Simple criteria for optimization of trapped particle confinement in stellarators [*]

V.V. Nemov[1], S.V. Kasilov[1], W. Kernbichler[2], G.O. Leitold[2]

[1] *Institute of Plasma Physics, National Science Center "Kharkov Institute of Physics and Technology", Akademicheskaya Str. 1, 61108 Kharkov, Ukraine*

[2] *Institut für Theoretische Physik - Computational Physics, Technische Universität Graz, Petersgasse 16, A–8010 Graz, Austria*

Introduction

Improving the collisionless α-particle containment in stellarators is one of the key issues in stellarator optimization. The most consequent approaches for the investigation of this problem are realized in codes which follow particle orbits and, therefore, allow for direct computation of particle losses (e.g., [1]). To increase computing efficiency, of course, also simple criteria which address this problem properly are of big interest (e.g., minimization of the geodesic curvature of the magnetic field line, residuals in the magnetic spectrum of quasi-symmetric systems, effective ripple, WATER parameter, etc. (see, e.g., [2]). The present work proposes a set of new criteria which are based on the computation of the bounce averaged ∇B-drift velocity of trapped particles across magnetic surfaces. For this purpose, two new target functions which are further denoted as Γ_v and Γ_w are introduced. For a given magnetic field the pertinent optimization parameters are numerically calculated using a field line following code. The proposed criteria are applied to a number of stellarator configurations.

Basic formulae

In accordance with [3] the velocity of the bounce averaged ∇B drift of the trapped particles across magnetic surfaces, v_{an}, can be presented as

$$v_{\text{an}} = \frac{\delta \psi}{\tau_b \langle |\nabla \psi| \rangle} = \frac{v^2}{2\omega_{c0} \langle |\nabla \psi| \rangle} \frac{\partial g_j / \partial b'}{\partial \hat{I}_j / \partial b'} \, , \tag{1}$$

with ψ being the magnetic surface label, τ_b being the bounce period and $\delta \psi$ being the increment in ψ during τ_b. The quantities g_j and $\hat{I}_j$ are given by

$$g_j = \frac{1}{3} \int_{s_j^{min}}^{s_j^{max}} \frac{\mathrm{d}s}{B} \sqrt{1 - \frac{B}{B_0 b'}} \left(4\frac{B_0}{B} - \frac{1}{b'} \right) |\nabla \psi| k_G, \ \ \hat{I}_j = \int_{s_j^{min}}^{s_j^{max}} \frac{\mathrm{d}s}{B} \sqrt{1 - \frac{B}{B_0 b'}}, \tag{2}$$

where B_0 is some reference magnetic field, $b' = v^2/(J_\perp B_0)$, $J_\perp = v_\perp^2/B$, $\omega_{c0} = eB_0/(mc)$, and $k_G = (\mathbf{h} \times (\mathbf{h} \cdot \nabla)\mathbf{h}) \cdot \nabla \psi / |\nabla \psi|$ is the geodesic curvature of a magnetic field line with the unit vector $\mathbf{h} = \mathbf{B}/B$. The quantities s_j^{min} and s_j^{max} correspond to the turning points of the trapped particle in the jth segment of the magnetic field line in which the particle is trapped. Averaging

is performed by the rule $\langle A \rangle = \lim_{L \to \infty} \int_0^L \frac{ds}{B} A / \int_0^L \frac{ds}{B}$. For a certain particle v_{an} (1) is a function of it's energy w and it's perpendicular adiabatic invariant $J_\perp$. To determine an integral effect of the ∇B drift one has to integrate v_{an}^2 over the phase space volume for the distribution of the trapped particles, f, and perform an average over the magnetic surface according to

$$F_n^* = \frac{1}{S} \oint_{(S)} dS \int v_{an}^2 f \, d\Gamma_{w,J_\perp}, \quad d\Gamma_{w,J_\perp} = \sum_{\sigma = \pm 1} \frac{\pi B}{m |v_\parallel|} dJ_\perp dw = \frac{2\pi B}{m |v_\parallel|} dJ_\perp dw, \quad (3)$$

