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# A Uniform Framework to Study Resistive Wall Modes

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## 1. Introduction

For advanced tokamaks, it is important to raise the plasma pressure $\beta_N$ above the Troyon no-wall limit, which is usually set by the pressure driven ideal external kink instability. A passive wall, with finite conductivity, converts the ideal kink mode into the resistive wall mode (RWM), growing at time scale of the wall eddy current decay time. Therefore, a key issue in expanding the operational space for advanced tokamaks is to stabilize the RWM by either *toroidal plasma rotation* or *active feedback*, or two combined. The physics of damping mechanisms for the RWM in rotating plasmas can be investigated by studying resonant field amplification (RFA), or so-called active MHD spectroscopy for the RWM.

Toroidal calculations, using the MHD stability code MARS-F [1], have shown that the plasma response is well described by low-order rational functions. These frequency dependent transfer functions describe fully the plasma dynamics for both feedback and RFA, thus offer a *uniform* way to study the RWM. This approach is useful to analyze the RFA, the rotation as well as the feedback experiments.

## 2. Plasma response model

For feedback stabilization of the RWM, the simplest control logic is current control, where the measured flux $\Psi_s$ through magnetic loops is used to drive the current $I_f$ in (saddle) feedback coils. For a given feedback gain $K$, the feedback logic reads $I_f = -K\Psi_s/M_{sf}$, where the free-space mutual inductance $M_{sf}$, between the sensor loop and the feedback coil, is introduced to normalize the feedback gain.

We introduce a frequency dependent transfer function $P(s)$

$$P(s) = \frac{\Psi_s(s)}{M_{sf}I_f}, \tag{1}$$

where $s$ is the Laplace variable. This transfer function describes fully the plasma response to the feedback current $I_f$. Moreover, this definition offers a natural way to describe the resonant field amplification, where $I_f$ is an excitation current producing standing/traveling waves as error fields. The plasma response model (PRM) is described by this transfer function.

We will show, in both the cylindrical theory and toroidal calculations, how to construct the PRM and how to use it to study the RWM.

**A. Fitzpatrick-Aydemir model.** First we consider the Fitzpatrick-Aydemir model [2].

$$\begin{aligned}
d\Delta\Psi_a &= [(\gamma - j\Omega_\phi)^2 + \nu(\gamma - j\Omega_\phi)]\Psi_a \\
&= -(1-\kappa)(1-md)\Psi_a + \sqrt{1-(md)^2}\Psi_w, \\
d\Delta\Psi_w &= \gamma S\Psi_w \\
&= -(1+md)\Psi_w + \sqrt{1-(md)^2}\Psi_a + 2md\Psi_c,
\end{aligned}$$

where $\Psi_w$ and $\Psi_a$ are the radial magnetic flux on the wall and the plasma surface, respectively. $\Psi_c$ is the flux produced by the feedback coil on the wall position in free-space. $\Omega_\phi$

is the plasma toroidal rotation frequency, normalized according to Ref. [2]. $\nu$ is the viscous dissipation coefficient. In this model the dissipation is introduced in a thin viscous layer close to the plasma boundary. $\kappa$ is the stability index for the RWM without the plasma rotation, such that $\kappa = 0$ corresponds to the no-wall marginal stability, and $\kappa = 1$ corresponds to the ideal-wall marginal stability. The parameter $d = \frac{1}{m}\frac{(r_w/a)^{2m}-1}{(r_w/a)^{2m}+1}$, where $r_w$ denotes the radial distance of the wall, $a$ the plasma minor radius, $m$ the poloidal Fourier harmonic. The parameter $S$ is proportional to the wall time $\tau_w$.

We mention that a similar model has been proposed by M.S. Chu et al. [3]. Both models assume effectively the *single mode* approximation for the RWM. In [4], we constructed the PRM using a cylindrical theory with *multiple* modes, in the absence of plasma rotation.

Using the radial flux $\Psi_w$ or $\Psi_a$ as sensor signals, we derive the corresponding transfer functions according to the definition (1)

$$\begin{aligned} P_w(s) &= \frac{2md[(s-j\Omega_\phi)^2+\nu(s-j\Omega_\phi)+(1-\kappa)(1-md)]}{(Ss+1+md)[(s-j\Omega_\phi)^2+\nu(s-j\Omega_\phi)+(1-\kappa)(1-md)]-[1-(md)^2]}, \\ P_a(s) &= \frac{2md\sqrt{1-(md)^2}}{(Ss+1+md)[(s-j\Omega_\phi)^2+\nu(s-j\Omega_\phi)+(1-\kappa)(1-md)]-[1-(md)^2]}. \end{aligned}$$

Defining an internal poloidal sensor as $\Psi_\theta \equiv -\frac{r}{m}\frac{d\Psi}{dr}|_{r_w-}$, we obtain the corresponding transfer function

$$P_\theta(s) \equiv \frac{\Psi_\theta}{\Psi_c} = \frac{\sqrt{1-(md)^2}}{md}P_a - \frac{1}{md}P_w.$$

Figure 1 shows transfer functions $P_w(j\omega)$ and $P_\theta(j\omega)$ in the complex plane, with $-\infty < \omega < \infty$ (Nyquist diagram). We plot transfer functions for both unstable plasma at slow rotation ($\Omega_\phi = 0, 0.1, 0.2$), and the plasma stabilized by fast rotation ($\Omega_\phi = 0.3, 0.4$). The Nyquist diagram shows both the stability margin and the phase variation of the open-loop plasma response. [More detail explanation can be found in [4].]

