Is the plasma at a plane probe unstable in the presence of fast electrons producing secondary electrons?

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Introduction

The stability of an unmagnetized plasma wall transition (PWT) in the presence of secondary electron emission (SEE) produced by the thermal plasma electrons was checked by Franklin et al. [1] for a collisionless plasma, Tskhakaya et al. [2,3] and Eibl [4] for a collisional plasma. Here, we analyze the stability of the presheath when the secondary electrons are produced at a plane probe surface by fast electrons in a collisional unmagnetized plasma. A dispersion relation is derived to analyze the three-stream instability produced by the interaction of the fast incoming electrons with the fast reflected electrons and the secondary electrons, embedded in the thermal-electron background.

Theory – *Presheath Instability Analysis*

We consider a time-dependent presheath characterized by the system of equations:

$$\frac{\partial f_e^{th}}{\partial t}(x,v) + v\frac{\partial f_e^{th}}{\partial x}(x,v) - \frac{eE}{m_e}\frac{\partial f_e^{th}}{\partial v}(x,v) = -\nu_e(f_e^{th} - \bar{f}_{e0}^{th})$$

$$\frac{\partial f_e^{f}}{\partial t}(x,v) + v\frac{\partial f_e^{f}}{\partial x}(x,v) - \frac{eE}{m_e}\frac{\partial f_e^{f}}{\partial v}(x,v) = 0 \qquad (1)$$

$$\frac{\partial f_e^{se}}{\partial t}(x,v) + v\frac{\partial f_e^{se}}{\partial x}(x,v) - \frac{eE}{m_e}\frac{\partial f_e^{se}}{\partial v}(x,v) = 0$$

Here $\bar{f}_{e0}^{th}$ represents the equilibrium distribution function for the thermal electrons and is given by a cut-off Maxwellian distribution function. To the system of equations (1) we will add Poisson's equation.

To linearize this system, we denote by $\bar{a}(x,v)$ the equilibrium value and by $\tilde{a}(x,t) = \hat{a}\exp[i(\omega t - kx)]$ the arbitrarily small perturbation in space and time, of any variable $a(x,t)$ which is characterizing our system (electron density n, average electron velocity u, electron distribution function f, electric field E). Here ω is the frequency, κ is

the wave number and $\hat{a}(x, v)$ is the amplitude of the linear eigenmode considered. With these notations, we obtain the dispersion relation for the presheath of our plasma containing thermal electrons, fast electrons and secondary electrons produced by them:

$$-\frac{e^2}{m_e k\,\varepsilon_0}\left[\int dv\,\frac{\partial \bar{f}_e^{th}/\partial \overline{v}}{\omega - \kappa v - iv_e} + \int d\overline{v}\,\frac{\partial \bar{f}_e^f/\partial v}{\omega - \kappa v} + \int dv\,\frac{\partial \bar{f}_e^{se}/\partial v}{\omega - \kappa v}\right] = 1 \qquad (2)$$

We will analyze here the case $\kappa v \ll v_e \ll \omega$. With this approximation we expand in Taylor series up to the first order the terms $(\omega - \kappa v - iv_e)^{-1}$ and $(\omega - \kappa v)^{-1}$. Here and below we consider a region near to the presheath entrance (CPE) and assume potential to be zero. In Eq. (2) $\bar{f}_e^{th}$ equals $\bar{f}_{e0}^{th}$ and is given by

$$\bar{f}_e^{th}(x, v) = A^{th}\exp\left[-\left(\frac{v}{v_{et}^{th}\sqrt{2}}\right)^2\right]H(v_c - v)\,, \qquad (3)$$

Where T_{pl}^{th} is the electron temperature in the bulk plasma and $v_{et}^{th} = \sqrt{k_B T_{pl}^{th}/m_e}$ and $\bar{n}_{e0}^{th}$ are the thermal velocity and the equilibrium thermal-electron density at CPE, respectively; A^{th} is a constant given by

$$A^{th} = \sqrt{2}\bar{n}_{e0}^{th}\cdot\left[v_{et}^{th}\sqrt{\pi}(1 + \text{erf}\sqrt{-eV_p/k_B T_{pl}^{th}})\right]^{-1}. \qquad (4)$$

