Structure of stochastic field lines near the separatrix in poloidal divertor tokamaks

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Poloidal divertor tokamaks are an important concept of the magnetic confinement of plasma. The magnetic configuration of these tokamaks contains a magnetic surface (a *magnetic separatrix*) sharply separating closed field lines on nested magnetic surfaces from open field lines hitting the walls of the fusion device. Any non-axisymmetric magnetic perturbations destroy the magnetic separatrix and create a zone of chaotic field lines near the separatrix, i.e., a stochastic layer. The structure of the stochastic layer mainly determines the spatial structure of plasma boundary and deposition patterns of heat and particles on divertor plates [1]. The nature of magnetic perturbations in plasmas may have a different origin: it may range from the magnetic fluctuations produced by plasma instabilities, field errors to external magnetic fields created by special coils [2]. The study of the stochastic layer near the separatrix becomes also important in the view of recent experiments in the DIII-D tokamak on the suppression of large edge-localized modes in high confinement (H-mode) plasmas affected by applied edge resonant magnetic perturbation [3].

In this presentation we describe two mapping methods to study the structure of stochastic magnetic field lines at the plasma edge formed by external magnetic perturbations in poloidal divertor tokamaks. Both methods are based on the Hamiltonian formulation of field line equations in a toroidal geometry. In the first mapping method one uses the Hamiltonian equations of field lines in Boozer coordinates, the toroidal flux, ψ, the toroidal flux H, and the poloidal and toroidal angles, ϑ and φ, respectively:

$$d\vartheta/d\varphi = \partial H/\partial \psi, \qquad d\psi/d\varphi = -\partial H/\partial \vartheta. \tag{1}$$

In these coordinates the magnetic field $\mathbf{B}$ can be written in the Clebsch form, $\mathbf{B} = \nabla \psi \times \nabla \vartheta + \nabla \varphi \times H$. The Hamiltonian can be presented as a sum, $H(\psi, \vartheta, \varphi) = H_0(\psi) + \varepsilon H_1(\psi, \vartheta, \varphi)$, where $H_0(\psi) = \int q^{-1}(\psi) d\psi$ is the unperturbed Hamiltonian, $q(\psi)$ is the safety factor determined by the equilibrium plasma, $\varepsilon H_1(\psi, \vartheta, \varphi)$ corresponds the perturbation, which can be expanded in Fourier series in poloidal and toroidal angles, ϑ and φ:

$$H_1(\psi, \vartheta, \varphi) = \sum_{mn} H_{mn}(\psi) \cos(m\vartheta - n\varphi + \chi_{mn}). \tag{2}$$

The Hamiltonian equations (1) are integrated using the symplectic mapping method developed in [4]. The map, defined as

$$(\vartheta_{k+1}, \psi_{k+1}) = \hat{M}(\vartheta_k, \psi_k), \tag{3}$$

relates the variables (ϑ_k, ψ_k) at the poloidal section $\varphi = \varphi_k = 2\pi k/N, (k = 0, \pm 1, \pm 2, \ldots; N \geq 1$ with the ones $(\vartheta_{k+1}, \psi_{k+1})$ at φ_{k+1}. This method has been described in [4] and it has been extensively applied to study field line structures in the TEXTOR-DED [5].

The second method is based on the mapping of the toroidal angle, φ, and the relative poloidal flux, $h = H - H_s$, where H_s is the poloidal flux on the separatrix. It is defined as

$$(\varphi_{k+1}, h_{k+1}) = \hat{M}(\varphi_k, h_k), \tag{4}$$

where (φ_k, h_k) are values of φ and h at the $k-$th crossing point of field line with the section Σ_s. This section consists of two stripes (segments in the poloidal plane) along the ξ and η axes which transversely cross each other along the X-line. The mapping scheme, as well as, the section Σ_s and the axes ξ and η are shown in Fig. 1. The general form of the mapping (4) is given in [6]. We use its simplified form