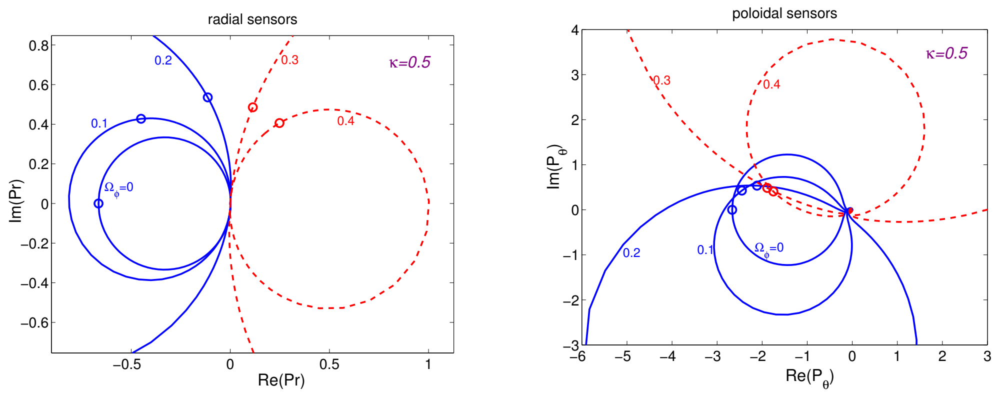


*Figure 1: Nyquist diagram of transfer functions for radial (left) and poloidal (right) sensors. The Fitzpatrick-Aydemir model is assumed with parameters $r_w = 1.2, m = 3, S = 100, \kappa = 0.5, \nu = 1$. Transfer functions are shown for both unstable (solid lines) and stable (dashed lines) plasmas.*

Knowing the plasma response model $P(s)$, we can study the stability and the performance of the closed loop for a given controller $K(s)$. The growth/damping rates of the closed loop are equal to the roots of the characteristic equation $1+K(s)P(s)=0$. The stability margin is shown in Fig. 2 in the $\kappa-\Omega_\phi$ plane, in the presence of both feedback and plasma rotation. The synergy between feedback and rotation is observed for both radial and poloidal sensors, in the sense that increasing the (proportional) feedback gain shrinks the unstable region in the $\kappa-\Omega_\phi$ plane. However, at weak damping (e.g. $\nu=0.1$), a slow rotation destabilizes the RWM, such that a larger critical gain is needed to stabilize the mode. Such a destabilization effect disappears at strong dissipation (e.g. $\nu=1$). For both poloidal and radial sensors, strong plasma dissipation results in *full* synergy between feedback and the plasma rotation, for all rotation frequencies. This is in a qualitative agreement with toroidal calculations with realistic plasma conditions [5].

We note that feedback with radial sensors requires much larger gains than poloidal sensors. Radial sensors are less effective to suppress the unstable region close to the ideal-wall limit.

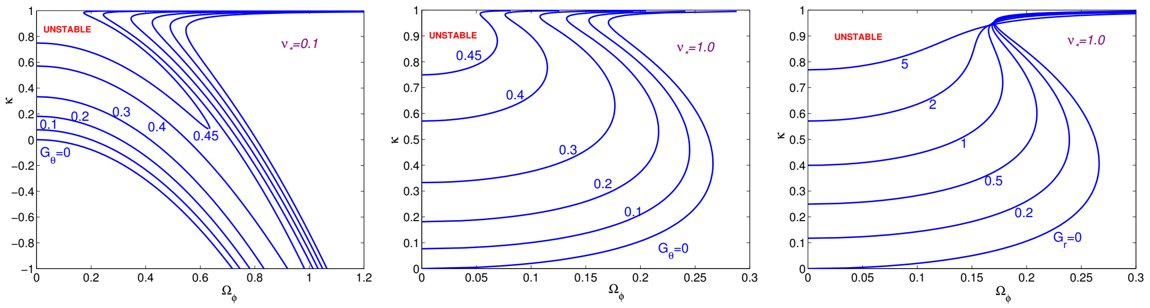


*Figure 2: Combined effect of feedback and rotation to stabilize the RWM, assuming a weak damping model with $\nu=0.1$ (left) or a strong damping model with $\nu=1$ (middle,right). Critical rotation is plotted against stability index $\kappa$, for various feedback gains using poloidal sensors just inside the resistive wall (left,middle) and radial sensors on the wall (right).*