We denote by $V_p\,(<0)$ the potential values at the probe surface. Hence the cut-off velocity at the CPE can be written as $v_c = \sqrt{-2eV_p/m_e}$. $\bar{f}^f$ is the equilibrium fast-electron distribution function at the CPE having, the form

$$\bar{f}^f(x, v) = A^f\exp\left\{-\left[\frac{v + v_{sh}}{\sqrt{2}v_{et}^f}\right]^2\right\}H(-v) + A^f\exp\left\{-\left[\frac{v - v_{sh}}{\sqrt{2}v_{et}^f}\right]^2\right\}H(v)H(v_c - v),\ (5)$$

where v_{sh} and $v_{et}^f = \sqrt{k_B T_{pl}^f/m_e}$ are the shift and thermal velocities of the fast electrons, respectively, A^f is a constant given by

$$A^f = \sqrt{2}\bar{n}_{e0}^f\left\{\sqrt{\pi}v_{et}^f\left[1 + 2\text{erf}\sqrt{E_0/k_B T_{pl}^f} + \text{erf}\left(\sqrt{-eV_p/k_B T_{pl}^f} - \sqrt{E_0/k_B T_{pl}^f}\right)\right]\right\}^{-1}, \qquad (6)$$

Where E_0 and $\bar{n}_{e0}^f$ are the initial energy and the density of the fast electrons at the CPE, respectively. $\bar{f}^{se}$ is the equilibrium secondary electron distribution functions at the CPE

$$\bar{f}^{se}(x, v) = A^{se}\exp\left\{-\frac{v^2}{2\left(v_{et}^{se}\right)^2}\right\}H(v - v_c)\,, \qquad (7)$$

Where $v_{et}^{se} = \sqrt{k_B T_{pl}^{se} / m_e}$ and A^{se} given by

$$A^{se} = \sqrt{2}\bar{n}_{e0}^{se} \left\{ \sqrt{\pi} v_{et}^{se} \left[1 + \mathrm{erf}\left(v_c / \sqrt{2} v_{et}^{se} \right) \right] \right\}^{-1}. \tag{8}$$

After substitution of the Eqs. (3, 5) and (7), the dispersion relation (2) becomes:

$$
\varepsilon(\omega,\kappa) \approx 1 + \frac{e^2}{m_e \varepsilon_0 \kappa} \cdot \left(\frac{I_1}{\omega} + \frac{\kappa I_2}{\omega^2} + \frac{\kappa^2 I_3}{\omega^3} \right) + i\,\frac{e^2}{m_e \varepsilon_0 \kappa} \left[\frac{\nu_e}{\omega^2} \left(I_{th}^1 + \frac{2\kappa I_{th}^2}{\omega} \right) \right.
$$
$$
\left. + \frac{3\kappa^2 I_{th}^3}{\omega^2} \right) - \frac{\pi}{\kappa} \left[\left(\frac{\partial \bar{f}^{th}}{\partial v} \right)\Big|_{v=\omega/\kappa} + \left(\frac{\partial \bar{f}^f}{\partial v} \right)\Big|_{v=\omega/\kappa} + \left(\frac{\partial \bar{f}^{se}}{\partial v} \right)\Big|_{v=\omega/\kappa} \right] = 0 \tag{9}
$$

Where

$$
I_1 = \bar{f}_e^{th}(v_c) + \bar{f}_e^f(v_c) - \bar{f}_e^f(v_c), \quad I_2 = v_c I_1 - n, \quad I_3 = v_c^2 I_1 - 2nu;
$$
$$
I_{th}^1 = \bar{f}_e^{th}(v_c), \quad I_{th}^2 = v_c \bar{f}_e^{th}(v_c) - n, \quad I_{th}^3 = v_c^2 \bar{f}_e^{th}(v_c) - nu; \tag{10}
$$
$$
n = \int f\,dv, \quad nv = \int vf\,dv;
$$

