

## Large scale flows and coherent structure phenomena in flute mode turbulence

Ingmar Sandberg<sup>1</sup>, Zhanna N.Andrushchenko<sup>2</sup> and Vladimir P.Pavlenko<sup>3</sup>

<sup>1</sup>National Technical University of Athens, Association Euratom-Hellenic Republic, Greece

<sup>2</sup>Swedish University of Agricultural Sciences, Uppsala, Sweden

<sup>3</sup>Uppsala University, Euratom-VR Fusion Association Uppsala, Sweden

**Abstract.** The properties of zonal and streamer flows in flute mode turbulence are studied. The stability criteria and the frequency of these flows are determined in terms of the spectra of turbulent fluctuations. Furthermore, it is shown that zonal flows can undergo a further nonlinear evolution leading to the formation of long-lived coherent structures. These structures consist of self-bound wave packets supporting stationary shear layers, and thus can be characterized as regions with a reduced level of anomalous transport.

**Basic equations.** Flute (or interchange) modes,  $k_{\parallel} = 0$ , are low frequency,  $\omega \ll \omega_{ci}$ , electrostatic oscillations of nonuniform magnetized plasma which become unstable due to combine effects of the density inhomogeneity and the curvature of the magnetic field. To describe flute modes we use the two-fluid equations for an inhomogeneous magnetized plasma with characteristic inhomogeneity scale length  $L_n$  along the radial axis  $x$ . For the slab geometry, we model the curved magnetic field by  $B(x) = B_0(1 - x/R)$  with unit vector  $\mathbf{b} = \hat{z} - (z/R)\hat{x}$ . Then, the magnetic-curvature-driven flute modes are described by the following set of dimensionless coupled equations for the perturbed electrostatic potential  $\Phi$  and density  $n$ :

$$(\partial_t - v_{ni}\partial_y)\nabla_{\perp}^2\Phi + v_g\partial_y n = \{\nabla_{\perp}^2\Phi, \Phi\} + \tau_i\nabla \cdot \{\nabla_{\perp}\Phi, n\} \quad (1)$$

$$(\partial_t + v_{ge}\partial_y)n + (v_{ne} - v_{ge})\partial_y\Phi = \{n, \Phi\} \quad (2)$$

where  $\{f, g\} = \hat{z} \times \nabla f \cdot \nabla g$  denotes the Poisson bracket. The electrostatic potential has been normalized by  $T_e/e$ , the time by  $\omega_{ci}$ , the length scales by  $\rho = c_s/\omega_{ci}$ , where  $c_s^2 = T_e/m_i$ , and the perturbed density by  $n_0$ . Furthermore,  $v_g = v_{ge} + v_{gi}$ . These equations include the diamagnetic drift,  $v_{nj} = T_j/(eB_0L_n)$ , the magnetic curvature drift,  $v_{gj} = 2T_j/(eRB_0)$ , of ions and electrons ( $j = i, e$ ), the finite ion Larmor radius effect is described by the term with  $\tau_i$ . The linear dispersion relation of flute mode is given by

$$\omega_k = -\frac{k_y(v_{ni} - v_{ge})}{2} \left( 1 \pm \varepsilon \sqrt{1 - \frac{k_{cr}^2}{k_{\perp}^2}} \right) \quad (3)$$

where  $\varepsilon \equiv (v_{ni} + v_{ge})/(v_{ni} - v_{ge})$ , and  $k_{cr}^2 \equiv 4v_g(v_{ne} - v_{ge})/(v_{ni} + v_{ge})^2$ . Modes of finite poloidal wave number with  $k_{\perp} \leq k_{cr}$  are linearly unstable.

