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# Alpha-particle Confinement and Conservation of Second Adiabatic Invariant

W.A. Cooper [1], M.Yu. Isaev [2], M.I. Mikhailov [2], J. Nührenberg [3], S. Okamura [4], V.D. Shafranov [2], A.A. Subbotin [2], C. Suzuki [4], K. Yamazaki [5], R. Zille [3]

[1] *CRPP, Association Euratom-Confédération Suisse, EPFL, Lausanne, Switzerland*
[2] *Russian Research Centre "Kurchatov Institute", Moscow, Russia*
[3] *Max-Planck-Institut für Plasmaphysik, IPP-EURATOM Association, Germany*
[4] *National Institute for Fusion Science, Oroshi-cho 322-6, Toki 509-5292, Japan*
[5] *Nagoya University, Furo-cho, Chikusa-ku, Nagoya 464-8603, Japan*

## Introduction

The improvement of $\alpha$-particle confinement is one of the main targets in the process of optimization of stellarator systems. As was demonstrated earlier, long-time fast-particle collisionless confinement can be significantly improved in quasi-isodynamic (qi) [1] stellarators with poloidally closed contours of the magnetic field strength $B$ on the magnetic surfaces and in quasiaxi- and quasihelically-symmetric (qa and qh) stellarators with toroidally closed contours of $B$ [2,3].

If qh (qa) conditions are fulfilled exactly, then the guiding center motion equations conserve the additional invariant $F_P + \rho_{||} I$ and the particles are confined if the orbits do not cross the plasma boundary (wall). Here $F_P$ and $I$ are external poloidal flux and current counted relative to the direction of quasi-symmetry, while $\rho_{||}$ is the guiding center radius. In these cases, the second adiabatic invariant, $\mathcal{J} = \oint V_{||} dl$ is constant on a magnetic surface. On the other hand, it is known that the conditions of quasi-symmetry cannot be satisfied exactly in the whole plasma volume [4].

In qi systems there is no additional integral of the drift equations of motion, while the second adiabatic invariant is constant on magnetic surfaces, too.

Investigations of fast particle confinement in tokamaks show that only a very small ripple (due to the finite number of toroidal field coils) is tolerable for good $\alpha$-particle confinement, although the $\mathcal{J}$-contours are well behaved even for significantly larger values of ripple components.

The behavior of $\mathcal{J}$-contours does not depend on the value of the magnetic field strength. On the other hand, the shape of the particle drift trajectories strongly depends on the gyro- to plasma-radius ratio. Thus, for configurations that are not exactly symmetric, it may occur that fast particles are not confined during the collisionless drift motion even in configurations with closed contours of the second adiabatic invariant.

In the present paper the correlation of $\alpha$-particle collisionless confinement with the behavior of contours of $\mathcal{J}$ is investigated for configurations with closed $\mathcal{J}$-contours. The problem of the validity of the second adiabatic invariant during the motion of fast particles in the plasma volume is discussed also.

## Results of numerical calculations for various systems

As already mentioned, for good long-time collisionless confinement of $\alpha$-particle it is necessary for the contours of the second adiabatic invariant to be closed inside the plasma volume. This is a necessary condition, but it may not be sufficient for the following reason. If the banana size is not sufficiently small in comparison with the plasma radius, then in symmetric configurations particles will be confined, but violations of symmetry can lead to "collisionless diffusion" [5]. Thus, it is interesting to analyse some configurations with closed $\mathcal{J}$ -contours in order to understand when violations of particle confinement occur in these systems. As examples, here the following configurations with closed $\mathcal{J}$ -contours are considered.

- A D-shaped tokamak with $R_0/a \approx 3$, $\iota(0) = 0.47$, $\iota(1) = 0.28$, $\beta = 0$. The toroidal ripple, $N = 18$, at the edge 0.003 is so small that the axisymmetry of $\mathcal{J}$ -contours is seemingly perfect and locally trapped orbits are absent.

- A QA tokamak [2], $N = 2$ configuration with prescribed profile of the rotational transform, $\iota(0) = 0.92$, $\iota(1) = 0.31$, $R_0/a \approx 4.4$, $< \beta >= 3.1\%$. The maximal amplitude of non-axisymmetric Fourier components of $B$ is $0.75$ (of $B_{-1,2}$).

