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A Toroidal Shell Model for Active Stabilization of Resistive Wall Modes and Its Application to KSTAR Plasmas

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Resistive wall modes (RWMs) are pressure driven external kink modes the growth times of which are converted into a nearby resistive wall time scale. In the presence of a conducting wall that is sufficiently close to the plasma, the ideal kink modes are suppressed while the RWMs prevail. Thus, the suppression of the remaining RWMs becomse a fundamental prerequisite to increase the plasma beta in next step devices such as ITER or KSTAR, which are envisioned to be operated in advanced tokamak (AT) modes. In the present work, we study the active feedback stabilization of the RWM by the exploitation of a sysmte of external feedback coils in a toroidal geometry. Our main purpose is to apply the formulation to evaluate the basic design requirements for the proposed KSTAR in-vessel control coil (IVCC) system. A similar work has been reported using a cylindrical geometry[1]. By the incorporation of the toroidal geometry, one can evaluate the beta limits in the presence of a feedback system, which was impossible in the cylindrical theory.

We consider a toroidal system consisting of a circular toroidal plasma with minor radius $r=a$, a resistive wall at $r=b$ (either poloidally partial or complete), and a system of discrete feedback coils at $r=f$. The wall and feedback coils are assumed to be concentric to the plasma. In the formulation, currents in the resistive wall and feedback coils are modeled by surface current distributions at $r=b$ and $r=f$, respectively. As a plasma model, we adopt a sharp boundary surface model, in which a plasma current is allowed to flow only at plasma surface, and the plasma pressure is constant inside the plasma boundary[2]. An elaborate surface model has been developed in Ref. 3 to study the effects of plasma rotation and dissipation on the RWM[3].

The pressure balance relation across the plasma-vacuum boundary,

$$(B_\theta / B_0)^2 = \beta_T + [1-(B_i / B_0)^2] / h_a^2, \qquad (1)$$

determines the equilibrium vacuum poloidal magnetic field. Here, B_θ is the vacuum poloidal magnetic field, B_0 is the toroidal magnetic field just outside the plasma at $\theta=\pi/2$, $\beta=2\mu_0 P/B_0^2$, P is the plasma pressure, B_i is the toroidal magnetic field just inside the plasma boundary, and $h_a=1+(a/R_0)\cos\theta$. The expansion of Eq. (1) by employing the high-beta assumption ($\beta/\varepsilon\sim 1$) determines the plasma equilibrium characterized by β/ε and averaged vacuum safety factor ($<q_a>$).

We represent the perturbed poloidal magnetic field using the poloidal flux,

$$\tilde{\vec{B}} = \vec{\nabla}\Psi \times \hat{z}.$$

In leading order, Ψ satisfies the Laplace equation both in the plasma region and vacuum regions, giving rise to the solutions,

$$\Psi_{p,1,2,3}(\hat{r},\theta,\phi,t) = \sum_{m,n} \{C_{p,1+,2+}\hat{r}^{|m|} + C_{1\text{-},2\text{-},3\text{-}}\hat{r}^{-|m|}\} \exp[\gamma t + i(m\theta - n\phi)] \qquad (2)$$

where γ denotes the growth rate, $m(n)$ is the poloidal(toroidal) mode number, the regions p, 1, 2, and 3 correspond to radial ranges, $0\leq r\leq a$, $a\leq r\leq b$, $b\leq r\leq f$, and $r\geq f$, respectively. The perturbed plasma displacement at the plasma boundary is given,

$$\xi(\theta,\phi,t) = \sum_{m,n} \xi_{mn} \exp[\gamma t + i(m\theta - n\phi)].$$

Determination of six unknown coefficients in Eq. (2) and the construction of an eigenvalue equation for γ require seven boundary conditions at $r=a$, b, and f. Also, a feedback law is to be set up determine the required surface current distribution at $r=f$. The six boundary conditions are as follows:

(i) two boundary conditions at $r=a$,

$$C^m_{\mathrm{p}+} = -n\varepsilon_a \xi^m,$$

$$C^m_{1+} + C^m_{1-} = \varepsilon_a \sum_k \xi^k \{\frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta \exp[i(k-m)\theta][\frac{k}{q_a(\theta)} - i\frac{\partial q_a^{-1}(\theta)}{\partial\theta} - n]\}/m = \varepsilon_a \sum_k V^m_k \xi^k \qquad (3)$$