**Coupled dynamics of flute mode turbulence and large scale flows.** To describe the dynamics of large-scale plasma flows that vary on longer space and time scale compared to small-scale

fluctuations, a multiple scale expansion is usually employed assuming that there is sufficient spectral gap separating the large and small scale motions. In what follows,  $[\tilde{\Phi}(\mathbf{r}, t), \tilde{n}(\mathbf{r}, t)]$ , denote the small-scale fluctuations and  $[\bar{\Phi}(\mathbf{r}, t), \bar{n}(\mathbf{r}, t)]$  the large scale ones. By averaging Eqs.(1,2), we get

$$(\partial_t - v_{ni} \partial_y) \nabla_{\perp}^2 \bar{\Phi} + v_g \partial_y \bar{n} = -\bar{R}^{\Phi} - \bar{R}^n \quad (4)$$

$$(\partial_t + v_{ge} \partial_y) \bar{n} + (v_{ne} - v_{ge}) \partial_y \bar{\Phi} = \overline{\tilde{n}, \tilde{\Phi}} \quad (5)$$

where  $R^{\Phi} = \{\tilde{\Phi}, \nabla_{\perp}^2 \tilde{\Phi}\}$  is the standard Reynolds force due to the polarization drift non-linearity, and  $R^n = \tau_i \{\tilde{n}, \nabla_{\perp}^2 \tilde{\Phi}\} + \tau_i \{\nabla_{\perp} \tilde{n}, \nabla_{\perp} \tilde{\Phi}\}$  is the diamagnetic Reynolds force due to the fluctuating ion pressure. Eqs.(4,5) describe the formation of large scale structures by the flute turbulence via Reynolds stresses. The flute mode propagation in weakly inhomogeneous media can be described by means of the wave kinetic equation for the wave-action density  $N_k(\mathbf{r}, t)$  in the  $\mathbf{r}$ - $\mathbf{k}$  space. The source of these slow spatial and temporal variations are large scale flows induced by the velocity and the density perturbations. The generalized wave-action density is found to be

$$N_k = k_{\perp}^4 \left( \frac{v_{ni} + v_{ge}}{v_g} \right)^2 \left| \frac{k_{cr}^2}{k_{\perp}^2} - 1 \right| |\Phi_k|^2 \quad (6)$$

Then, the WKB-type wave kinetic equation that describes the evolution of wave-action density  $N_k(\mathbf{r}, t)$  in the flute mode turbulence due to the interaction between the mean flows and the small-scale fluctuations is given by

$$\frac{\partial N_k}{\partial t} + \frac{\partial N_k}{\partial \mathbf{r}} \cdot \frac{\partial \omega_k^{NL}}{\partial \mathbf{k}} - \frac{\partial \omega_k^{NL}}{\partial \mathbf{r}} \cdot \frac{\partial N_k}{\partial \mathbf{k}} = \gamma_k N_k - \Delta \omega_k N_k^2 \quad (7)$$

The nonlinear frequency is defined by  $\omega_k^{NL} = \omega_k + \mathbf{k} \cdot \mathbf{V}_0$ , where the Doppler shift is due to the presence of the large scale flows with velocity  $\mathbf{V}_0 = \mathbf{V}_{\Phi} + \mathbf{V}_n$ , where

$$\mathbf{V}_{\Phi} = -\frac{1}{2} (\nabla \bar{\Phi} \times \mathbf{z}), \quad \text{and} \quad \mathbf{V}_n = -\frac{\tau_i}{4} (\nabla \bar{n} \times \mathbf{z}) \quad (8)$$

The non-linear frequency shift  $\Delta \omega_k$  in the rhs of Eq. (7) represents the part of the non-linear interactions among the flute modes which balance the linear growth rate. Considering small deviations of the spectrum function from the equilibrium,  $N_k = N_k^0 + \tilde{N}_k$ , the perturbed density  $\tilde{N}_k$  can be calculated by the linearization of the wave kinetic equation for a uniform equilibrium  $\partial N_k^0 / \partial \mathbf{r} = 0$ :

$$\tilde{N}_k^{res} = \frac{\partial}{\partial \mathbf{r}} (\mathbf{k} \cdot \mathbf{V}_0) \frac{\partial N_k^0}{\partial \mathbf{k}} R(\Omega, q, \delta \omega_k) \quad (9)$$

Here we assumed the local approximation, i.e.  $\partial \omega_k / \partial \mathbf{r} = 0$ , and  $\tilde{N}_k \sim \exp(i\mathbf{q}\mathbf{r} - i\Omega t)$ . The response function is defined by  $R(\Omega, q, \delta \omega_k) = i / (\omega - \mathbf{q} \cdot \mathbf{V}_g + i\delta \omega_k)$ , where  $\delta \omega_k$  is the total decorrelation frequency which includes the linear growth rate and a nonlinear shift, and  $\mathbf{V}_g = \partial \omega_k / \partial \mathbf{k}$  is the group velocity.