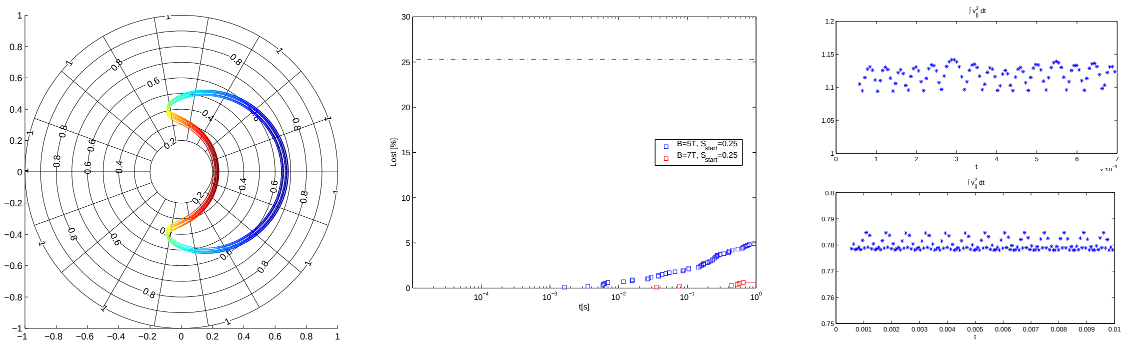
- A QI stellarator (see, e.g. [6]), $N = 6$ configuration without net toroidal current, with $\iota(0) = 0.86$, $\iota(1) = 0.98$, $< \beta >= 8\%$. The bumpy component here is about $25\%$.

- A QH stellarator [3], $N = 6$ configuration with zero net toroidal current, $< \beta >= 0$, $R_0/a \approx 12$, $\iota \approx 1.5$

Power-plant-sized configurations are considered with plasma volume $V = 1000m^3$ and magnetic field strength $5T$. Collisionless $3.5Mev$ $\alpha$ -particle orbits are investigated.

*Tokamak with a small bumpy component*

Fig. 1a shows the projection of a reflected-particle trajectory in polar representation $s, \theta$. This particle is started at $s = 0.25$, with $s$ the normalized toroidal flux. The banana size of this particle is large enough, so that "collisionless diffusion" can take place: after a long time ($\tau \approx 0.6sec$) this particle escapes from the plasma volume. Trapped particles with larger banana size escapes even earlier. The hystory of losses of 1000 $\alpha$ -particles started at $s = 0.25$ is shown in Fig. 1b. Increasing the magnetic field strength from $5T$ to $7T$ leads to significant improvement of the confinement. The two last graphs in Fig. 1c show the quality of conservation of the integral $\oint V_{||}^2 dt$ along the real particle trajectory. It is observed that the increase of $B$ from $5T$ to $7T$ diminishes the oscillations of this integral significantly and is in good correlation with the behavior of losses.

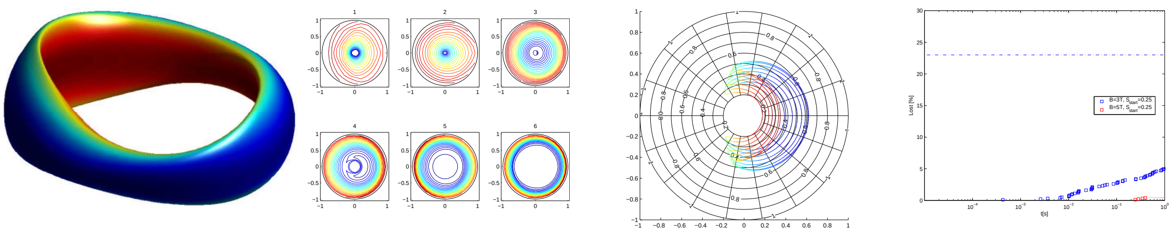



**a** **b** **c**

Fig. 1. Tokamak with a small bumpy component; **a** - projection of trapped $\alpha$ -particle trajectory for $B = 5T$; **b** - collisionless $\alpha$ -particle losses as a function of time of flight, for $B = 5T$ and $B = 7T$, the dashed line shows the fraction of reflected particles; **c** - time dependence of the integral $\oint V_{||}^2 dt$ for $B = 5T$ (top) and $B = 7T$ (bottom).