(ii) two B_r field continuity relations at $r=b$ and $r=f$,

$$\hat{b}^{|m|}C^m_{2+} + \hat{b}^{-|m|}C^m_{2\text{-}} = \Psi^m_b = \hat{b}^{|m|}C^m_{1+} + \hat{b}^{-|m|}C^m_{1\text{-}}, \qquad C^m_{3\text{-}} - (C^m_{2\text{-}} + \hat{f}^{2|m|}C^m_{2+}) = 0, \qquad (4)$$

and (iii) tangential magnetic field discontinuity relations at $r=b$ and and $r=f$,

$$b\left[\partial\Psi^m/\partial r\right]_{b-}^{b+} = \gamma\tau_w \Psi^m_b \qquad (5)$$

$$f\left[\partial\Psi^m/\partial r\right]_{f+}^{f-} = \mu_0 f K^m_\varphi = (1/|m|n\pi^2)\sum_{k,j=1}^{N}\sum_l Q^m_k g^k_j S^j_l \Psi^l_b. \qquad (6)$$

The matrices appearing in Eq. (6) represent the effects of feedback currents. They depend on the choice of the feedback algorithm employed. In the present work, we computed them by using the smart coil feedback law. Then, the matrix g^k_j is the proportional gain, and Q^m_k and S^j_l represent the sensor algorithm being employed. After a lengthy algebra, it can be shown that

$$Q_k^m = \left\{ \sin(m\Delta\theta_k/2)\sin(n\Delta\varphi_k/2)\exp[i(n\varphi_k - m\theta_k)] \right\}$$

$$S_l^j = (\exp(il\theta_j - in\varphi_j)/n)\sin(n\Delta\varphi_j/2)\{2\sin(m\Delta\theta_j/2) + (b/R_0)$$
$$\times[\exp(i\theta_j)(l/l+1)\sin((l+1)\Delta\theta_j/2) + \exp(-i\theta_j)(l/l-1)\sin((l-1)\Delta\theta_j/2)]\}.$$

All the coefficients in Eq. (2) can then be described as ξ by using Eqs. (3) to (6). The dispersion relation for γ is obtained from the pressure balance relation for an incompressible plasma at the plasma-vacumm interface,

$$[\vec{B}\cdot\vec{B}_1]_{a\text{-}\varepsilon}^{a+\varepsilon} = \text{-}[\xi\hat{r}\cdot\vec{\nabla}(\frac{B^2}{2})]_{a\text{-}\varepsilon}^{a+\varepsilon}. \qquad (6)$$

After a considerable amount of algebra, one can reduce Eq. (6) to the following matrix equation,

$$\left[L_m^j W_k^m \left((A^{-1})_\alpha^k B_l^\alpha - \delta_l^k \right) - J_l^j \right] \xi^l = Z_l^j \xi^l \qquad (7)$$

where the matrices are defined as,

$$L_m^j = \frac{|m|}{2\pi}\int_{-\pi}^{\pi} d\theta \exp[i(m-j)\theta](1/q_a(\theta)),\quad A_l^m = (\hat{b}^{|m|}(1+\gamma\tau_w/2|m|)\delta_l^m - \left(\frac{\hat{b}}{\hat{f}}\right)^{|m|}\frac{\hat{b}^{|l|}}{2|m|n\pi^2}D_l^m,$$

$$B_j^l = -(\hat{b}^{|l|}\gamma\tau_w/2|l|)\delta_j^l + \left(\frac{\hat{b}}{\hat{f}}\right)^{|l|}\frac{\hat{b}^{-|j|}}{2|l|n\pi^2}D_j^l,\quad W_k^m = (\delta_j^m + (A^{-1})_l^m B_j^l)^{-1}{}_k^m V_j^k,$$

$$D_j^l = \sum_{k,j=1}^{N} Q_k^m g_j^k S_l^j,\quad L_m^j = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta \exp[i(m-j)\theta](1/q_a^2(\theta) + (\beta/\varepsilon_a)\cos\theta).$$

Equation (7) is the desired form of the eigenvalue equation for γ with respect to the plasma displacement. The solution of det(Z)=0 represent the growth rates of RWMs modified by a set of feedback coil currents.