In a weakly nonlinear regime  $R(\Omega, q, \delta\omega_k) \rightarrow \pi\delta(\Omega - q \cdot V_g)$ , while for a wide fluctuating spectrum  $R(\Omega, q, \delta\omega_k) \rightarrow 1/\delta\omega_k$ . The broad spectrum of large scale structures regulates the flute turbulence by the process of random shearing and thus governs the mechanism of self-regulation and saturation of the flute mode turbulence.

Long Term Dynamics of Zonal Flows. Equations describing the evolution of the zonal flow,  $q(q_x, q_y) = q(q_x, 0)$ , are obtained from Eqs. (4, 5) by calculating the Reynolds stress forces

$$\frac{\partial \overline{\Phi}_{q_x}}{\partial t} = \int k_x k_y \left( 1 - \frac{v_{ni}}{2v_g} k_{\perp}^2 \right) |\tilde{\Phi}_k|^2 d^2k \quad (10)$$

$$\frac{\partial \overline{n}_{q_x}}{\partial t} = 0 \quad (11)$$

The second term in the right hand side of Eq. (10) is attributed to the ion diamagnetic drift and to the finite ion Larmor radius and may lead to the suppression of the zonal flow generation. Combining Eqs. (10,11) and using Eqs. (6,8), we get relation which connects the zonal flow velocity with the spectra of the small scale fluctuations,

$$\frac{\partial V_{0,y}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial x} \int k_x k_y \zeta(k_{\perp}) |\tilde{\Psi}_k|^2 d^2k \quad (12)$$

Here  $\Psi_k = n_k + \alpha_k \Phi_k$ , parameters  $\alpha_k$  and  $\zeta(k_{\perp})$  are defined by

$$\alpha_k = k_{\perp}^2 \left[ \frac{v_{ni} + v_{ge}}{2v_g} \right] \left( 1 - \sqrt{1 - \frac{k_{cr}^2}{k_{\perp}^2}} \right), \quad \zeta(k_{\perp}) = \frac{1}{2k_{\perp}^2} \frac{v_g v_{ni}}{(v_g + v_{ni})^2} \left( \frac{2}{k_{\perp}^2} \frac{v_g}{v_{ni}} - 1 \right) \left| \frac{k_{cr}^2}{k_{\perp}^2} - 1 \right|^{-1}$$

Inserting now Eq. (9) into (12) and assuming the zonal flow variation to be of the form  $V_{0,y} \sim \exp(iq_x x)$ , we obtain

$$\frac{\partial V_{0,y}}{\partial t} = q_x^2 D_{xx} V_{0,y}$$

This equation determines the stability of the zonal flow since  $\gamma_{zf} = q_x^2 D_{xx}$ . The coefficient  $D_{xx}$  is given by

$$D_{xx} = -\frac{1}{2} \int k_x k_y^2 \frac{\partial N_k}{\partial k_x} \zeta(k_{\perp}) R(\Omega, q_x, \delta\omega_k) d^2k \quad (13)$$

