

## Neoclassical tearing modes in the presence of sheared flows

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Neoclassical tearing modes (NTMs) pose a potential threat to long pulse tokamak experiments and future tokamak based reactors, such as ITER [1], where their onset can cause soft disruptions and prevent the achievement of high  $\beta$ . There is considerable current research interest in elucidating the evolutionary characteristics of these modes and the conditions for their destabilization [2-5]. Among some of the outstanding issues related to them, the interaction of equilibrium shear flows with NTMs continues to be of paramount importance since sheared velocity flows are known to be widely prevalent in tokamak devices and can be generated by neutral beams, ion cyclotron heating and self-consistent drift turbulence. A number of past studies have examined the effect of flows on tearing modes, particularly in the linear regime and for simplified geometries [6]. There have also been a few nonlinear studies [7, 8] but the problem is quite complex, particularly in realistic toroidal geometries. We have recently begun [5] a detailed numerical study on the nonlinear evolution of NTMs in the presence of sheared equilibrium flows with the help of a modified version of a finite difference code NEAR [10]. The numerical model and the code have been tested in the past for the effect of flows on resistive linear tearing modes [9] and subsequently extended and benchmarked for nonlinear resistive and neoclassical tearing modes [5]. In this paper we report on further findings of our numerical work and also present a model analytic calculation that provides some physical insight into the numerical results.

Our numerical simulations are based on the solutions of a set of reduced MHD equations that were proposed by Kruger *et al* [10] and which in the limit of  $\beta \sim \delta^{1/2}$  ( $\delta \ll 1$ ), take the form,

$$\frac{\partial \Psi}{\partial t} - (b_0 + b_1) \cdot \nabla \phi_1 - b_1 \cdot \nabla \phi_0 = \eta \tilde{J}_{\parallel} - \frac{1}{ne} b_0 \cdot \nabla \cdot \Pi_e \quad (1)$$

$$\begin{aligned} \nabla \cdot \left( \frac{\rho}{B_0} \frac{d}{dt} \frac{\nabla \phi_1}{B_0} \right) + (V_1 \cdot \nabla) \left( \nabla \cdot \left( \frac{\rho}{B_0} \frac{\nabla \phi_0}{B_0} \right) \right) &= (B_0 \cdot \nabla) \frac{\tilde{J}_{\parallel}}{B_0} + (B_1 \cdot \nabla) \frac{J_{T\parallel}}{B_0} \\ &+ \nabla \cdot \frac{B_0 \times \nabla p_1}{B_0^2} + \nabla \cdot \frac{B_0}{B_0^2} \times \nabla \cdot \Pi \end{aligned} \quad (2)$$

$$\frac{dp_1}{dt} + (V_1 \cdot \nabla) p_0 + \Gamma p_T \nabla \cdot V_1 = (\Gamma - 1) \left[ \eta J_{T\parallel}^2 - \Pi : \nabla V + \Pi_e : \nabla \frac{J}{ne} - \nabla \cdot q \right] \quad (3)$$

$$\rho \frac{d\tilde{V}_{\parallel}}{dt} + (V_1 \cdot \nabla) V_{\parallel 0} = -b_0 \cdot \nabla p_1 - b_1 \cdot \nabla p_T - b_0 \cdot \nabla \cdot \Pi \quad (4)$$

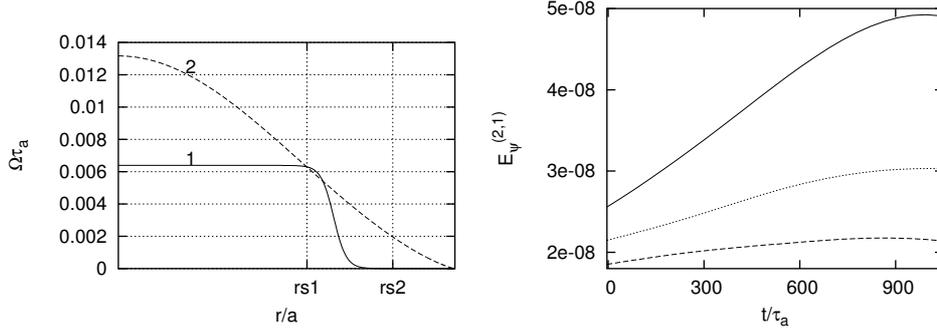


Figure 1: Equilibrium toroidal flow profiles (on the left) and nonlinear evolution of the (2, 1) NTM (on the right) for those profiles: no flow (solid curve), flow profile-1 (heavy dotted curve), flow profile-2 (dashed curve).