$$h_{k+1} = h_k \mp \varepsilon F^{(\pm)}(\varphi_k, h_{k+1}, h_k),$$
$$\varphi_{k+1} = \varphi_k \pm \pi [q(h_k) + q(h_{k+1})] \pm \varepsilon G^{(\pm)}(\varphi_k, h_{k+1}, h_k), \tag{5}$$

where

$$F^{(\pm)}(\varphi_k, h_{k+1}, h_k) = \mathrm{Im} \sum_n n R_n(h_{k+1}) \exp(in[\varphi_k \pm \pi q(h_k)] + \chi_n),$$
$$G^{(\pm)}(\varphi_k, h_{k+1}, h_k) = -\mathrm{Re} \sum_n \frac{dR_n(h_{k+1})}{dh_{k+1}} \exp(in[\varphi_k \pm \pi q(h_k)] + \chi_n). \tag{6}$$

Here $q(h)$ is the safety factor as a function of the relative poloidal flux h. The coefficients $R_n(h) = K_n(h) + iL_n(h)$ are the Melnikov type integrals

$$R_n(h) = \int_{-\pi q(h)}^{\pi q(h)} V_n(h, \tau) e^{in\tau} d\tau, \tag{7}$$

taken over the functions $V_n^{(j)}(h, \tau) \equiv H_n(z^{(j)}(h, \varphi - \varphi_0), p_z^{(j)}(h, \varphi - \varphi_0))$ along the unperturbed field lines near the $j-$th separatrix. The integrals $R_n(h)$ can be presented as a sum of regular, $R_n^{(reg)}(h)$, and oscillatory, $R_n^{(osc)}(h)$, parts, i.e., $R_n(h) = R_n^{(reg)}(h) + R_n^{(osc)}(h)$.

The regular part, $R_n^{(reg)}(h)$, is a smooth function of the relative poloidal flux h. The oscillatory part, $R_n^{(osc)}(h)$, is a rapidly oscillating function of h, with a local period of oscillations. The zeros of $R_n^{(reg)}(h)$, i.e., $R_n^{(reg)}(h_{mn}) = 0$, coincide with the resonant poloidal fluxes of primary

resonances, $q(h_{mn}) = m/n$. Since field lines are mostly affected near the primary resonances where oscillatory terms of the integrals $R_n(h)$ vanish, we can retain only the smooth regular parts $R_n^{(reg)}(h)$.

To illustrate the mapping methods we have chosen a simplified model of the plasma: the equilibrium plasma is modeled by three–current loops. The parameters of the model are chosen to have the plasma shape close to the JET plasma configuration: the major radius of the magnetic axis is $R_0 = 3.1$ m and the toroidal field $B_0 = 2$ T, the plasma current $I_p = 2$ MA. The external magnetic perturbations are created by N pair of loop coils with opposite flowing currents and located along the equatorial plane at the major radius $R_c = 5.2$ m.
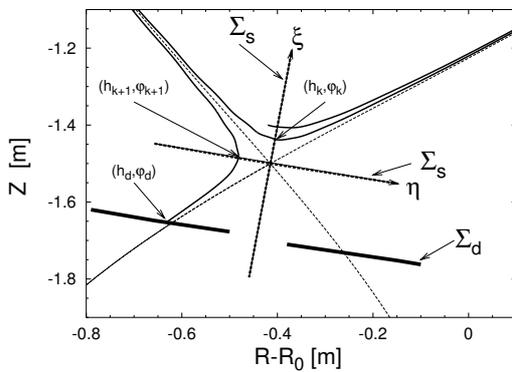

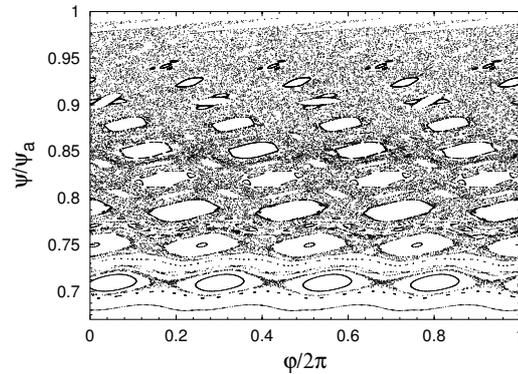Figure 1: Scheme of mapping of field lines to the section Σ_s and to the divertor plates.

