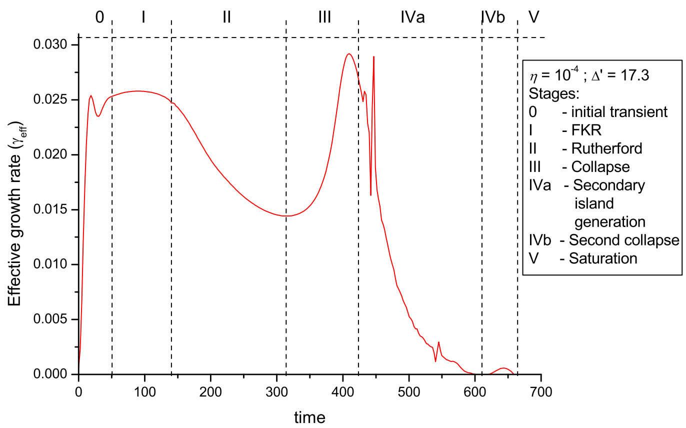

FIG. 1: Plot of the effective growth rate $\gamma = d/dt \ln \psi_1(X)$ vs. time for a strongly driven tearing mode showing five distinct evolution stages. $\psi_1(X)$ is the perturbed flux measured at the X-point.

$$\partial_t\omega + \boldsymbol{v}_\perp \cdot \boldsymbol{\nabla}\omega = \boldsymbol{B}_\perp \cdot \boldsymbol{\nabla} j_\parallel, \quad (1)$$

$$\partial_t\psi + \boldsymbol{v}_\perp \cdot \boldsymbol{\nabla}\psi = \eta\nabla^2\psi \quad (2)$$

These equations are solved in a two-dimensional periodic box $L_x \times L_y$ using a pseudo-spectral code at resolutions up to 3072×4096. The total magnetic field is $\boldsymbol{B} = B_z\boldsymbol{e}_z + \boldsymbol{B}_\perp$, the in-plane magnetic field is $\boldsymbol{B}_\perp = \boldsymbol{e}_z \times \boldsymbol{\nabla}\psi$, the in-plane velocity is $\boldsymbol{v}_\perp = \boldsymbol{e}_z \times \boldsymbol{\nabla}\phi$, $\omega = \boldsymbol{e}_z \cdot (\boldsymbol{\nabla} \times \boldsymbol{v}_\perp) = \nabla^2\phi$ and $j_\parallel = \boldsymbol{e}_z \cdot (\boldsymbol{\nabla} \times \boldsymbol{B}) = \nabla^2\psi$. We impose the equilibrium configuration

$$\psi^{(0)} = \psi_0/\cosh^2(x), \qquad \phi^{(0)} = 0. \tag{3}$$

We choose the units of field strength in such a way that the maximum value of the unperturbed in-plane magnetic field $B_y^{(0)} = d\psi^{(0)}/dx$ is $B_{y,\max}^{(0)} = 1$. This is accomplished by setting $\psi_0 = 3\sqrt{3}/4$. All lengths are scaled so that $L_x = 2\pi$. Time is scaled by the in-plane Alfvén time $L_x/2\pi B_{y,\max}^{(0)}$. To the equilibrium (3) we add an initial perturbation $\psi^{(1)} = \psi_1(x)\cos(ky)$, where $k = mL_x/L_y$ and m is an integer that we always set to $m = 1$. For the equilibrium (3), the instability parameter is [6]

$$\Delta' = \frac{\psi_1'(+0) - \psi_1'(-0)}{\psi_1(0)} = 2\sqrt{k^2+4}\,\frac{15 + 2k^2 - k^4}{k^2(k^2+4)}. \tag{4}$$

The equilibrium is tearing-unstable if $\Delta' > 0 \Leftrightarrow k < \sqrt{5}$. Δ' is varied by changing k, i.e., L_y.

Our work, details of which will be published elsewhere, presents, for the first time, a unified picture of the resistive tearing mode evolution for arbitrary values of the parameters (η, Δ'), showing under which conditions different evolutionary stages succeed each other. We have performed a thorough quantitative analysis of the onset criteria, the times-scales, and the geometric properties of each stage. In this paper we simply highlight some of our main results and conclusions. These are:

1. For intermediate (i.e. large but finite) values of Δ' such that the constant-ψ approximation is invalid, the linear growth rate of the tearing mode is given by:

$$\gamma = \tau_H^{-2/3}\tau_\eta^{-1/3} - 1.18\ \left(\Delta' L_\perp\right)^{-1}\tau_H^{-1} \tag{5}$$

This expression reduces to the $m = 1$ growth rate in the limit of $\Delta' \to \infty$ but provides a better approximation in the finite Δ' regime.