$$\begin{aligned} \nabla \cdot q = & -\chi_{\perp} \nabla^2 p_1 - (\chi_{\parallel} - \chi_{\perp}) [b_1 \cdot \nabla (b_0 \cdot \nabla p_0) + b_0 \cdot \nabla (b_0 \cdot \nabla p_1 + b_1 \cdot \nabla p_0) \\ & + b_0 \cdot \nabla (b_1 \cdot \nabla p_1) + b_1 \cdot \nabla (b_1 \cdot \nabla p_0) + b_1 \cdot \nabla (b_0 \cdot \nabla p_1)] \end{aligned} \quad (5)$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + V \cdot \nabla ; V = \Omega(\psi) R^2 \nabla \zeta + V_1 = \frac{B_0 \times \nabla \Phi_0}{B_0^2} + V_{0\parallel} b_0 + \frac{B_0 \times \nabla \Phi_1}{B_0^2} + \tilde{V}_{\parallel} b_T$$

and other notations are standard (for a more detailed discussion see [5, 10]). The equilibrium toroidal velocity which is conveniently expressed in terms of a function  $\Omega(\psi)$  is ordered such that  $V_0/V_A \sim \varepsilon \ll 1$  so that the flows are restricted to the sub-Alfvénic range.

As is well known, the neoclassical tearing mode is driven unstable by a perturbation of the bootstrap current. To study the evolution of neoclassical tearing modes it is therefore necessary to retain the stress tensor terms in the reduced MHD equations in order to provide the drive term. It is also important to keep the heat flow terms in the pressure evolution equation and ensure pressure equilibration on the flux surfaces. For the neoclassical viscous stress tensor we have used the following closure ansatz [5, 9],

$$\nabla \cdot \vec{\Pi}_j = \rho_j \mu_j \langle B^2 \rangle \frac{v_j \cdot \nabla \theta}{(B \cdot \nabla \theta)^2} \nabla \theta, \quad (6)$$

where  $j = i, e$  and  $\mu_j$  is the viscous damping frequency of each species  $j$ . Before investigating the effect of flows on NTMs we have benchmarked the NEAR code by reproducing these characteristic features of the NTMs and paid particular attention to pressure equilibration [5]. The equilibrium profile is generated by numerically solving the Grad-Shafranov equation with the help of an equilibrium code called TOQ [11]. The typical ratio of  $\chi_{\parallel}/\chi_{\perp}$  in most of our runs has been of the order of  $10^6$  or more and the Reynolds number  $S$  has been kept at  $10^5$  or higher. In Fig.1 we have shown toroidal flow in the system using equilibrium flow profiles 1 and 2 and the

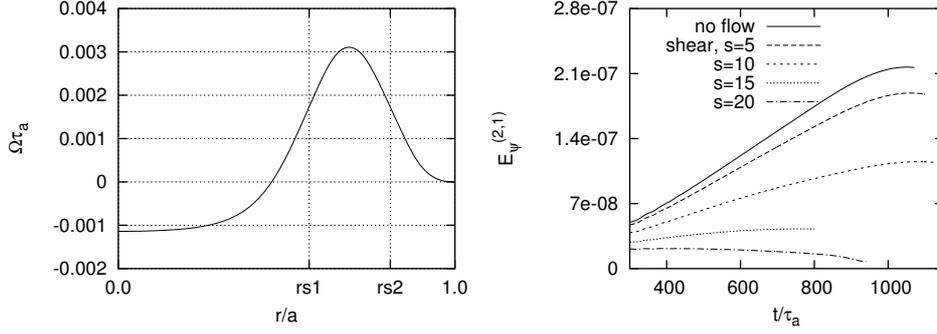


Figure 2: Equilibrium toroidal flow profile with positive flow shear at (2,1) surface with no differential flow and the nonlinear evolution of the (2, 1) NTMs with this profile for different flow shear

