Effect of toroidal flow and flow shear on the quasi-interchange instability in tokamaks with weak magnetic shear

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Introduction Tokamak experiments in the "hybrid" scenario [1], as well as some experiments in spherical tokamaks [2], have q-values very close to unity in a wide area in the plasma core. Static, toroidal equilibria of this kind are susceptible to a pressure-driven, internal kink instability with an eigenfunction of "quasi-interchange" character [3]. In many experiments (especially in spherical tokamaks) the toroidal plasma rotation driven by the neutral beams is, however, quite substantial [1-2]. It is therefore of interest to examine the consequences of such plasma flows for the quasi-interchange (QI) instability. The present work investigates the effect of toroidal flow and, especially, toroidal flow shear on the QI mode using a compressible, ideal MHD description of the plasma. The work extends the analysis in Ref. [4], where weakly subsonic, rigid rotation was found to be strongly stabilizing.

Equilibrium We consider a toroidal plasma with circular cross section and large aspect ratio, $\varepsilon = r/R_0 << 1$, where r is the minor radius in the plasma and R_0 the major radius of the plasma center. Furthermore, we assume that the plasma rotates toroidally with a frequency of order $\Omega \sim \omega_s \sim \varepsilon\omega_A$, where $\omega_s = (\Gamma p_0/\rho_0)^{1/2}/R_0$ is the sound frequency and $\omega_A = B_0/R_0(\mu_0\rho_0)^{1/2}$ the Alfvén frequency (p_0 and ρ_0 denote the plasma pressure and density, respectively, B_0 the toroidal magnetic field and Γ = 5/3 the adiabatic index). Assuming that the equilibrium magnetic surfaces are isothermal, the Shafranov shift Δ is described by the equation [4] $d\Delta/dr = -\, r(\beta_p + l_i/2)/R_0$, with the plasma inductance l_i and the poloidal beta value β_p given by

$$l_i(r) = \frac{2q^2}{r^4}\int_0^r \frac{r'^3}{q^2}dr', \quad \beta_p(r) = -\frac{2\mu_0 q^2 R_0^2}{r^4 B_0^2}\int_0^r r'^2 \frac{d}{dr'}\left(p_0 + M_s^2 p_0\right)dr', \tag{1a, b}$$

respectively. $M_s = \left(\rho_0\Omega^2 R_0^2/2p_0\right)^{1/2}$ in Eq. (1b) is the sonic Mach number. Thus, the rotation contributes to the Shafranov shift by increasing the "effective" poloidal beta value. When $M_s \sim 1$, the effect of the dynamical pressure $M_s^2 p_0 = \rho_0\Omega^2 R_0^2/2$ is of the same order of magnitude as the effect of the static pressure. Here we specify the equilibrium profiles as follows: We let $q = 1$ in the region $0 \le r \le r_1$ of weak magnetic shear, whereas in the edge region $r_1 \le r \le a$ we assume that $q(r)$ increases quadratically with r from unity at $r = r_1$ up to $q_a = 4$ at the plasma edge $r = a$ [4]. Furthermore, $\rho_0(r)$ is assumed constant, and $p_0(r)$ and $\Omega(r)$ are chosen as

$$p_0(r) = p_0(0)\left(1 - r^2/a^2\right), \;\; \Omega(r) = \Omega_0\left(1 - r^2/r_\Omega^2\right)^{1/2}. \tag{2a, b}$$

This leads to a constant value of β_p in the region $0 \leq r \leq r_1$. More specifically, we obtain the effective (total) beta value as $\beta_p = \beta_{p0} + \beta_{p\Omega}$, where the static and dynamical parts of β_p are given by $\beta_{p0} = \mu_0 p_0(0) R_0^2 / a^2 B_0^2$ and $\beta_{p\Omega} = \Omega_0^2 R_0^2 / 2\omega_A^2 r_\Omega^2$, respectively. Examples of the q- and Ω-profiles used here are shown in Fig. 1. The rotation shear is described by the parameter r_Ω, which is allowed to vary in the range $r_1 \leq r_\Omega \leq \infty$. We point out that the rotation profile is of importance only in the region of weak magnetic shear, so the dashed parts of the Ω-curves in Fig. 1 are redundant for the QI stability problem looked at here. With other forms of the profiles of $\rho_0(r)$, $p_0(r)$ and $\Omega(r)$ we expect quantitative, but not qualitative, differences from the results of the stability analysis below.