where $f = f(w, \psi)$ and $d\Gamma_{w,J_\perp}$ is the phase space volume element. Transforming the integration over the magnetic surface area to an integration along the magnetic field line and using the mono-energetic distribution function $f = nm^{3/2}\delta(w - E)/(4\sqrt{2}\pi\sqrt{E - e\Phi})$, one obtains

$$F_n^* = \frac{\sqrt{2}}{4\pi} \Gamma_w \frac{v^2 \rho_L^2}{R_0^2} n, \quad (4)$$

where $\rho_L = v/\omega_{c0}$ is the characteristic Larmor radius, R_0 is the major radius of the torus and

$$\Gamma_w = \frac{\pi R_0^2}{\sqrt{2}} \lim_{L_s \to \infty} \left(\int_0^{L_s} \frac{ds}{B} \right)^{-1} \frac{1}{\langle |\nabla \psi| \rangle^2} \int_{B_{min}^{abs}/B_0}^{B_{max}^{abs}/B_0} db' \sum_{j=1}^{j_{max}} \left(\frac{\partial g_j}{\partial b'} \right)^2 \left(\frac{\partial \hat{I}_j}{\partial b'} \right)^{-1}. \quad (5)$$

The quantity Γ_w (5) is calculated by integration over the magnetic field line length, s, over the sufficiently large interval $[0, L_s]$, and by integration over the pitch angle variable b'. The quantities B_{min}^{abs} and B_{max}^{abs} are the absolute minimum and maximum of B within the interval $[0, L_s]$. The index j is numbering the field line intervals $[s_j^{min}, s_j^{max}]$ where $b' - B/B_0 \geq 0$ for a given $J_\perp$ and w.

Upon substituting $v_{an}^2 \equiv 1$ in (3) one obtains the phase space volume of the trapped particles averaged over a magnetic surface with the weight f:

$$V^* = n \frac{2\sqrt{2}}{\pi} \Gamma_s, \quad \Gamma_s = \frac{\pi}{2\sqrt{2}} \frac{1}{\langle |\nabla \psi| \rangle} \left\langle |\nabla \psi| \sqrt{1 - \frac{B}{B_{max}^{abs}}} \right\rangle. \quad (6)$$

Using the definition $\hat{v}_{an}^2 = F_n^*/V^*$ one can formulate the mean square average of the bounce averaged ∇B drift velocity of trapped particles across the magnetic surface as

$$\hat{v}_{an} = \frac{1}{2\sqrt{2}} \Gamma_v \frac{\rho_L}{R_0} v, \quad \Gamma_v = \sqrt{\Gamma_w/\Gamma_s}. \quad (7)$$

For the bounce averaged poloidal motion of the trapped particles one obtaines in Boozer coordinates (ψ, θ, φ) the following expression

$$\frac{d\theta}{dt} = \frac{v^2 B_0}{2\omega_{c0}} \left[\frac{\partial \hat{G}_j/\partial b'}{\partial \hat{I}_j/\partial b'} - \frac{2}{3} \frac{(F' + \iota I')}{(F + \iota I)} \frac{\partial \hat{K}_j/\partial b'}{\partial \hat{I}_j/\partial b'} + \frac{2e}{mv^2} \Phi' \right], \quad (8)$$

$$\hat{G}_j = \frac{1}{3} \int_{s_j^{min}}^{s_j^{max}} \frac{ds}{B} \sqrt{1 - \frac{B}{B_0 b'}} \left(4\frac{B_0}{B} - \frac{1}{b'} \right) \frac{1}{B} \frac{\partial B}{\partial \psi}, \quad \hat{K}_j = \int_{s_j^{min}}^{s_j^{max}} \frac{ds}{B} \sqrt{1 - \frac{B}{B_0 b'}} \left(1 - \frac{B}{B_0 b'} \right), \quad (9)$$

where $F(\psi)$ and $I(\psi)$ are the poloidal and toroidal current functions, $\iota(\psi)$ is the rotational

transform in units of 2π, $2\pi\psi$ is the toroidal magnetic flux, and $\Phi(\psi)$ is the electrostatic potential.