Figure 1 shows the critical β/ε for n=1 and n=2 as a function of $<q_a>$. A complete resistive wall at b=1.2a is assumed. The threshold β/ε (~ 0.09) is smaller

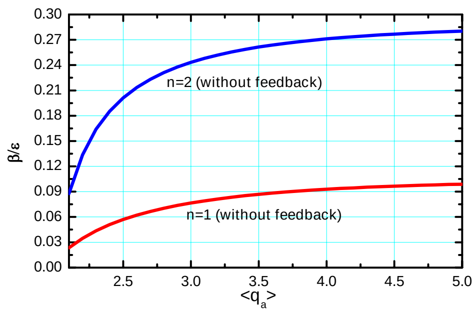

Fig 1. Critical β vs. q_a for a complete wall.

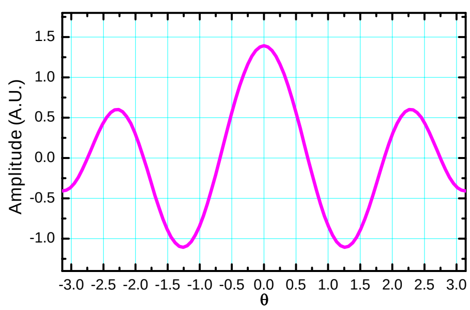

Fig.2. An unstable eigenfunction.

than that of the ideal kink mode (~ 0.21)[2]. The threshold β/ε does not change as the wall position varies, and the unstable modes grow in the resistive wall time scale. Thus, they are identified as RWMs. Figure 2 represents a poloidal structure for an unstable eigenmode. The ballooning nature of the unstable eigenmode with the dominant m=2 harmonic component is evident in Fig. 2.

After these benchmark calculations, we applied the formulation to the proposed KSTAR plasmas employing in-vessel control coil system (IVCC) for the stabilization of RWMs[4]. Only four middle coils are used in this study although 12 control coils are available in its full operation. The stability boundaries for n=1 and n=2 modes are the same as in Fig. 1, with higher growth rates than those for the complete wall case. Figure 3 shows the critical β/ε for three different values of assumed radial feedback coil locations (f=0.65, 0.7, 0.8). As f is increased, the efficacy of the feedback system also increases due to the increment of the area that is affected by the feedback currents. It was also found that there is a limitation of achievable β/ε regardless of gain values. The critical β/ε is approximately 0.11 (g_p ~ 300) when $<q_a>$=3.5 (Fig. 3). Thus, we expect ~ 40 % increase of β/ε at the expense of ~ 10 kA-turns of feedback currents. The efficacy of the feedback system decreases as $<q_a>$ increases due to the onset of overstability, as can be seen in Fig. 4.

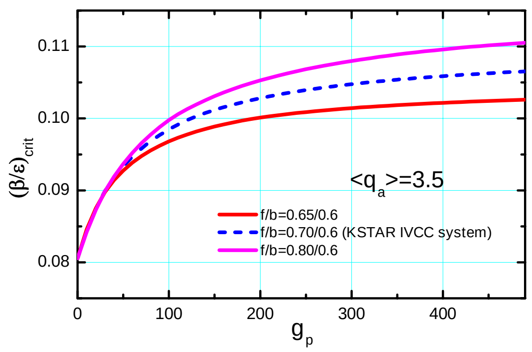

Fig 3. Critical β/ε vs.gain for three different values of feedback coil position.

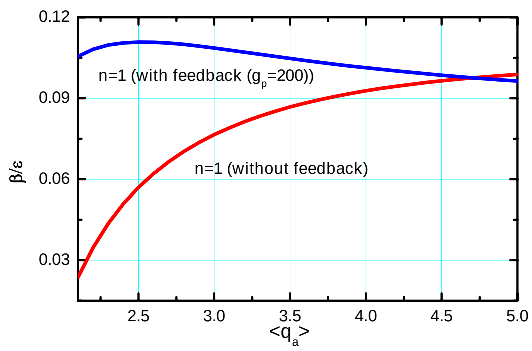

Fig.4. Critical β/ε vs. $<q_a>$

In summary, we have shown that the proposed KSTAR in-vessel coil system is capable of increasing the beta limit up to 140% using only four middle coils.

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