Stability analysis Since the pressure- and rotation-profiles in Eqs. (2a, b) lead to a constant $\beta_p = \beta_{p0} + \beta_{p\Omega}$ in the region of weak magnetic shear, the stability equation for the QI mode derived for a rigidly rotating plasma in Ref. [4] is valid also here. Thus, neglecting the second term in the inertia operator in Eq. (9b) in Ref. [4], the stability problem of the QI mode for the profiles in Eqs. (2a, b) can be formulated as [4]

$$\int_0^{\hat{r}_1} \frac{\hat{r}^5 d\hat{r}}{Q(\hat{r},\omega)} = \frac{\hat{r}_1^4}{\beta_p^2}\frac{1-C}{3+C}, \text{ where } Q = -\hat{\omega}_D^2 + \hat{\Omega}^2 M_s^2 - \frac{\left(4\hat{\omega}_D^2 + \hat{\Omega}^2/2\right)\hat{\Omega}^2 + 2\hat{\omega}_D^2\hat{\omega}_s^2}{\hat{\omega}_s^2 - \hat{\omega}_D^2}. \tag{3a, b}$$

Here, C is a numerical constant related to the solution of the $m = 2$ side-band equation in the edge region [4], $\omega_D(r) \equiv \omega + \Omega(r)$ is the Doppler-shifted mode frequency, and the normalized quantities in (3) are defined as $\hat{r} = r/a$, $\hat{\omega}_D = \omega_D/\varepsilon_a\omega_A$, $\hat{\omega}_s = \omega_s/\varepsilon_a\omega_A$, $\hat{\Omega} = \Omega/\varepsilon_a\omega_A$, where $\varepsilon_a = a/R_0$. In addition to $\omega_D(r)$ and $\Omega(r)$, r-dependence of $Q(\hat{r},\omega)$ comes from $\hat{\omega}_s^2 = \Gamma\beta_{p0}\left(1-\hat{r}^2\right)$ and $M_s^2 = \hat{\Omega}_0^2\left(1-\hat{r}^2/\hat{r}_\Omega^2\right)/2\beta_{p0}\left(1-\hat{r}^2\right)$.

In the present work, the eigenvalue equation (3) has been solved numerically, and the main features of the eigenfrequency ω are illustrated in Figs. 2-6. Two main regimes (and mechanisms) of rotational stabilization of the QI instability can be identified. Firstly, if the rotation is rigid or weakly sheared ($r_1/r_\Omega \sim 2/3$ or smaller, see Fig. 1), the QI mode is stabilized by the Brunt-Väisälä (BV) mechanism explained in Ref. [4]. This is shown in Fig. 2 which illustrates the complex eigenvalue $\omega = \omega_r + i\omega_i$ (in normalized form) as a function of the (normalized) rotation frequency at the axis for the plasma parameters $r_1 = 0.5a$, $\beta_{p0} = 0.3$ and $r_\Omega = a$. Notice the splitting of ω_r (represented by the Doppler-shifted mode frequency at the magnetic axis) at the point of BV-stabilization. Secondly, if the rotation is strongly sheared ($r_1/r_\Omega \sim 1$), a substantial reduction of the growth rate takes place already for central rotation frequencies smaller than the rigid rotation frequency required for BV-stabilization. The dependence of ω_r and ω_i on the central rotation frequency in such a case is illustrated in Fig. 3 for a plasma with $r_1 = 0.2a$, $\beta_{p0} = 0.3$ and $r_\Omega = 0.225a$. Notice that there is no splitting of ω_r (dashed, blue curve) in this case (no BV effect) and that there is a limited range of Ω_0 that gives a reduced growth rate ω_i (solid, red curve). Fig. 4 illustrates the growth rate and the loss of BV-stabilization (solid, red curves) in a plasma with the same parameters as in Fig. 2 as the shear increases from $r_\Omega/a = 1000$ ($\approx$ rigid rotation) up to $r_\Omega/a = 0.6$. As the flow shear becomes of order $r_1/r_\Omega \sim 1$, however, the growth rate is very strongly reduced, as shown by the dotted, green curve in the figure, calculated for $r_\Omega/a = 0.51$. This shear stabilization is, however, lost already for $r_\Omega/a > 0.52$-0.53. Fig. 5 illustrates the same effect in the case $r_1 = 0.2a$. Finally, the β_{p0}-dependence of the critical rotation frequency for BV-stabilization of the QI mode in the case of weak, or moderate, flow shear for both $r_1/a = 0.2$ and $r_1/a = 0.5$ is shown in Fig. 6. Notice that $\Omega_{0,crit}$ is normalized with the width r_1 of the region of low magnetic shear, leading to approximately similar stability boundaries for the two cases $r_1/a = 0.2$ (red) and $r_1/a = 0.5$ (blue). This is due to the scaling $\Omega_{0,crit} \sim r_1$ found in the case of rigid rotation in Ref. [4], which is seen to be (approximately) valid also if the rotation is weakly sheared. Furthermore, the β_{p0}-scaling $\Omega_{0,crit} \sim \beta_{p0}^{3/4}$ of the critical rotation frequency, strictly valid in the case of rigid rotation [4], is fairly accurate also for weakly sheared rotation.

