Profile robustness and routes to turbulence in the helimak configuration

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The Helimak is the simplest toroidal plasma configuration that exhibits an MHD-equilibrium. It is very suitable for studies of low-frequency gradient driven instabilities and their routes to turbulence in the presence of magnetic field curvature and shear, and related to this, cross-field turbulent transport and formation of universal plasma profiles [1].

The configuration consists of a purely toroidal magnetic field B_φ with a weak vertical magnetic field component B_y superposed, and the key parameter is the magnetic pitch ratio $r_B = |B_y|/B \sim 10^{-2}$. It is convenient to introduce cylindrical coordinates (x, φ, y), where $x = R - R_0$ denotes the radial distance along the major radius from the minor axis, y is the (vertical) distance in the direction along the major torus axis, and φ is the toroidal angle. The helical field lines ascend a vertical distance $\Delta y_B \approx 2\pi R r_B$ between two successive intersections with the poloidal plane $\varphi = 0$. In a broad range of operating regimes the equipotential surfaces are elongated in the y direction into a slab-like structure.

For very low magnetic field strength B the equipotential surfaces (which are $\mathbf{E} \times \mathbf{B}$-flow surfaces) are not closed, and the plasma can escape to the wall by steady state advection. Increasing B will tend to close these surfaces and increase the electron pressure gradient until threshold for electrostatic flute interchange instability is attained. For higher values of B the pressure profile remains resilient (see Fig. 1), and it is believed that this is due to a dynamic balance between forced steepening of the profile due to increased effective plasma production in the core and the relaxation of the profile through increased anomalous transport caused by the unstable waves. We show that this dynamics can be described by an autonomous system of three ordinary differential equations with many of the same properties as the famous Lorenz equations [2], and
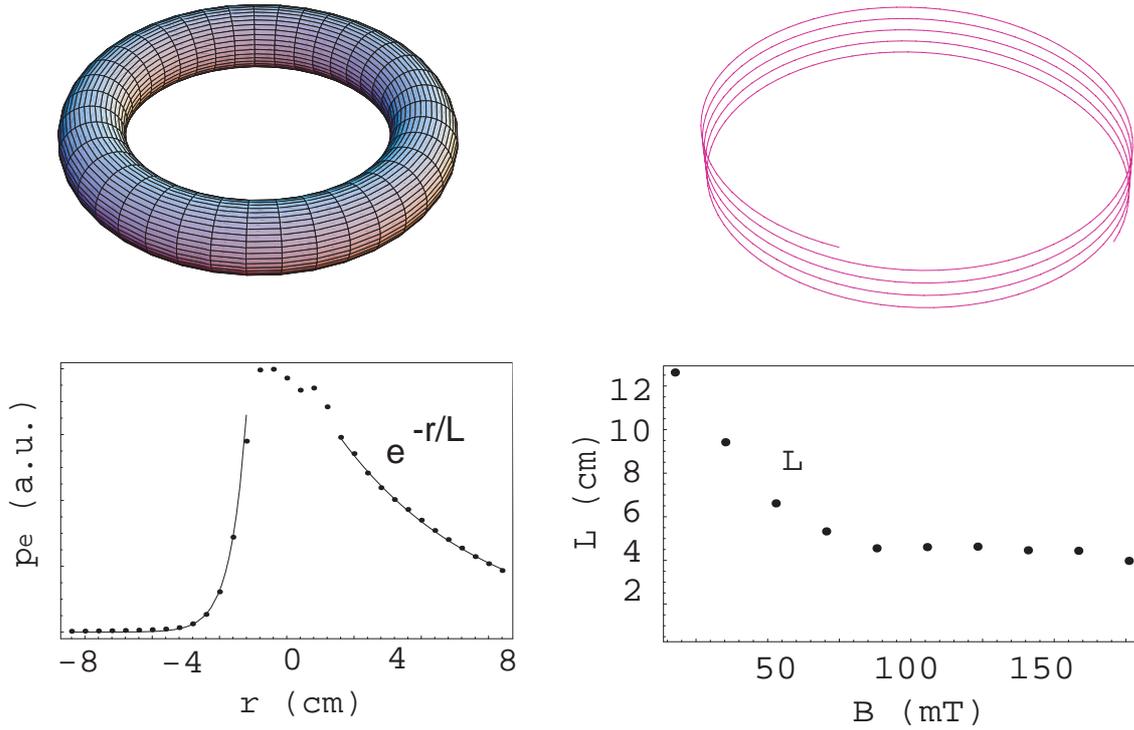
Figure 1: Upper left: Topology of the toroidal plasma column. Upper right: Spiral structure of a magnetic field line. Lower left: Radial profile of electron pressure profile in the equatorial plane, with their exponential fits on the high-field side ($r < 0$) and the low-field side ($r > 0$). Lower right: Profile scale length on the low-field side versus B-field strength. Instbility threshold is at $B \approx 50$ mT.

gives rise to a period doubling route to low-dimensional chaos. The equations describing flute fluctuations driven unstable on the low-field side take the form [3]

$$\partial_t \nabla_\perp^2 \phi + [\phi, \nabla_\perp^2 \phi] + \zeta \partial_y n = \nu_i \nabla_\perp^2 \phi \,, \tag{1}$$

$$\partial_t n + [\phi, n] + (\kappa - \zeta)\partial_y \phi + \zeta \partial_y n = 0 \,. \tag{2}$$