Figure 2: Poincaré section of field lines in the toroidal plane (φ, ξ) (see Fig. 1).

Poincaré section of field lines in the toroidal plane (φ, ξ) obtained by the mapping (3) is shown in Fig. 2 for the perturbation current $I_c = 100$ kA and the toroidal mode $n = 4$. (The coordinate ξ is replaced by the normalized toroidal flux ψ/ψ_a, where ψ_a is the toroidal flux at the separatrix). As seen from Fig. 2 the stochastic layer formed near the separatrix covers sufficiently large area which is about 27% of the total toroidal flux. It consists of chains of overlapping islands and is not fully chaotic. The sizes of these islands decrease with approaching the separatrix. At the area very close to the separatrix there are four stripes along with field lines leaving the plasma region. The direction of stripes in Poincaré sections is determined by the iteration direction of the mapping. Fig. 2 describes when field lines are iterated in the positive direction of φ. The direction of stripes changes to the opposite if field lines are run in the negative direction of φ.

The fine structure of the stochastic layer in the area close to the separatrix and the stripes can be revealed by the so–called *laminar plot*. The latter is the contour plot of a number N_p poloidal turns for field lines which connect the left and right divertor plates. The separatrix
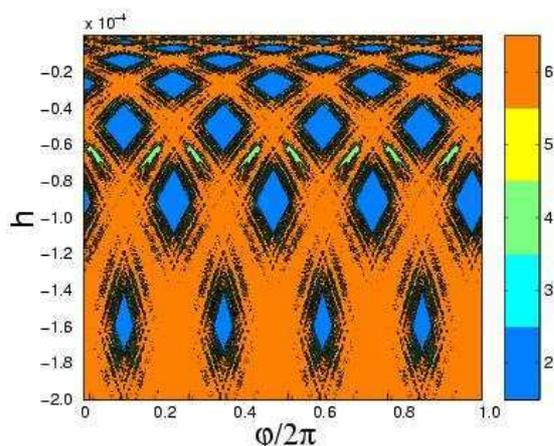
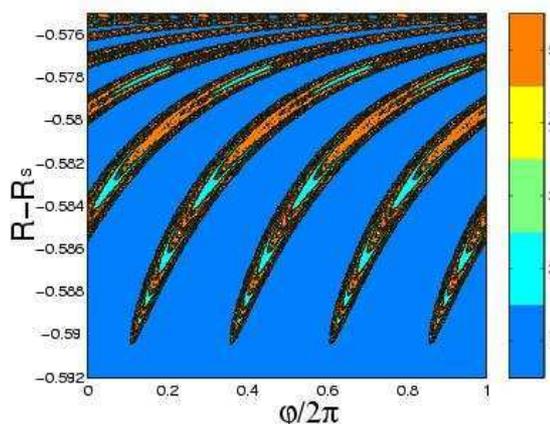Figure 3: Laminar plot of field lines in the (φ, ξ) plane ($h < 0$) of the section Σ_s.

Figure 4: Magnetic footprint structure on the divertor plate in the (φ, R) plane.

mapping (5) is a computationally effective tool for calculating N_p. One step of the separatrix map corresponds to one poloidal turn. It runs almost three orders of magnitude faster than the iterative mapping (3). The laminar plot of field lines obtained for the perturbation current $I_c = 100$ kA is shown in Fig. 3.

The separatrix mapping is also applied to study the magnetic footprints on the divertor plates. The latter determine heat and particle deposition patterns on the divertor target plates since electrons and ions predominantly follow field lines. The magnetic footprint on the left divertor plate corresponding to the stochastic layer in Figs. 2, 3 is shown in Fig. 4. It reveals the fine structure of footprints showing areas with different connection lengths measured in a number of poloidal turns N_p. The blue area with $N_p = 1$ corresponds to field lines which do not enter the plasma confined region. The spiral-like areas with $N_p \geq 2$ correspond to field lines entering the plasma region, and therefore electrons and particles following these field lines are predominantly deposited along these spiral-like structures.

References

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