Results

The optimization targets Γ_w and Γ_v have been calculated for a number of stellarators using a field line tracing code being able to operate with configurations given in real space coordinates as well as in Boozer coordinates: The idealized W7-X standard high-mirror configuration [4] as well as the quasi-helically symmetric (QHS) stellarator [5] (see Fig. 1) and two quasi-isodynamical (QI) stellarator configurations with poloidally closed contours of B on magnetic surfaces [6] (see Fig. 2). For a conventional stellarator with circular cross section of magnetic surfaces and a sufficiently large aspect ratio the calculation of the factors Γ_w, Γ_v and Γ_s results in $\Gamma_w^{(conv)} = \Gamma_s^{(conv)} = \sqrt{\varepsilon_h}$ and $\Gamma_v^{(conv)} = 1$ with ε_h being the helical ripple. It follows from Fig. 1 that for W7-X and QHS the parameters Γ_w and Γ_v are much smaller than those for the conventional stellarator. Compared to the result for the conventional stellarator, the parameter Γ_v is approximately 5 times smaller for W7-X and even 10 times smaller for QHS. These results correlate with results of the effective ripple computations.

The QI configurations are the result of an optimization of quasi-poloidally symmetric configurations with respect to collisionless particle confinement (1st QI configuration) and subsequent optimization towards poloidal closure of the $J_\parallel$-contours (2nd QI configuration [6]). It turnes out that effective ripple for the 1st QI configuration is smaller than that for the 2nd configuration. However, from the viewpoint of poloidal closure of the $J_\parallel$-contours the 2nd configuration has a clear advantage. It follows from Fig. 2 that the 2nd configuration is also preferable from the viewpoint of the Γ_w and Γ_v parameters.

Using Eqs. (1) and (8) angles between the $J_\parallel$-contours and the magnetic surface cross section are assessed as

$$\gamma_{J\parallel} = \mathrm{arctg}\,\frac{v_{an}}{r d\theta/dt}, \;\; \gamma_c = \frac{2}{\pi}\,\gamma_{J\parallel}. \tag{10}$$

For the magnetic surfaces at r/a=0.718 it is found that $|\gamma_c| \le 0.22$ and $|\gamma_c| \le 0.17$ for the 1st and the 2nd QI configurations, respectively, (Φ'=0). Fig. 3 shows the corresponding results for a conventional l=2 stellarator with the magnetic field in the form of [7] in Boozer coordinates. It follows from Fig. 3 that for a rather big fraction of trapped particles $J_\parallel$-contours are perpendicular to the magnetic surface (γ_c=1 for $d\theta/dt$=0).

Summary

The main optimization target proposed here is the mean square average over the pitch angle and over the magnetic surface of the bounce averaged ∇B drift velocity across magnetic surfaces, $\hat{v}_{an}$, which is cast into the dimensionless factor Γ_v (Eq. (7)). Another useful target is Γ_w (Eq. (5)) which corresponds to an integral effect of the square of the ∇B drift. Both parameters are calculated using a field line tracing code. Minimization of Γ_v and Γ_w corresponds to the optimization of the trapped particle collisionless confinement being of big importance for α-particle confinement. A numerical analysis of velocities of the radial and poloidal drifts (Eqs. (1) and (8), respectively) allows for a quick assessment of $J_\parallel$-contours in the vicinity of a given magnetic surface. This has to be done for a variety of positions of local minima of B

along a field line and for various pitch angles of trapped particles.