The zonal flow gets unstable when  $D_{xx} > 0$ . This instability can be interpreted as a result of the resonant interaction between zonal flow and the small scale modulations of the turbulence. In the flute turbulence, where it is usually  $k_x (\partial N_k^0 / \partial k_x) < 0$ , the zonal flow may become unstable due to the contribution of the modes with  $k_{\perp}^2 < 2v_g / v_{ni}$ . Part of these modes is linearly unstable. However, when  $v_{gi} / v_{ni} < \sqrt{3\tau_i^2 + 2\tau_i} - 2\tau_i$  (for  $\tau_i < 2$ ), it turns that  $k_{cr}^2 > 2v_g / v_{ni}$  and subsequently, the modes responsible for the instability of the zonal flow may contribute significantly to the value of the integral (13). If  $k_x (\partial N_k^0 / \partial k_x) > 0$ , it is more likely that the zonal flow is stable. For perturbations with  $\Omega \ll q_x V_{gx}$ , we can take into

account the non-resonant response  $\tilde{N}_k^{(1)}$  of the turbulent spectra over perturbations of the induced zonal flow. In this case the solution to the linearized wave kinetic equation (7) yields

$$\tilde{N}_k^{(1)} = k_y V_{0y} \left( \frac{\partial \omega_k}{\partial k_x} \right)^{-1} \frac{\partial N_k^0}{\partial k_x}$$

Substituting the later expression into Eq. (12), we obtain the oscillation frequency of zonal flow,  $\Omega_{zf} \cong -u_x q_x$ , where

$$u_x = \frac{1}{2} \int k_x k_y^2 \left( \frac{\partial \omega_k}{\partial k_x} \right)^{-1} \frac{\partial N_k^0}{\partial k_x} \zeta(k_\perp) d^2 k$$

As the amplitude of zonal flow grows, nonlinear effects become significant. Using the derived expression of  $\tilde{N}_k^{(1)}$ , we can determine iteratively the next order nonlinear response  $\tilde{N}_k^{(2)}$  for the non-resonant interaction

$$\tilde{N}_k^{(2)} = \frac{1}{2} (k_y V_{0y})^2 \left( \frac{\partial \omega_k}{\partial k_x} \right)^{-1} \frac{\partial}{\partial k_x} \left[ \left( \frac{\partial \omega_k}{\partial k_x} \right)^{-1} \frac{\partial N_k^0}{\partial k_x} \right]$$

Including the total response,  $\tilde{N}_k = \tilde{N}_k^{res} + \tilde{N}_k^{(1)} + \tilde{N}_k^{(2)}$ , into Eq.(12), we obtain the nonlinear equation which describe the evolution of zonal flow

$$u_x \frac{\partial^2}{\partial x^2} V_{0y} + b_x \frac{\partial^2}{\partial x^2} V_{0y}^2 - D_{xx} \frac{\partial^3}{\partial x^3} V_{0y} = \frac{\partial}{\partial t} \frac{\partial}{\partial x} V_{0y} \quad (14)$$

The coefficient  $b_x$  in the nonlinear term is given by

$$b_x = \frac{1}{4} \int k_x k_y^3 \left( \frac{\partial \omega_k}{\partial k_x} \right)^{-1} \frac{\partial}{\partial k_x} \left[ \left( \frac{\partial \omega_k}{\partial k_x} \right)^{-1} \frac{\partial N_k^0}{\partial k_x} \right] \zeta(k_\perp) d^2 k$$

Eq.(14) admits localized solutions. The simplest solution is of the kink type and it is given by

$$V_{0y} = \frac{1}{2} \left\{ V_{1y} + V_{2y} + (V_{1y} - V_{2y}) \tanh \left[ x b (V_{1y} - V_{2y}) / (2 D_{xx}) \right] \right\}$$

where  $V_{2y} = -V_{1y} - (u_{0x} + u_x) / b_x$ . This solution describes the transient region between two different values of the flow. So, the cooperative effects of the wave motion, steeping and instability result in the possibility to the formation of stationary or moving kink solitons. The values of parameters which determine characteristic lengths of these structures are defined by the value of the group velocity and by the spectral density of the background fluctuations. The above analysis demonstrates the self-organization properties of flute mode-zonal flow coupled system.

**Summary.** The properties of large scale flows excited in an electrostatic turbulence of flute type are investigated by means a wave kinetic equation for a weakly inhomogeneous media. The resonant interaction between mean flow and turbulence may lead to the stabilization of the large-scale flows. The nonlinear evolution of zonal flow may result in the formation of stationary coherent structures in the transition layer between surfaces of different flow velocities.