32nd EPS Conference on Plasma Phys. Tarragona, 27 June - 1 July 2005 ECA Vol.**29C**, P-2.130 (2005)

Dust-lattice waves: Role of charge variations and anisotropy of dust-dust interaction

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The system of charged dust particles in a plasma with ion flow (e.g., in a plasma sheath) represents a non-Hamiltonian system [1] because of the non-reciprocal interaction $[\phi(\mathbf{r}) \neq \phi(-\mathbf{r})]$ (wake field) and spatial charge variations: the system cannot be described by a Hamiltonian, and, hence, no energy is conserved. Thus, the system of dust particles can exhibit some properties unusual for Hamiltonian systems.

One of these properties was demonstrated in experiment [2], namely, the melting of the 2D dust crystal due to the coupling instability of vertical and horizontal (longitudinal) dust-lattice modes. While the physics of phase transitions in Hamiltonian systems is usually related to the competition between thermal and interaction energies (condition for a phase transition is some relation between the temperature and the interparticle distance), the physics of the melting observed in experiment [2] is quite different: the dust crystal melted when the neutral pressure was small enough so that the dust-neutral friction was insufficient to suppress the growth of oscillations of particles in a dust lattice.

In Ref. [2], the melting was explained by the theory of Ref. [3], namely, by non-hamiltonicity due to interaction anisotropy only. In Ref. [4], it was shown that the simultaneous presence of vertical charge variations (i.e., charge variations due to vertical displacements) can substantially increase the effect. In the present paper, we consider the simultaneous presence of the following three effects: interaction anisotropy, vertical charge variations and horizontal charge variations (i.e., charge variations due to perturbations of the distances between particles). Although in experiments the horizontal charge gradient is believed to be much smaller than the vertical one, we will show that it can be nevertheless significant: to compare the effects, one should compare the product of the horizontal charge gradient and the vertical electric field of the sheath with the product of the vertical charge gradient and the horizontal electric field acting on a particle from the neighboring particle. Moreover, the simultaneous presence of both charge gradients gives rise to a new effect: the coupling instability can now be triggered even in the absence of the interaction anisotropy.

We consider an infinite string of dust particles with equilibrium charge $(-Q) < 0$ and equilibrium separation L. The equilibrium positions of dust particles are on the horizontal X-axis. The Z-axis is directed vertically downward. We consider the motion of particles to be in the XZ-plane. The forces acting on the dust particles are the gravity force, the force of the electric field of the sheath, the dust-dust interaction, and the dust-neutral friction. In the XZ plane, the electric field of the sheath is directed vertically downward and depends only on the coordinate z: $E = E(z)$. As for the dust-dust interaction, we consider only interaction with neighboring dust particles and apply the following model: each dust particle induces the electrostatic potential given by

$$\phi_n(x,z) = (-Q_n) f(|x-x_n|, z-z_n) \tag{1}$$

where $(-Q_n) < 0$ is the momentary value of the charge of the n-th particle, x_n and z_n are the coordinates of the n-th particle, $f(|x-x_n|, z-z_n)$ is some function of the specified arguments. We use a power series expansion of this function near $|x-x_n| = L$, $z-z_n = 0$:

$$f(|x-x_n|, z-z_n) = f_0 + (|x-x_n|-L) f_x + (z-z_n) f_z$$
$$+\frac{1}{2}(|x-x_n|-L)^2 f_{xx} + \frac{1}{2}(z-z_n)^2 f_{zz} + (|x-x_n|-L)(z-z_n) f_{xz} + \ldots \tag{2}$$

Concerning the dust charge variations, we apply the following model: the linear perturbations of the dust charge are given by

$$\delta Q_n = (\delta x_{n+1} - \delta x_{n-1}) Q_x + (\delta z_n) Q_z \tag{3}$$

where the symbol δ is used to designate the deviations from the equilibrium values: $\delta Q_n = Q_n - Q$ is the perturbation of the absolute value of the n-th particle charge, $\delta x_n = x_n - nL$ is the perturbation of the x-coordinate of the n-th particle, $\delta z_n = z_n$.

To normalize distances, we use some arbitrary length λ. This length λ can be associated, for example, with the Debye radius or the length of the dust-dust interaction. The set of our dimensionless parameters is as follows:

$$e_0 = E(0)\frac{\lambda^2}{Q}; \qquad e_1 = \left.\frac{dE(z)}{dz}\right|_{z=0} \frac{\lambda^3}{Q}; \qquad \kappa = \frac{L}{\lambda}; \qquad q_x = \frac{Q_x \lambda}{Q}; \qquad q_z = \frac{Q_z \lambda}{Q};$$
$$\sigma_x = f_x \lambda^2; \qquad \sigma_z = f_z \lambda^2; \qquad \sigma_{xx} = f_{xx}\lambda^3; \qquad \sigma_{zz} = f_{zz}\lambda^3; \qquad \sigma_{xz} = f_{xz}\lambda^3 \tag{4}$$

