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Clustering and pinch of impurities in plasma edge turbulence

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Introduction The knowledge of the mechanisms that control the impurity content of fusion plasmas is of crucial importance for the success of future fusion reactors. Recently, progress has been made towards understanding the turbulent transport of charged particles in magnetized plasmas, which in many cases is believed to be responsible for the anomalously high levels of transport observed in experimental devices [1, 2].

In this contribution we investigate the turbulent transport of impurities in a generic model for plasma edge turbulence, focusing on the compressible features that result from impurity-particle inertia. We find that the impurity density correlates with the vorticity of the electric drift, and we also discover a radial pinch that scales linearly with the mass–charge ratio of the impurities [3].

Model equations As a basic model for the ambient plasma turbulence, we consider the two-dimensional Hasegawa–Wakatani (HW) model for resistive drift-wave turbulence at the plasma edge [4], which under the gyro-Bohm normalization may be written in the form

$$\mathrm{D}_t(n-x) = C(\varphi - n) + \mu_n \nabla_\perp^2 n, \tag{1a}$$

$$\mathrm{D}_t \omega = C(\varphi - n) + \mu_\omega \nabla_\perp^2 \omega, \tag{1b}$$

with $\omega \equiv \nabla_\perp^2 \varphi$ and $\mathrm{D}_t \equiv \partial_t + \hat{\mathbf{z}} \times \nabla_\perp \varphi \cdot \nabla_\perp$. Here, n denotes the fluctuating component of the plasma density, φ is the electrostatic potential, and ω is the vorticity of the electric drift velocity $\mathbf{v}_E = \hat{\mathbf{z}} \times \nabla_\perp \varphi$. The adiabaticity parameter C determines the character of the turbulence, ranging from adiabatic ($C \to \infty$) to hydrodynamic ($C \to 0$). The remaining parameters, μ_n and μ_ω, correspond respectively to the collisional diffusivity of electrons and the ion viscosity.

We regard the impurities as a passive species, that is, a species advected by the turbulence but not having any influence on it. Though only valid when the density of impurities is much lower than that of the bulk plasma, this approximation is suitable for revealing generic features of the turbulent transport. Moreover, we treat the impurities as a cold fluid, thus characterized by a particle density θ and a flow velocity $\mathbf{v}_\theta$. In our model, we consider this velocity to be the sum of the electric and polarization drifts. Therefore, under the same normalization applied to the turbulence model, the flow velocity of the impurities is given by

$$\mathbf{v}_\theta = \mathbf{v}_E + \mathbf{v}_p = \hat{\mathbf{z}} \times \nabla_\perp \varphi - \zeta \mathrm{D}_t \nabla_\perp \varphi, \tag{2}$$

where ζ is a small parameter proportional to the mass–charge ratio of the impurity particles. Hence, the impurity model that results from the continuity equation reads

$$\mathrm{D}_t\theta - \nabla_\perp\cdot\left(\theta\zeta\mathrm{D}_t\nabla_\perp\varphi\right) = \mu_\theta\nabla_\perp^2\theta, \tag{3}$$

where the term on the right-hand side arises from collisional diffusion.

The polarization drift, which represents the effect of impurity-particle inertia, constitutes a higher-order correction to the electric drift velocity. Nevertheless, we shall see that the polarization drift adds important qualitative features to the dynamics of impurities, giving rise to subtle compression effects.

Turbulent mixing and clustering The importance of inertia in the advection of particles by turbulent flows is well known in fluid dynamics, where it has for instance been shown to result in clustering in vortical structures [5]. In this section we investigate if corresponding clustering effects appear for impurities in magnetized plasmas. In order to most clearly reveal such a compression effect, we consider a homogeneous initial distribution of impurities, since the latter would remain unchanged in the incompressible case.

Simulations of the HW turbulence model (1) and the impurity model (3) were performed on a doubly periodic square domain of side length 40 using the semi-Lagrangian pseudospectral code described in [3]. The adiabaticity parameter C in the HW model was set to the transitional value 1, while the viscosity μ_ω and the diffusivities μ_n and μ_θ were given the value 0.02. The impurity density field θ was initialized with the constant value $\theta_0 = 1$, and the initial turbulent fields were taken from a HW saturated turbulent state. Simulations were carried out for different values of the impurity mass–charge ratio parameter ζ ranging from -0.01 to 0.05.