2. Rutherford's nonlinear analysis of the early nonlinear evolution of the tearing mode has been numerically validated for a range of values of the parameters (η, Δ') . A Rutherford nonlinear regime exits if the following condition is satisfied: $\ell_\eta < W \ll \min\{1/\Delta'\ , L_\perp\ , W_{\text{sat}}\}$. The first criterion on the RHS of this expression ensures both that the constant-ψ approximation is valid and that the collapse of the X-point does not happen while the mode is still in the linear regime. The second criterion simply states that the dissipation scale length is much smaller than the characteristic length of the equilibrium. The third condition ensures that between ℓ_η and the saturated width a window exists for the nonlinear evolution of the mode. Making use of the FKR result for ℓ_η this condition becomes equivalent to:

$$\eta \ll \min\{\frac{1}{\tau_H L_\perp \Delta'^3}\ ,\ \frac{L_\perp^{3/2}}{\tau_H \Delta'^{1/2}}\ ,\ \frac{W_{\text{sat}}^{5/2}}{\tau_H L_\perp \Delta'^{1/2}}\} \tag{6}$$

3. A collapse of the X-point occurs if $W_{crit} > 8.2/\Delta'$, in qualitative agreement with the theory of Waelbroeck [7] (see Fig. 2). Since, according to the theory of Escande & Ottaviani [8] and Militello & Porcelli [9], $W_{sat} \sim \Delta'$, this implies that a collapse of the X-point will always occur for $\Delta' > const$. For the parameters of our simulations, we

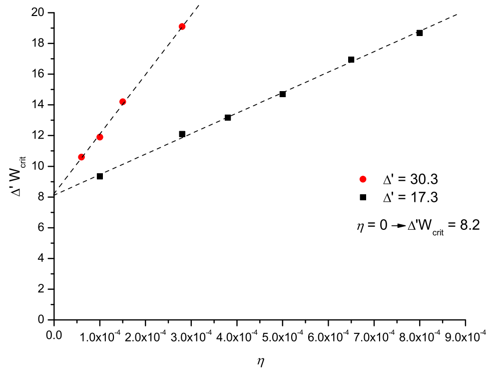

FIG. 2: The critical island width for collapse vs. η at fixed $\Delta' = 17.3,\ 30.1$. Dashed lines are linear fits. $W_{\rm crit}$ is measured at the time when $d\gamma/dt = 0$ in the nonlinear regime (i.e., at $t \approx 314$ in the run of Fig. 1.)

obtain $\Delta' > 6$ for collapse.

4. In simulations for which the above criterion is satisfied, a current sheet forms, giving rise to to Sweet-Parker exponential reconnection. During this stage, reconnection proceeds with a rate $\sim \eta^{1/2}$. The collapse thus causes a speed-up of the nonlinear growth rate, which, for low enough values of η, can exceed the linear rate.

5. The current sheet appears to be unstable to tearing modes if its aspect ratio $A > 50$. Based on the data available to us at this stage, the aspect ratio of the current sheet formed by the collapse of the X-point can be expressed as: $A \approx S^{1/2}\,[1 - const/(\Delta' L_\perp)]$. If, however, for given values of (η, Δ'), the aspect ratio exceeds ~ 50 such current sheets will be unstable.

6. The instability of the current sheet proceeds in the following way. First, a linear island forms whose O-point coincides with the X-point of the original mode. As this island develops and enters its nonlinear evolution, the new X-points can also collapse and give rise to current sheets. Although secondary islands have been reported in numerical work by other authors (e.g. [10, 11]) the details of their evolution had so far not been clarified. This finding gives support to the linear stability calculation of Bulanov *et al.* [12].

7. The secondary island grows until the attraction forces between the secondary and the main islands become strong enough to initiate the coalescence process. Due to the intrinsic symmetry of our configuration, the secondary island is forced to split, with each halve moving in opposite ways and being absorbed by the main island.

8. The saturation amplitude is well described by the the recent theories of Escande & Ottaviani and Militello & Porcelli for low values of Δ'. The theory of White *et al.* [13] provides a worse approximation in this regime. For large enough values of Δ' such that a collapse occurs, a jump occurs in the saturation amplitude. It thus seems that X-point like saturation stages are defined by $\psi_{sat} \sim \Delta'$, whereas current sheet like saturation stages occur at constant amplitudes which, for sufficiently low values of the resistivity, depend only on the size of the simulation domain and initial configuration.

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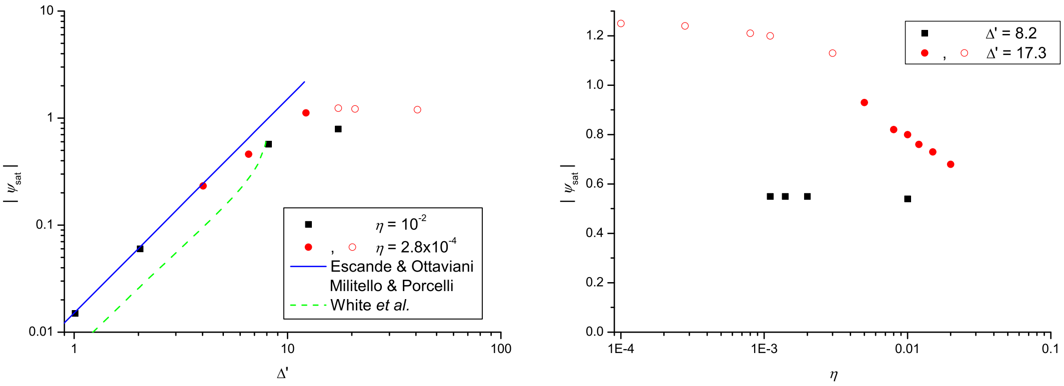

FIG. 3: Left: Saturation amplitude vs. Δ' for different values of η. Dark blue line represents the solution of refs. [8, 9], the light blue line the prediction of the model of White *et al.* [13]. Right: Saturation amplitudes vs. η for $\Delta' = 8.2, 17.3$. In both figures, red empty circles denote runs for which the final island width is larger than the simulation box.

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