From Eq. (9) it readily follows that the real part of the frequency ω_r is the solution of the third order equation $\mathrm{Re}\,\varepsilon(\omega_r,\kappa) = 0$, using the assumption $\mathrm{Re}\,\omega \gg \mathrm{Im}\,\omega = \delta\omega$. For the growth rate we obtain:

$$
\delta\omega = -\mathrm{Im}\,\varepsilon(\omega_r,\kappa) \left[\frac{\partial\,\mathrm{Re}\,\varepsilon(\omega_r,\kappa)}{\partial\omega_r} \right]^{-1} = -\frac{\pi\omega^2}{\kappa} \left[\left(\frac{\partial \bar{f}^{th}}{\partial v} \right)\Big|_{v=\frac{\omega_r}{\kappa}} + \left(\frac{\partial \bar{f}^f}{\partial v} \right)\Big|_{v=\frac{\omega_r}{\kappa}} + \left(\frac{\partial \bar{f}^{se}}{\partial v} \right)\Big|_{v=\frac{\omega_r}{\kappa}} \right.
$$
$$
\left. -\frac{\nu_e \kappa}{\pi\omega_r^2} \left(I_{th}^1 + \frac{2\kappa I_{th}^2}{\omega_r} + \frac{3\kappa^2 I_{th}^3}{\omega_r^2} \right) \right] \cdot \left[I_1 + \frac{2\kappa I_2}{\omega_r} + \frac{3\kappa^2 I_3}{\omega_r} \right]^{-1} \tag{11}
$$

and the group velocity corresponding to Eq. (11) is

$$
v_g = \frac{\partial\omega}{\partial\kappa} = \left[\frac{\partial\,\mathrm{Re}\,\varepsilon(\omega_r,\kappa)}{\partial\kappa} \right] \cdot \left[\frac{\partial\,\mathrm{Re}\,\varepsilon(\omega_r,\kappa)}{\partial\omega_r} \right]^{-1}
$$
$$
= \frac{\omega_r}{\kappa} \cdot \left[I_1 + \frac{\kappa^2 I_3}{\omega_r^2} \right] \cdot \left[1 + \frac{2\kappa I_2}{\omega_r} + \frac{3\kappa^2 I_3}{\omega_r} \right]^{-1}. \tag{12}
$$

The instability condition can be formulated by two criterions: $\delta\omega > 0$, $v_g / \delta\omega \ll \delta x$, corresponding the instability conditions for a nonuniform media [5]. Here, δx is the typical gradient scale length.

As an example, we consider the following case $n_f / n_i = 4 \cdot 10^{-4}$, $m_e v_{sh}^2 / 2T_f = 50$ and $\delta x \approx 0.01\,\mathrm{m}$. Results are plotted in the figures below. One can see, that there exists a region in $\kappa\lambda_D$, space where the conditions for instability are fulfilled.
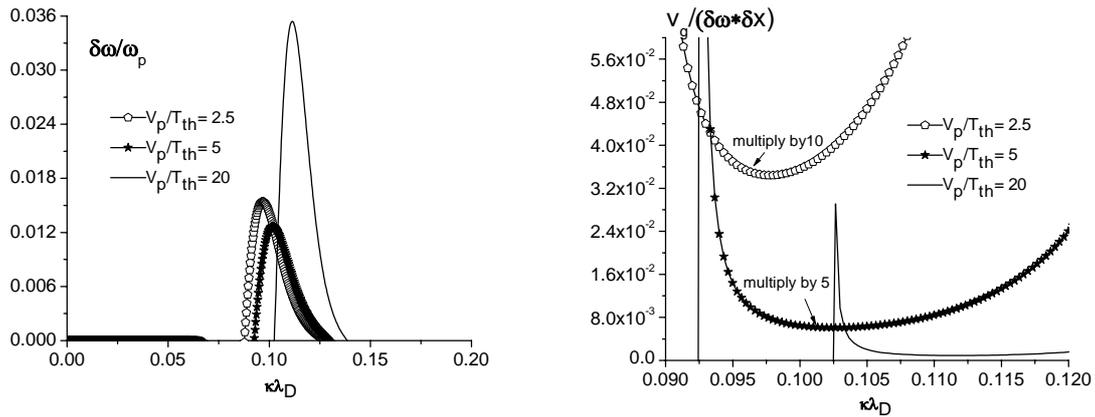
Conclusions We derived analytically the dispersion relation for the PWT problem, for plasma containing three particle streams: the incoming fast electron, the reflected fast electron and the secondary electron produced by the primary fast electrons at a plane probe surface. We show that for realistic parameters this kind of plasma sheath is *unstable*.

References

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