In Fig. 1 we present snapshots of the vorticity ω of the electric drift velocity and the impurity density field at $t = 50$ for the case $\zeta = 0.01$. The scarce visual difference between the two fields

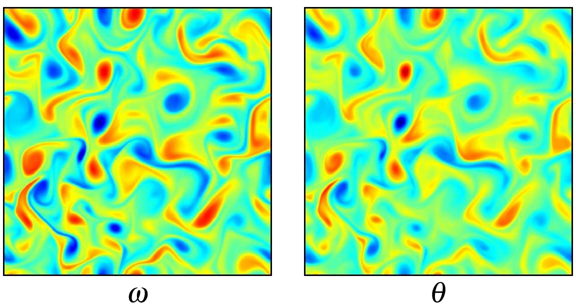

Figure 1: Vorticity ω and density θ of impurities 50 time units after the simulation is started with a uniform distribution of impurities with $\zeta = 0.01$.

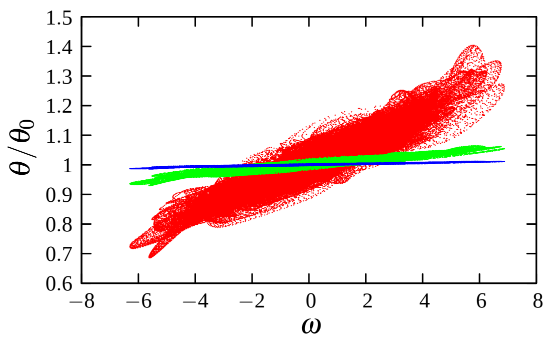

Figure 2: Scatter plot of vorticity ω and relative density θ/θ_0 of impurities at $t = 100$ for $\zeta = 0.05$ (red), $\zeta = 0.01$ (green), and $\zeta = 0.002$ (blue).

indicates the presence of a strong correlation between the vorticity and the impurity density. This correlation is even more apparent in Fig. 2, which shows a scatter plot of vorticity and relative impurity density θ/θ_0 at $t = 100$ for three different values of ζ. The plot clearly suggests an approximate linear relation of the form $\theta/\theta_0 \approx 1 + K\omega$, with the regression coefficient K depending on ζ. Actually, further analysis reveals that K is in turn nearly proportional to ζ.

This behavior can easily be explained using an argument of turbulent mixing, or turbulent equipartition [6]. Performing basic algebraic manipulations, we may rewrite the impurity model in the form

$$\mathrm{D}_t(\ln\theta - \zeta\omega) = \zeta\nabla_\perp \ln\theta \cdot \mathrm{D}_t\nabla_\perp\varphi + \frac{\mu_\theta}{\theta}\nabla_\perp^2\theta. \tag{4}$$

Assuming that the normalizations are adequate and that the relative fluctuations of impurity density are of order ζ, the right-hand side of this last equation is of order $\zeta^2 + \zeta\mu_\theta$, whereas the fluctuating component of the quantity inside the derivative on left-hand side is of order ζ. Hence, $\ln\theta - \zeta\omega$ is an approximate Lagrangian invariant, and thus will be homogenized by the turbulence. The smallness of the relative density fluctuations and mass conservation then yield

$$\theta/\theta_0 \approx 1 + \zeta\omega. \tag{5}$$

Hence, the present analysis fully explains the behavior observed in our simulations, showing that, as a consequence of their inertia, impurity particles of positive charge tend to cluster in positive vortices, the opposite taking place for negatively charged impurities.

Net transport and pinch We lastly investigate the appearance of impurity pinch velocities as a result of the compressibility introduced by the polarization drift. As a measure of the net radial transport of impurities, we consider the global radial impurity flux due to the electric drift,

$$\Gamma_\theta \equiv \int \theta\mathbf{v}_E \cdot \hat{\mathbf{x}}\,\mathrm{d}\mathbf{x} = -\int \theta\partial_y\varphi\,\mathrm{d}\mathbf{x}. \tag{6}$$

We also define an associated overall radial drift velocity $V_\theta \equiv \Gamma_\theta / \int \theta\,\mathrm{d}\mathbf{x}$, and begin by monitoring this quantity in the preceding simulations.