References

[1] W. Lotz et al., *Plasma Phys. Control. Fusion* **34**, 1037 (1992)

[2] Long-Poe Ku, Recent Progress in Configuration Development for Compact Stellarator Reactors, Report at Aries Meeting January 9, 2003; UCSD, San Diego,CA

[3] V. V. Nemov, S. V. Kasilov, W. Kernbichler and M. F. Heyn, Phys. Plasmas **6**, 4622 (1999)

[4] C. Nührenberg, *Phys. Plasmas* **3**, 2401 (1996)

[5] J. Nührenberg and R. Zille, *Phys. Lett. A* **129**, 113 (1988)

[6] A. A. Subbotin et al., *29th EPS Conference on Plasma Phys. and Contr. Fusion,* Montreux, Switzerland, 17-21 June 2002, ECA Vol. **26B**, P5.086 (2002)

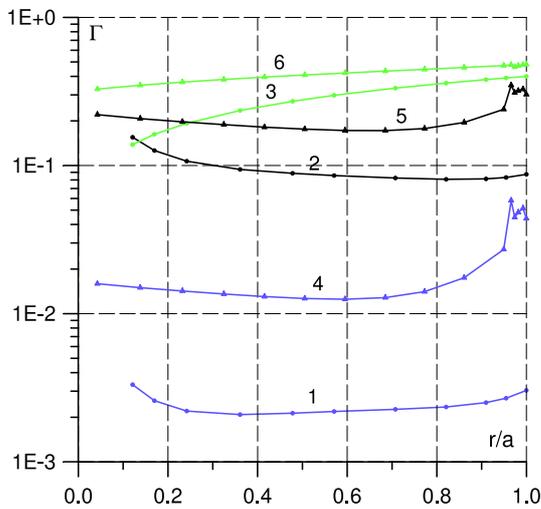[7] A. H. Boozer and G. Kuo-Petravic, *Phys. Fluids* **24**, 851 (1981)

Fig.1. Factors Γ_w, Γ_v and Γ_s for the magnetic configurations of W7-X (number of the field periods n_p=5) and QHS (n_p=6): 1, 2 and 3: Γ_w, Γ_v and Γ_s, respectively, for QHS; 4, 5 and 6: Γ_w, Γ_v and Γ_s, respectively, for W7-X.
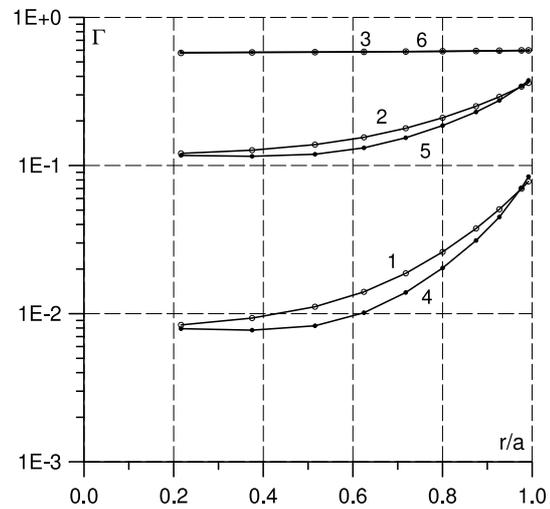
Fig.2. Factors Γ_w, Γ_v and Γ_s for the QI magnetic configurations (n_p=6): 1, 2 and 3: Γ_w, Γ_v and Γ_s, respectively, for the 1st configuration; 4, 5 and 6: Γ_w, Γ_v and Γ_s, respectively, for the 2nd configuration.
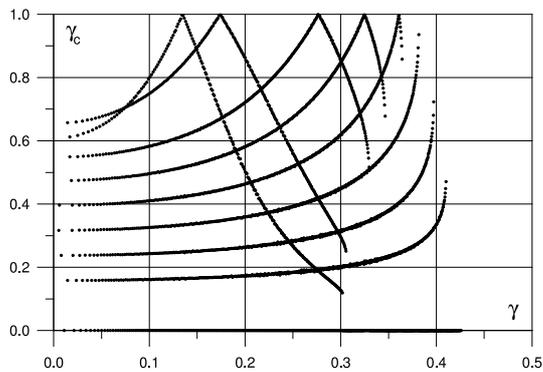

Fig.3. Parameter γ_c as function of pitch angle γ for l=2 stellarator (n_p=8) for various minima of B for the magnetic surface close to the boundary; $\gamma = v_{\|0}/v_{\perp 0}$, $v_{\|0}$ is $v_\|$ in a point of a local minimum of B. The curves represent characteristic results. The electric field is assumend to be zero, $\Phi' = 0$.