**B. Toroidal calculations.** We also use the stability code MARS-F [1] to compute the plasma response model for realistic equilibria. Figure 3 shows a generic example of the PRM computed for a JET-like shaped plasma. We assume a semi-kinetic damping model [6] for the interaction between the rotating plasma and the RWM, and using external mid-plane saddle coils for feedback control. Radial sensors are used for a moderately unstable RWM with $\beta_N$ in the middle between no-wall and ideal-wall limits. The plasma is stable with rotation frequencies larger than a critical value $\omega_{\rm ROT}^{\rm cr}/\omega_A$=0.017. [Both the rotation and the Alfvén frequency $\omega_A$ is defined in the plasma center.] The Nyquist diagram for the stable plasma ($\omega_{\rm ROT}/\omega_A$=0.03 or 0.06) is similar to that from Fig. 1, indicating that a single mode gives reasonable approximation of the RWM dynamics for stable plasmas. However, the Nyquist diagram for unstable plasma (slow rotation $\omega_{\rm ROT}/\omega_A$=0 or 0.01) is more complex than that from Fig. 1. In fact, we find that an accurate approximation of such a response involves at least three modes.

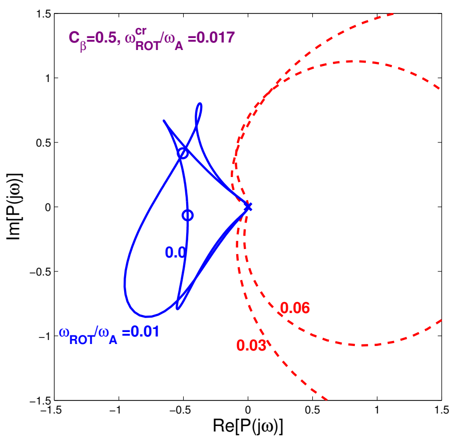


*Figure 3: Nyquist diagram of the plasma response computed for a JET-like shaped plasma. The response of both unstable (solid lines) and stable (dashed lines) plasmas is presented.*

Figure 4 shows the poles location in the complex plane, for the response of unstable plasma ($\omega_{ROT}/\omega_A$=0.01) and stable plasma ($\omega_{ROT}/\omega_A$=0.03). For a comparison, shown also are growth rates of unstable RWM without feedback and with increasing $\omega_{ROT}/\omega_A$ from 0 up to 0.06, as well as the (intrinsically) stable RWM without feedback and plasma rotation. One of the poles in the Padé approximation always corresponds to the RWM growth rate modified by the plasma rotation. For the unstable plasma, the other two poles represent the lumped effect of all the stable RWM. These two poles are close to the origin, hence give significant contribution to the transfer function. On the other hand, for the stable plasma, the second pole is far away from the origin, giving minor contribution to the transfer function at low frequency range. Therefore, the stable plasma is well approximated by a single pole.

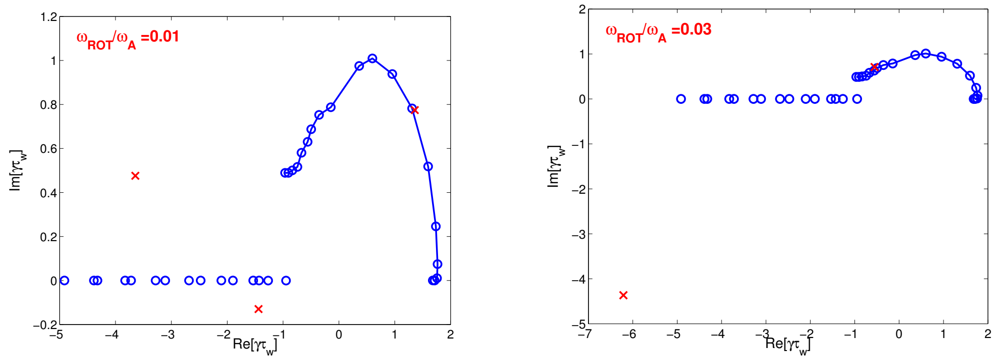


*Figure 4: Padé approximation for both unstable (left) and stable (right) plasma response. 'o' - growth/damping rates of the RWM with plasma rotation but without feedback. 'x' - poles in the Padé approximation of transfer functions, computed in the feedback control of the unstable RWM (left) and in the resonant field amplification for the stable RWM (right). In the latter case, the unstable branch of RWM is stabilized by a strong plasma rotation, as shown by the solid lines.*

## 3. Summary

The plasma response model, defined in terms of frequency dependent transfer functions, offers a uniform framework to describe and analyze RWM for both active feedback control of the unstable RWM, and the resonant field amplification due to the stable RWM. This approach decouples the RWM MHD physics study from the controller design. The output of the model is directly related to the measurable signals in RFA experiments. Moreover, direct comparison of the transfer functions offers a good way to benchmark numerical codes.

Analysis of the Fitzpatrick-Aydemir model (or Chu's model), based on this approach, shows that these simple, single-mode models give qualitative description of rotational and feedback stabilization of the RWM. The models with strong dissipation agree better with toroidal results including a semi-kinetic damping. Toroidal calculations show that the response of unstable plasma involves *several* modes, whereas the response of rotationally damped plasma is adequately described by a *single* mode at low frequency range.

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