*Quasi-axisymmetric tokamak*

Similar computations were made for a two-period quasi-axisymmetric tokamak [2] which is a configuration with toroidally closed $B$-contours on magnetic surfaces, too. This is a configuration with net toroidal current and nonzero external rotational transform ($\iota_{extern}(1) \approx 0.14$). The 3D view of this configuration is shown in Fig. 2a. The color here defines the value of the magnetic field strength. Inspite of the 3D geometry of the configuration, there are no locally trapped particles here, and the $\mathcal{J}$-contours are very well closed inside the plasma volume. These contours are shown in Fig. 2b in $s, \zeta$ coordinates. The coincidence of these contours with magnetic surfaces is not so perfect as in the ripple tokamak considered above. An example of a particle trajectory is shown in Fig. 2c. The banana size of the particle is smaller than in the ripple tokamak, mainly because of the larger rotational transform. The radial motion of banana could be connected with deviation of $\mathcal{J}$ from the magnetic surface, rather than with "collisionless diffusion". This is confirmed by direct calculation: it is seen from Fig. 2d that for $B = 5T$ the losses are very small, and even for smaller value of the magnetic field, $B = 3T$ the losses are not very large.

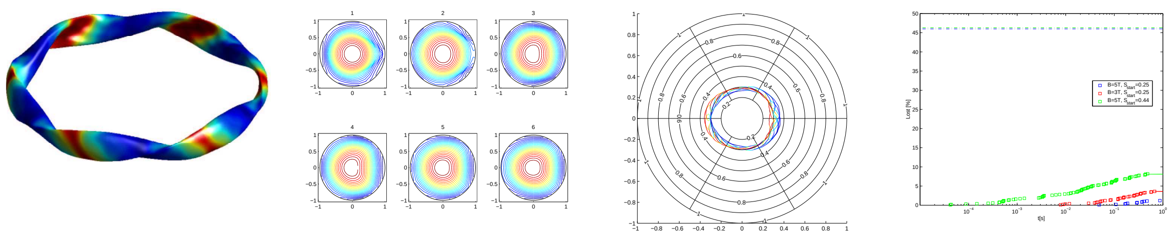


**a** **b** **c** **d**

Fig. 2. QA tokamak; **a**- 3D view of the boundary magnetic surface, the color defines the magnetic field strength; **b**- constant-$\mathcal{J}$ contours in polar representation $s, \zeta$ for a set of increasing $B_{reflect}$, 1 - corresponds to deeply trapped, 6 - to barely reflected orbits; **c**- projection of a reflected $\alpha$-particle trajectory; **d**- collisionless $\alpha$-particle losses as a function of time of flight, for $B = 5T$ and $B = 3T$.

*Quasi-isodynamic stellarator*

The third configuration considered is of a different type. It is a quasi-isodynamic configuration with poloidally closed contours of $B$ on magnetic surfaces.



**a** **b** **c** **d**

Fig. 3. QI stellarator; **a**- 3D view of the boundary magnetic surface; **b**- constant-$\mathcal{J}$ contours in polar representation $s, \theta$ for a set of increasing $B_{reflect}$; **c**- projection of a reflected $\alpha$-particle trajectory; **d**- collisionless $\alpha$-particle losses as a function of time of flight, $B = 3T$ and $B = 5T$, $s_{start} = 0.25$ (1/2 of minor plasma radius) and for $B = 5T$, $s_{start} = 0.44$ (2/3 of plasma minor radius).

The 3D view of this configuration is shown in Fig. 3a. It is seen that surfaces $B$=const cut the plasma column. The approximate quasi-isodynamicity leads to closure of the second adiabatic invariant contours which are now shown in $s, \theta$ coordinates (see Fig. 3b). As the reflected particles are trapped inside one period, the banana size is significantly smaller here than in the above configurations (see Fig. 3c). The results of direct calculations of particle drift motion are shown in Fig. 3d. It is seen that even for $B = 3T$ the particles started at 1/2 of small radius are confined well enough.

*Quasi-helically symmetric stellarator*

Analogous consideration shows that in quasi-helically symmetric configuration described in Ref. [3] with aspect ratio $A = 12$ and rotational transform about $1.5$, the banana size is smaller than in the above QA configuration and larger than in the QI system. The collisionless $\alpha$-particle confinement is good, even for $B = 3T$. The losses here are connected with residual local maxima of $B$ on magnetic surfaces, that lead to losses of barely reflected particles.

**Conclusions**

In all three types of advanced stellarators (qa, qh, qi), good long-time collisionless $\alpha$-particle confinement can be realized for power-plant-size parameters.

For the same value of volume and magnetic field strength, the smallest banana size occurs in qi configurations. In the qh configuration, the banana size is larger than in qi, but smaller than in qa systems. One of the reasons is that the distance between the reflection points is smallest in the qi configuration (one period). In qh configurations, this distance is larger but still small because of large rotational transform relative to the direction of quasi-symmetry. The largest banana size is in qa systems.

For small banana size, the correlation between $\mathcal{J}$ contour behavior and particle confinement is better, so that the optimization toward closure of $\mathcal{J}$ contours can be used for the improvement of particle confinement.

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