In Fig. 3 we present the evolution of the radial drift velocity for four different values of ζ. In every case, the evolution of the radial drift comprises a strong transient burst, after which the drift enters a saturated quasistationary regime. Moreover, the radial drift velocity is seen to have a definite sign opposite to that of ζ, that is, opposite to the charge of the impurity particles. It follows that, within our model, positively charged impurities are subject to a continuous radially inward drift. In Fig. 4 we show the time-averaged radial drift velocities $\overline{V_\theta}$ in the saturated regimes, computed using the values between $t = 25$ and $t = 150$. The variation of the time-

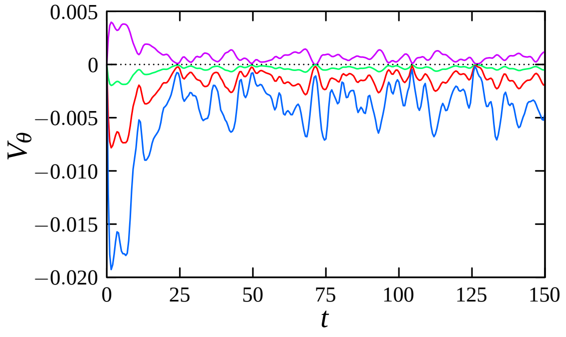

Figure 3: Evolution of the radial drift velocity V_θ of impurities for $\zeta = -0.01$ (purple), $\zeta = 0.005$ (green), $\zeta = 0.02$ (red), and $\zeta = 0.05$ (blue).

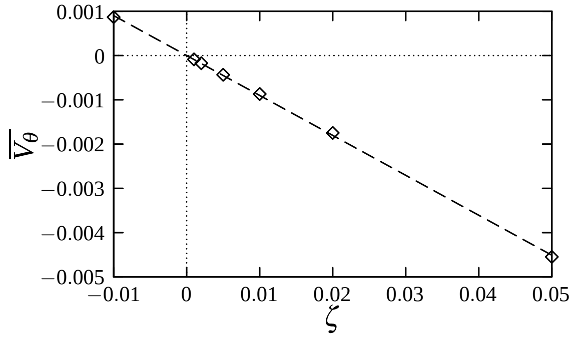

Figure 4: Time-averaged radial drift velocity $\overline{V_\theta}$ of impurities as a function of ζ. The fitted line corresponds to $\overline{V_\theta} = -0.090\zeta$.

averaged radial drift with ζ is seen to be remarkably linear, meaning that the present drift scales linearly with the mass–charge ratio of the impurity particles.

The reason for this peculiar pinch effect is not initially apparent. With the aim of understanding its onset, we analyze the evolution equation for the radial impurity flux. Within a periodic domain, the evolution of Γ_θ under the impurity model (3) is governed by

$$\frac{\mathrm{d}\Gamma_\theta}{\mathrm{d}t} = -\int \theta \mathrm{D}_t(\partial_y \varphi)\,\mathrm{d}\mathbf{x} - \int \zeta\theta(\partial_y\varphi)\mathrm{D}_t\omega\,\mathrm{d}\mathbf{x} + O(\zeta^2+\zeta\mu_\theta) \equiv \Lambda_1+\Lambda_2+O(\zeta^2+\zeta\mu_\theta). \quad (7)$$

In the case of uniform impurity density, Λ_2 is the only nonvanishing contribution to $\mathrm{d}\Gamma_\theta/\mathrm{d}t$, being therefore responsible for the initial development of the radial drift. In fact, using the HW vorticity equation, it can be shown that Λ_2 is approximately given by $-\zeta C\theta_0\Gamma_n$, where Γ_n is the sole source of energy in the HW model. Since Γ_n is in practice positive definite, it follows that Λ_2 is indeed a source for a net radial drift opposite to the type of charge of the impurities. As for the saturation of the drift, the eventual relation $\theta/\theta_0 \approx 1+\zeta\omega$ predicted in the previous section yields an approximate cancellation of $\Lambda_1+\Lambda_2$, which leads to the quasistationary regime of the radial pinch. Hence, the present argument identifies the mechanism for the onset of the radial pinch and likewise explains its dependence on the mass–charge ratio of the impurity particles.

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