Conclusions It has been shown that the Brunt-Väisälä mechanism, discussed in detail in Ref. [4], is capable of stabilizing the quasi-interchange instability in tokamaks with $q \approx 1$ in the plasma core also if the toroidal rotation is weakly sheared. Since the flow shear leads to an enhanced "effective" poloidal beta value β_p (static plus dynamic), however, weakly sheared rotation requires somewhat higher central rotation frequencies for stabilization as compared with rigid rotation. At sufficiently strong flow shear, Brunt-Väisälä stabilization is in general not possible for reasonable central rotation frequencies. On the other hand, if the rotation is very strongly sheared ($r_1/r_\Omega \sim 1$), the shear itself is capable of reducing the growth rate of the instability to very small values.

[1] Litaudon X *et al* 2004 *Plasma Phys. Control. Fusion* 46 A19

[2] Menard J E *et al* 2003 *Nucl. Fusion* 43 330, and *Proc. 30th EPS Conf. on Contr. Fusion and Plasma Phys.* (*St Petersburg, 2003*) vol 27A, P-3.101

[3] Wesson J A 1986 *Plasma Phys. Control. Fusion* 28 243; Hastie R J and Hender T C 1988 *Nucl. Fusion* 28 585; Waelbroeck F L and Hazeltine R D 1988 *Phys. Fluids* 31 1217

[4] Wahlberg C 2005 *Plasma Phys. Control. Fusion* 47 757

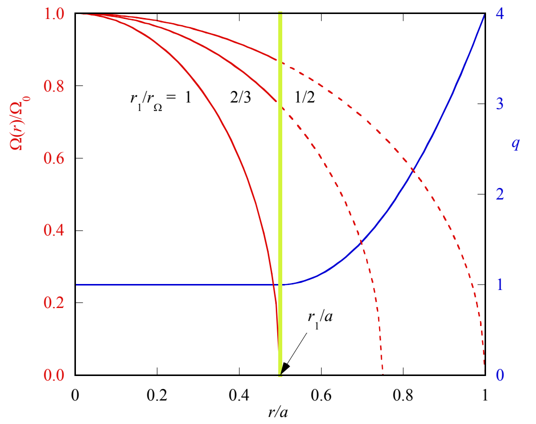

Fig.1 Examples of the rotation (Ω) profile and the q-profile used in the numerical solution of the eigenvalue equation (3).

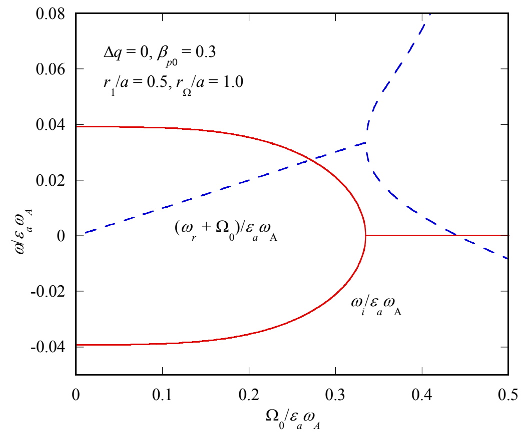

Fig. 2 The real (dashed) and imaginary(solid) parts of the eigenvalue ω as functions of Ω_0. The figure illustrates the stabilization of the mode by the Brunt-Väisälä mechanism.