Here n and ϕ are perturbed dimensionless electron density and potential, respectively, $\kappa = \rho_s/L$ is the dimensionless inverse pressure scale length, $\zeta = 2\rho_s/L$ is the dimensionless B-field curvature, and ν_i is the dimensionless ion-neutral collision frequency. The convective nonlinearities take the form of Poisson brackets $[f, g] \equiv \partial_x f \partial_y g - \partial_y f \partial_x g$, where x is the radial coordinate and y is the vertical coordinate in the direction of
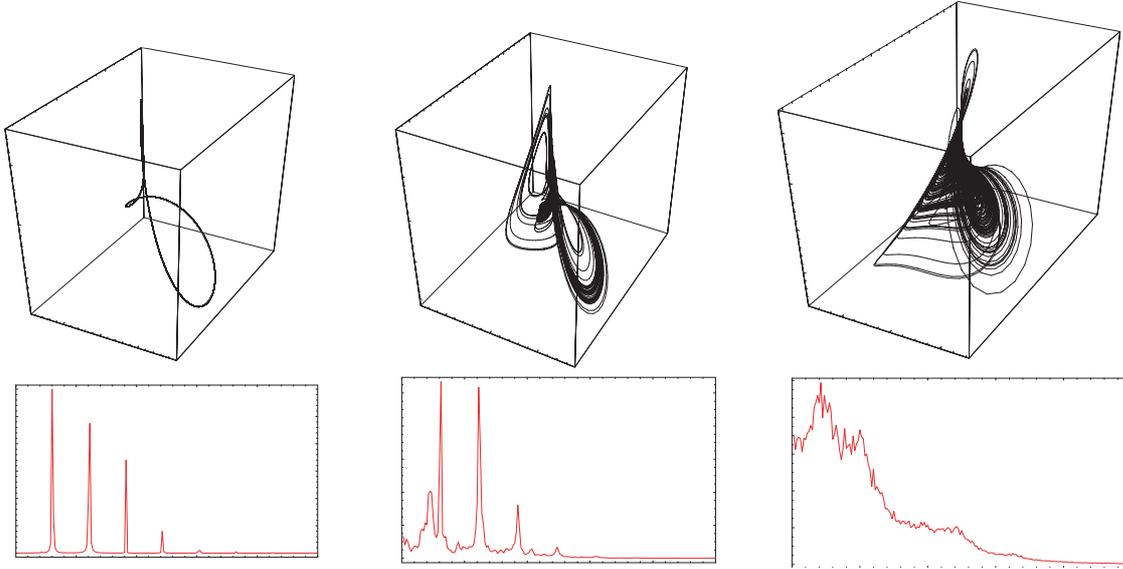
Figure 2: Attractors for $(A(t), B(t), C(t))$ and power spectrum for $A(t)$ for $\nu = 1$ and three different values for s. Left: $s = 0.25$. Middle: $s = 0.2629$. Right: $S = 1$.

wave propagation. Because a magnetic field line rises a distance Δy_B for every toroidal turn, a vertical magnetic wave number $k_y = 2\pi/\Delta y_B$ is selected. By linearizing Eqs. 1 and 2, neglecting the term $\zeta \partial_y n$ (which turns the slowly propagating flute mode into a stationary convection cell), and writing $\phi = A(t) \sin(k_y y)$, $n = B(t) \cos(k_y y)$, these equations reduce to the linear, ordinary differential equations

$$d_t A = -\nu_i A - \frac{\zeta}{k_y} B \tag{3}$$

$$d_t B = -\kappa k_y A + \zeta k_y A \,, \tag{4}$$

which yields a purely growing instability if the threshold condition $\kappa > \zeta$ is satisfied. Assuming a time-dependent density gradient, we can write a mass conservation equation giving the time-evolution of this normalized gradient $C(t) \equiv \kappa(t)/\zeta$ in terms of the anomalous flux due to the flute convection cells and the effective plasma production rate s. After some rescalings Eqs. (3), (4) and this mass conservation equation reduce to the dynamical system:
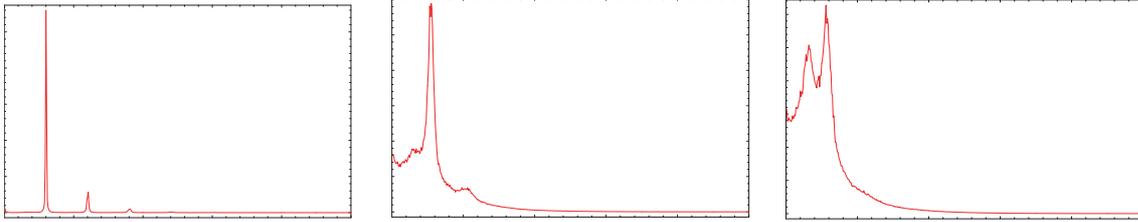
Figure 3: Experimental power-spectra for increasing magnetic field strength B. Left figure is for B just above instability threshold.

$$d_t A = -\nu A - B \,, \tag{5}$$

$$d_t B = -CA + A \,, \tag{6}$$

$$d_t C = AB + s \,. \tag{7}$$

This dynamical system has many similarities to to the Lorenz equations, such as the occurence of two unstable fixed points; $A = \pm\sqrt{s/\nu}\,$, $B = \mp\sqrt{s\nu}\,$, $C = 1\,$. Physically realistic values for the dimensionless damping rate and source are $\nu \sim s \sim 1$. The behavior for $\nu = 1$ and increasing s are shown in Fig. 2. From periodic solutions there is a period doubling route to chaos. Chaos sets in when the orbit starts to alternate between encircling one or the other of the two fixed points as shown in the middle figure.

Direct comparison with experiemntal time-series is non-trivial because the model does not include the vertical plasma drift, which implies an additional Doppler shifted frequency. Still some qualitative features like spectral broadening are common for the chaos spectra shown in Fig. 2 and the experimental spectra depicted in Fig. 3. Correcting for this effect we hope to reconstruct the attractor and calculate the largest positive Lyapunov exponent and the attractor dimension from the experimental data.

References

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