island width evolution for three different cases - the top curve is without any flow, the bottom curve is for flow profile 1 (pure differential flow) and the intermediate curve is for flow profile 2 (differential flow + shear). It is observed that differential flow has a stabilizing influence and leads to a lower level of mode saturation as shown by the lowest curve on the right hand panel of Fig.1. For profile 2 we find a decrease in the stabilization effect (the intermediate curve in the right panel of Fig.1) indicating that negative velocity shear has a destabilizing effect. To study the effect of the sign of the flow shear we have used a different flow profile as shown in the left panel of Fig.2 and we find that as we increase the positive flow shear, it stabilizes the NTMs.

Next, to gain some analytic understanding of the nature of the sheared flow contributions we have carried out a Rutherford type [12] calculation for the island evolution in the presence of flows [3]. We omit details of the calculation but point out that the flow effects are incorporated in the polarization drift term through the electrostatic potential  $\Phi$  [3] as well as in the convective derivative term of the vorticity equation. Our final island evolution equations take the form,

$$0.41 \frac{\partial W}{\partial t} = D_R^{neo} \left[ \frac{\Delta'_c}{4} + 0.58 \frac{\sqrt{\epsilon} \beta_\theta L_q}{W} \frac{W^2}{W^2 + W_\chi^2} - 6.35 \frac{D_I}{\sqrt{W^2 + 0.65 W_\chi^2}} \right. \\ \left. + \frac{L_s^2}{k_\theta^2 v_A^2} \left( 2.3 \frac{(\omega - \omega_E)(\omega - \omega_E - \omega_*)}{W^3} + 0.71 \frac{\omega_E'^2}{W} \right) - 0.77 \frac{L_s}{k_\theta v_A} \frac{\bar{v}_{\parallel 0}}{v_A} \frac{\omega_E'}{W} \right] \quad (7)$$

$$0.82 \frac{\partial}{\partial t} \left[ W(\omega - \omega_E) + \frac{\omega_E'}{2} W^2 \right] = -12.6 \frac{\mu_e}{W} (\omega - \omega_E) - \frac{1}{4\sqrt{2}} \left( \frac{nsV_A}{R^2 q} \right)^2 W^4 \Delta'_s \quad (8)$$

where,  $D_R^{neo} = c^2/4\pi\sigma_{neo}$  is the magnetic diffusion coefficient calculated using the neoclassical resistivity,  $\beta_\theta = 8\pi p_e/B_\theta^2$ ,  $L_p = -(d \ln p/dr)^{-1}$ ,  $L_q = (d \ln q/dr)^{-1}$ ,  $\bar{v}_{\parallel 0}$  is the average parallel flow velocity,  $\omega_E = k_\theta c \Phi'_0(r=r_s)/B_0$  is the drift frequency due to the equilibrium electric field and is the flow contribution,  $\omega_E' = k_\theta c \Phi''_0(r=r_s)/B_0$  is the flow shear contribution. In Eqn.(7)

the second term on the RHS is the perturbed bootstrap current term which drives the mode unstable when it is larger than the ( $\Delta'_c < 0$ ) term. The third term is the GGJ term [13] arising from the pressure curvature and it is stabilizing. The next term is the usual polarization current term modified by the flow contribution. The fifth term is the contribution from velocity shear and is seen to be of a destabilizing nature. The sixth term is the parallel flow term which depends on the sign of the flow shear. Eqn.(8) is the evolution equation for the mode frequency.

Our numerical results indicate that differential flow has a strong stabilizing influence on the nonlinear evolution of neoclassical tearing modes whereas negative velocity shear has a destabilizing effect and positive velocity shear has a stabilizing effect. While a quantitative comparison with the analytic model is not possible at this stage some qualitative features of the results can be understood. The destabilization effect of negative shear flows is consistent with the findings of earlier linear studies as well as the Rutherford model derived here. The stabilization effect of positive flow shear is also consistent with our model calculation. The stabilization due to differential flow can't be explained from the present single helicity calculation and is best understood in terms of the influence of differential flow on toroidal mode coupling and equilibrium modifications of the pressure profile caused by the centrifugal effects of flow. Some of these aspects are currently being investigated both numerically and through appropriate analytic modeling.

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