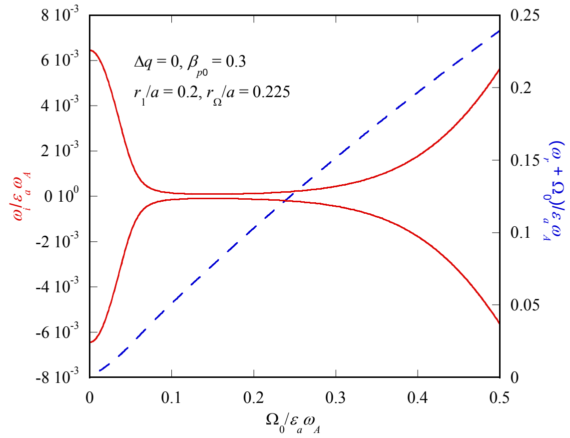

Fig. 3 The real (dashed) and imaginary(solid) parts of the eigenvalue ω as functions of Ω_0. The figure illustrates the reduction of the growth rate by very strong flow shear.

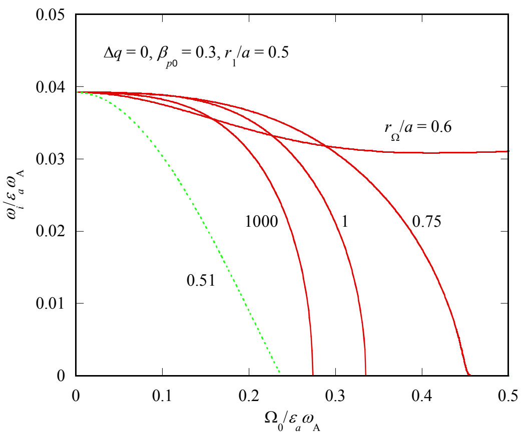

Fig. 4 Growth rate as a function of Ω_0 for various values of the flow-shear parameter r_Ω/a. The solid (red) curves with $r_\Omega/a \geq 0.75$ correspond to Brunt-Väisälä stabilization and the dotted (green) curve to shear stabilization

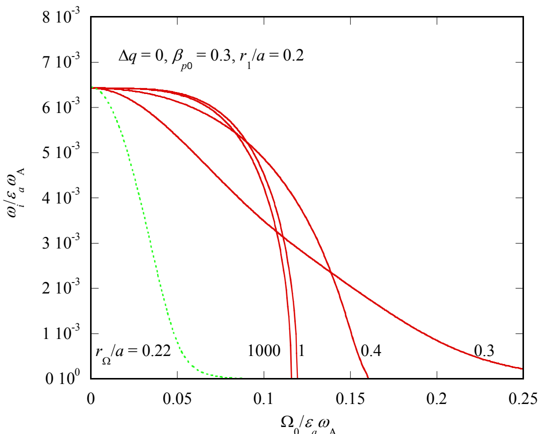

Fig. 5 Growth rate as a function of Ω_0 for various values of the flow-shear parameter r_Ω/a. The solid (red) curves with $r_\Omega/a \geq 0.4$ correspond to Brunt-Väisälä stabilization and the dotted (green) curve to shear stabilization

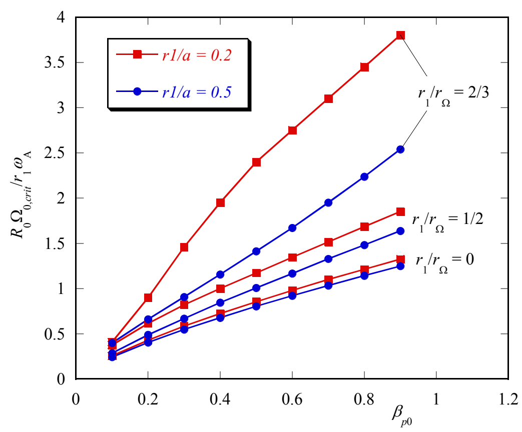

Fig. 6 Critical rotation frequency as a function of β_{p0} for a few values of the flow-shear parameter r_Ω/a and for r_1/a = 0.2 (red) and r_1/a = 0.5 (blue).