Assuming the perturbations are proportional to $\exp(ikn\kappa - i\omega t)$ where the time t is normalized by $\lambda^{3/2}\sqrt{M}/Q$ (M is the dust mass), we obtain the dispersion relation

$$[\omega^2 + i\gamma\omega - \Omega_h^2(k)][\omega^2 + i\gamma\omega - \Omega_v^2(k)] = U_c(k) \tag{5}$$

where $\gamma > 0$ describes the dust-neutral friction (γ is the dimensionless damping rate in the absence of any forces except the dust-neutral friction, in which case $\omega_{1,2} = 0$, $\omega_{3,4} = -i\gamma$), and

$$\Omega_h^2(k) = 4\left[\sigma_{xx}\sin^2(k\kappa/2) + \sigma_x q_x \sin^2(k\kappa)\right] \tag{6}$$

$$\Omega_v^2(k) = e_1 + e_0 q_z + 4\sigma_{zz}\sin^2(k\kappa/2) + 4\sigma_z q_z \cos^2(k\kappa/2) \tag{7}$$

$$U_c(k) = 4\sin^2(k\kappa)\left[-\sigma_{xz}^2 + \sigma_{xz}\sigma_x q_z - \sigma_{xz} q_x e_0 + \sigma_x q_x q_z e_0 \right.$$
$$\left. -4\sigma_{xz}\sigma_z q_x \cos^2(k\kappa/2) + 4\sigma_x\sigma_z q_x q_z \cos^2(k\kappa/2)\right] \tag{8}$$

We have two modes (characterized by $\Omega_h(k)$ and $\Omega_v(k)$) damped by the dust-neutral friction and coupled with each other through the coupling coefficient $U_c(k)$. The frequencies $\Omega_h(k)$ and $\Omega_v(k)$ are respectively the horizontal and vertical frequencies in the following sense. If we assume the particles can move only along the X-axis (i.e., we use $\delta z_n \equiv 0$ instead of considering forces in the vertical direction) and $\gamma = 0$, $\Omega_h(k)$ will be the frequency of these horizontal oscillations. Analogously, if we assume the particles can move only vertically (i.e., we use $\delta x_n \equiv 0$) and $\gamma = 0$, the frequency of these vertical oscillations will be $\Omega_v(k)$. The first four terms in the coupling coefficient $U_c(k)$ (8) are related respectively to:

1. only interaction anisotropy (considered in Ref. [3]),
2. interaction anisotropy and vertical charge variations (considered in Ref. [4]),
3. interaction anisotropy and horizontal charge variations,
4. vertical and horizontal charge variations.

In the laboratory experiments, all these four terms can be comparable with each other, while two remaining terms are negligible if $e_0 \gg |\sigma_z|$ (i.e., if the force of the electric field of the sheath is much greater than the vertical component of the dust-dust interaction in the equilibrium state).

It is easy to perform the stability analysis of the dispersion relation (5) with any arbitrary real functions $\Omega_h^2(k)$, $\Omega_v^2(k)$, $U_c(k)$, using the inequality $\gamma > 0$ only (e.g., $\Omega_h^2(k)$, $\Omega_v^2(k)$ can be negative, and $U_c(k)$ can be large). The instability conditions for a given k are as follows:

- when $[\Omega_v^2(k) - \Omega_h^2(k)]^2 + 4U_c(k) < 0$, we have oscillatory instability if

  $$|[\Omega_v^2(k) - \Omega_h^2(k)]^2 + 4U_c(k)| > 2\gamma^2[\Omega_v^2(k) + \Omega_h^2(k)] \tag{9}$$

  otherwise the system is stable;

- when $[\Omega_v^2(k) - \Omega_h^2(k)]^2 + 4U_c(k) > 0$, we have nonoscillatory instability if any of the following is satisfied

$$\Omega_h^2(k) + \Omega_v^2(k) < 0 \tag{10}$$

$$\Omega_h^2(k)\Omega_v^2(k) < U_c(k) \tag{11}$$

otherwise the system is stable.

For a system with weak coupling, the oscillatory instability can occur only in the case of *crossing* of the horizontal $\Omega_h(k)$ and vertical $\Omega_v(k)$ frequencies at some k. The instability condition is

$$U_c(k_{cross}) < 0; \qquad |U_c(k_{cross})| > \gamma^2 \Omega_{cross}^2 \tag{12}$$

where $(k_{cross}, \Omega_{cross})$ is the crossing point. In this case, the oscillatory instability occurs in a small interval of wave numbers near the crossing point.

We give a numerical example with parameters of experiment [2] (for more details, see our paper [5]). We assume that (i) the dust-dust interaction is the sum of the screened Coulomb potential and the non-screened dipole field (dipole is directed downward), in which case $\sigma_x = -(\kappa + 1)\exp(-\kappa)/\kappa^2$, $\sigma_z = -p/\kappa^3$, $\sigma_{xx} = (\kappa^2 + 2\kappa + 2)\exp(-\kappa)/\kappa^3$, $\sigma_{zz} = -(\kappa+1)\exp(-\kappa)/\kappa^3$, $\sigma_{xz} = 3p/\kappa^4$ (the length λ used to normalize distances in (4) is assumed to be the screening length; $p > 0$ is the dipole moment in units of $Q\lambda$), and (ii) the charge gradients and the interaction anisotropy (i.e., the parameter p) are small. We use $Q = 15500e$, $\lambda = 0.5$ mm, $M = 5.5 \times 10^{-10}$ g, vertical frequency of a single particle $\omega_v/2\pi = 15.5$ Hz, argon pressure 2.8 Pa. This gives $e_1 \approx 12$, $e_0 \approx 24$, $\gamma \approx 0.1$. Thus, the crossing of modes occurs for $\kappa < 0.96$, while for $\kappa < 0.66$ the vertical mode becomes unstable due to mutual repulsion of particles. Therefore, the melting can occur in the range $0.66 < \kappa < 0.96$. In the case of no charge variations, the melting becomes possible at some κ from this range if p is greater than about 10^{-2}. For this p, all the first four terms in the coupling coefficient (8) are of the same order at $q_z \sim 10^{-1}$ and $q_x \sim 10^{-2}$. Thus, each of these four terms can be sufficient to trigger the oscillatory instability observed in experiment [2].

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